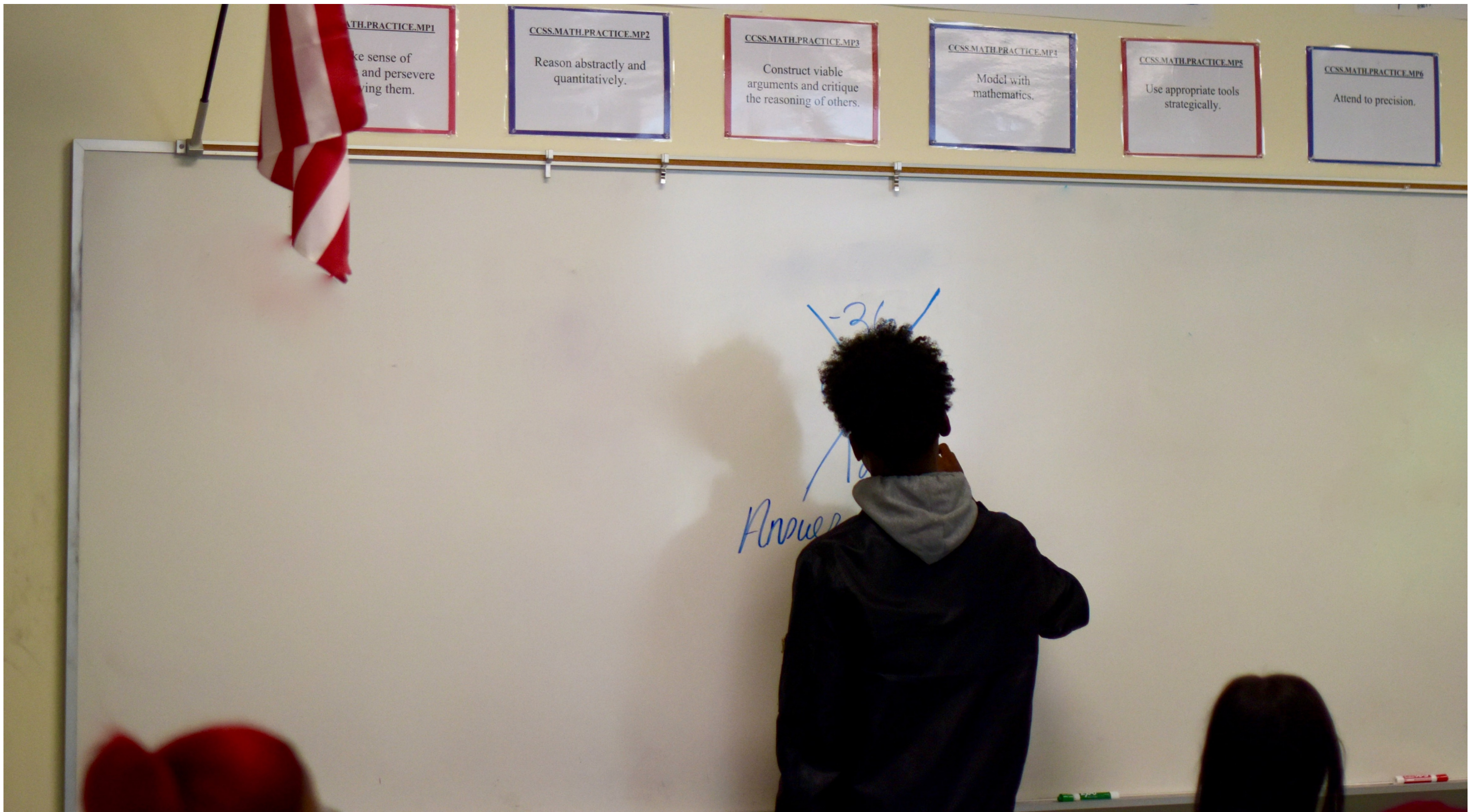

Ratios: A Guide to Grade 7 Mathematics Standards

UnboundEd

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7.RP.A | Analyze proportional relationships and use them to solve real-world and mathematical problems.

Welcome to the UnboundEd Mathematics Guide series!These guides are designed to explain what new, high standards for mathematics say about what students should learn in each grade, and what they mean for curriculum and instruction. This guide, the first for Grade 7, includes three parts. The first part gives a "tour" of the standards for Ratios & Proportional Relationships using freely available online resources that you can use or adapt for your class. The second part shows how Ratios & Proportional Relationships relate to other concepts in Grade 7. And the third part explains where Ratios & Proportional Relationships are situated in the progression of learning from Grades 3-8.

Part 1: What do these standards say?

The standards for Grade 7 contain a number of important ideas, so why begin this series with Ratios & Proportional Relationships? For starters, these standards are part of a major cluster, meaning they deserve a significant amount of class time over the course of the school year. (It's generally a good idea to prioritize major standards within the year to make sure they get the attention they deserve.) Proportional relationships are particularly important because they're a crucial step on the path to algebra, connecting multiplication and division from the elementary grades to linear equations, slope, and other concepts in Grade 8 and high school.[1]

Proportional relationships are also a great way to start the year because they link directly to what students should learn about ratios and rates in Grade 6. Moveover, some of the other work in Grade 7 (such as the standards for Expressions & Equations and Geometry) is easier for students to understand once they have an understanding of proportional relationships. If you're wondering where to start your year, proportional relationships are a solid bet.

In Grade 7, the standards in the Ratios & Proportional Relationships (RP) domain are grouped together into one cluster (called RP.A, since it's the first and only cluster). Let's take a look at precisely what the standards say, and then we'll discuss each one more thoroughly.

7.RP.A | Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.A.1

Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.*

7.RP.A.2

Recognize and represent proportional relationships between quantities.

7.RP.A.2.A

Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

7.RP.A.2.B

Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

7.RP.A.2.C

Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.*

7.RP.A.2.D

Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1, r)$ where r is the unit rate.

7.RP.A.3

Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

The order of the standards doesn't indicate the order in which they have to be taught. (Standards are only a set of expectations for what students should know and be able to do by the end of each year; they don't prescribe an exact sequence or curriculum.) These standards, for example, are all related, but they can be taught in the order listed, in another order, or in an integrated fashion.

Ratios, rates and proportional relationships: Essential concepts

As adults, we use proportional relationships all the time without naming the ideas involved. But teaching students who are new to proportional relationships requires precise knowledge of what they're being expected to learn. Let's pause for a moment to think about the concepts mentioned in the standards. (It's not important that students memorize these exact definitions, but it's good to have a clear idea of the concepts that the standards expect students to master.)

- A **ratio** is a pair of non-negative numbers, *A*:*B*, which are not both 0 (such as 1:2, 5:3, etc.).[2] Any two quantities can be associated in a ratio. Some examples of ratios that associate two quantities in this way are:
  - I paid 1 dollar for every 4 pounds of flour. The ratio of dollars to pounds of flour is 1:4.
  - For every 3 girls in the class, there are 2 boys. The ratio of girls to boys is 3:2.
  - In the birdhouse at the zoo, the ratio of wings to beaks is 2:1.
- Ratios are most useful in situations in which other, equivalent ratios have meaning. **Equivalent ratios** are the ratios obtained by multiplying the numbers in a ratio by the same positive number. So, for a class where the ratio of girls to boys is 3:2, we can generate equivalent ratios of 6:4, 9:6, 12:8, and so on. In context, this means that the class might have 6 girls and 4 boys, 9 girls and 6 boys, 12 girls and 8 boys, and so on. The number of students and the size of each group of boys and girls might change, but the ratios 3:2, 6:4, 9:6, and 12:8 are all equivalent. All of these are "in the same ratio."
- Ratios have companion unit rates. A **unit rate** of two quantities in a ratio is the number of units of the first quantity for every 1 unit of the second quantity. For example, each of these statements contains a ratio followed by its associated unit rate:
  - A recipe uses 1 cup of sugar for every 2 cups of flour, so there is 1/2 cup of sugar for every cup of flour.
  - Juice costs 96 cents for every 32 ounces, which is a rate of 3 cents for each ounce.
  - A car travels 120 miles every 2 hours, which is a rate of 60 miles per hour.
- A **proportional relationship** is a collection of pairs of numbers that are in equivalent ratios. A proportional relationship is described by an equation of the form $y = kx$, where k is a positive constant, often called a constant of proportionality (see below). The graph of a proportional relationship on the coordinate plane is a ray with the endpoint at (0, 0). An example of a proportional relationship represented in a table is:

Apples (lbs)	3	6	9	12	1	3/2
Cost ($)	2	4	6	8	2/3	1

- The **constant of proportionality** is the constant unit rate between the pairs of quantities in a proportional relationship. For example, in the table above, the constant of proportionality between the first row and the second row is 2/3, because 3 · (2/3) = 2, 6 · (2/3) = 4, 9 · (2/3) = 6, and so on.

Unit rates associated with ratios of fractions

The standards for Grade 6 introduce students to the concept of a unit rate as the number of units of one quantity "for every 1" or "per 1" of a second quantity. (**6.RP.A.2**) Unit rates in Grade 6 are limited to rates of two whole numbers. So students might see examples like those described in the section above: A recipe with 1 cup of sugar for every 2 cups of flour implies a unit rate of 1/2 cup of sugar for every cup of flour. Students should also be solving problems involving unit rates of whole numbers. (**6.RP.A.3**) For example, if we want to make a larger batch with the same recipe—say, using 11 cups of flour—how much sugar will we need?

Now, in Grade 7, students are challenged to apply their knowledge of unit rates to situations involving fractional quantities. (**7.RP.A.1**) The standard gives us a nice example here: A person walks 1/2 mile in each 1/4 hour. How many miles per hour is this person walking? To do this, students will need to rely on more than just the procedures for dividing fractions; they should realize that computing unit rates of fractions is an extension of the same concepts and structures they already know. Moreover, they can use the same representations, such as tables and tape diagrams, that they've used in the past to solve unit rate problems.

This task might work well in an early lesson on fractional unit rates because the numbers involved aren't too abstract. A student could represent or draw a picture of the solution to this problem without much difficulty.

Track Practice

Angel and Jayden were at track practice. The track is 2/5 kilometers around.

- Angel ran 1 lap in 2 minutes.
- Jayden ran 3 laps in 5 minutes.

a. How many minutes does it take Angel to run one kilometer? What about Jayden?
b. How far does Angel run in one minute? What about Jayden?
c. Who is running faster? Explain your reasoning.

The goal here is for students to find a unit rate for each runner: either the number of kilometers per 1 minute, or the number of minutes per 1 kilometer. Initially, they may need to start by making a table and reading the unit rate (see the solution section of the task for an example of how this might look). They might also create a tape diagram or use a number line to visualize how many minutes go into "every 1" kilometer. As students become more familiar with contexts that invite unit rate reasoning, though, they'll naturally begin to develop more efficient arithmetic strategies for finding unit rates.

Proportional relationships

In Grade 6, students learned the concept of a ratio (**6.RP.A.1**) and used collections of equivalent ratios to solve problems. (**6.RP.A.3**) Though they probably didn't use the term proportional relationship, this is exactly what they were working with. Now, they'll formalize their understanding of proportional relationships and explore the properties of these relationships more deeply. (**7.RP.A.2**) This is the critical advancement over work in Grade 6, and an important precursor to work with linear equations in Grade 8.

"Scale factor" in proportional relationships

The opening of standard **7.RP.A.2** gives us two charges: Students should be able to "recognize" and "represent" proportional relationships. This task uses a ratio table (a familiar representation from Grade 6) to begin investigating the characteristics of proportional relationships.

Grade 7, Module 1, Lesson 2: Example 1

A new self-serve frozen yogurt store opened this summer that sells its yogurt at a price based upon the total weight of the yogurt and its toppings in a dish. Each member of Isabelle's family weighed his dish, and this is what they found. Determine if the cost is proportional to the weight.

Weight (ounces)	**12.5**	**10**	**5**	**8**
Cost ($)	**5**	**4**	**2**	**3.20**

The cost ____ the weight.

Grade 7, Module 1, Lesson 2 Available from engageny.org/resource/grade-7-mathematics-module-1-topic-lesson-2; accessed 2015-05-29.

Students should be able to see how the two rows of the table are related by the same constant of proportionality (or unit rate) and that multiplying by a certain "scale factor" between columns of the table yields equivalent ratios.

- A student who recognizes the constant of proportionality or unit rate will be able to analyze the table and say, "This is a proportional relationship because there is a constant unit rate. Any weight divided by its corresponding cost is 2.5."
- A student who recognizes "scale factors" will be able to analyze the table and say, "This is a proportional relationship because any pair in the table can be generated from another pair in the table by multiplying. For example, 5 ounces for 2 dollars leads to 10 ounces for 4 dollars by multiplying by 2."

Both of these are hallmarks of proportional relationships, and students should be able to use these as tests to determine whether a relationship they've encountered is proportional or not.

Graphing proportional relationships

Students should also take these sorts of tables and plot the values on the coordinate plane. They should notice that the graph is a straight line through the origin, and should be able to explain why it looks that way. To see how students might come to this conclusion by solving a problem, let's take a look at this task:

Gym Membership Plans

In January, Georgia signed up for a membership at Anytime Fitness. The plan she chose cost $95 in start-up fees and then $20 per month starting in February. Edwin also signed up at Anytime Fitness in January. His plan cost $35 per month starting in February, and his start-up fees were waived.

a. Create tables for both Georgia and Edwin that compare the number of months since January to the total cost of their gym memberships. Continue this table for one year.

b. Plot the points from the two tables in part (a) on a coordinate plane.

c. Decide if either or both gym memberships are described by a proportional relationship, and write an equation representing any such relationship. Explain how parts (a) and (b) could be used to support your answer.

Let's focus on parts (a) and (b) here, and the first aspect of part (c). Students are graphing two relationships here, one of them proportional and one not proportional. (The commentary and solutions for the task show what the table and graph for each relationship could look like.) In examining the table, students might notice how only one relationship has a constant of proportionality, and in examining the graph, they can observe what that proportional relationship looks like when it appears on the coordinate plane. You can take this task a few steps further by asking questions like:

- Why is the graph astraight line?
- What does point (0, 0) mean in this situation, and why does the graph of the proportional plan start there?
- What do the other points on the graph mean?In particular, what does the point (1, 35) mean inEdwin's plan? (Students should make the connection between that point and the constant of proportionality (35/1) from their work with the table.)

Equations of proportional relationships

Once students have plenty of experience identifying the constant of proportionality in tables and graphs, they can use these representations to write equations for proportional relationships. The lesson below shows how it might be done:

Grade 7, Module 1, Lesson 8: Example 1

Example 1: Do We Have Enough Gas to Make It to the Gas Station?

Your mother has accelerated onto the interstate beginning a long road trip, and you notice that the low fuel light is on, indicating that there is a half a gallon left in the gas tank. The nearest gas station is 26 miles away. Your mother keeps a log where she records the mileage and the number of gallons purchased each time she fills up the tank. Use the information in the table below to determine whether you will make it to the gas station before the gas runs out. You know that if you can determine the amount of gas that her car consumes in a particular number of miles, then you can determine whether or not you can make it to the next gas station.

Mother's Gas Record

Gallons	Miles Driven
8	224
10	280
4	112

a. Find the constant of proportionality, and explain what it represents in this situation.

Gallons	Miles Driven	
8	224	$\frac{224}{8} = 28$
10	280	$\frac{280}{10} = 28$
4	112	$\frac{112}{4} = 28$

The constant of proportionality, k, is 28. The car travels 28 miles for every one gallon of gas.

b. Write equation(s) that will relate the miles driven to the number of gallons of gas.

$y = 28x$ or $m = 28g$

In Example 1, we can see students interpreting the constant of proportionality in a table (for every gallon, the car goes 28 miles), and moving from there to an equation. This is an abstract concept and can be difficult for students to do for the first time, so to help them, you might consider extending the table like so:

Gallons	Miles Driven
8	224
10	280
4	112
1	
g	

It might also help to write the equation by writing unevaluated expressions in the "Miles Driven" column, once students have identified the constant of proportionality:

Gallons	Miles Driven
8	8 × 28
10	10 × 28
4	4 × 28
1	1 × 28
g	

As they find a pattern in the table ("the number of miles is always the number of gallons times 28"), they can write an expression, $28g$, for the number of miles the car can go on any number of gallons g. From there, you can help them introduce a second variable to represent the number of miles driven (say, m), and they have their first equation.

$$\underline{28}g = m$$

↑ constant of proportionality (unit rate)

After trying this in a few different situations (examining the table to find the constant of proportionality is good practice), you might ask, "What do you notice about all of these equations?" Students should be able to explain that the constant of proportionality relates the two variables in each equation.

Multistep ratio and percent problems

In Grade 6, students learned the concept of a percent as a rate "per 100" and used ratio-style reasoning to solve percent problems. (**6.RP.A.3.C**) (This is different from the way percent was treated in the past, when it was often introduced as a completely new idea, isolated from others. Now, however, students should view percents as a specialized application of ratios and rates.) Most of that work was single-step, meaning that students had to interpret a situation in terms of ratios and perform a calculation or two in order to find an equivalent percent. A couple of examples:

- 5 of the 25 girls on Alden Middle School's soccer team are seventh-grade students. Find the percentage of seventh-graders on the team.
- Of the 25 girls on the Alden Middle School soccer team, 40% also play on a travel team. How many of the girls on the middle school team also play on a travel team?

These were good tasks for students new to percents, since they allowed them to understand percent problems as ratio situations and practice using helpful representations like double number lines and tape diagrams. Now the stakes are higher; students are going to have to solve problems that require them to calculate percentages or equivalent ratios in the midst of other procedures. (**7.RP.A.3**) And because of the increased complexity of problems, the level of sense-making that students have to demonstrate is higher. Let's take a look at an example task. (If you have a moment, look through the example solutions on the linked page as well; there are multiple methods of solving.)

Gotham City Taxis

The taxi fare in Gotham City is \$2.40 for the first $\frac{1}{2}$ mile and additional mileage charged at the rate \$0.20 for each additional 0.1 mile. You plan to give the driver a \$2 tip. How many miles can you ride for \$10?

You can see there's a lot going on here: Students need to interpret proportional "components" of the problem (such as the "additional mileage" charge), as well as nonproportional components (such as the first half-mile charge and the tip). And notice the complexity of unit rates implied here: Students won't have seen anything like this in Grade 6, since any unit rate solution involves finding a rate of two fractions (recall that in Grade 6 unit rates were limited to non-complex fractions). You might help students begin to tackle a problem like this by having them read it several times, and then helping them rewrite or diagram the three pieces of the problem. Which components depend on the distance traveled, and which don't? How do you know? Once students are able to distinguish the different parts of the taxi fare, both in terms of cost and distance, they're well-positioned to find the solution.

This is just one example of Grade 7 work with multistep problems. To get a more comprehensive picture of what this standard looks like, we recommend browsing its entire gallery of problems from Illustrative Mathematics. One item of particular interest could be this one involving successive discounts, which is an interesting variation on percent problems:

Double Discounts

Emily has a coupon for 20 percent off of her purchase at the store. She finds a backpack that she likes on the discount rack. Its original price is \$60 but everything on the rack comes with a 30 percent discount. Emily says

Thirty percent and twenty percent make fifty percent so it will cost \$30.
a. Is Emily correct? Explain.
b. What price will Emily pay for the backpack?

At this point, you may be wondering: Is there a way to help students take on all of these problems? While it's not possible to teach students how to solve every single formulation of a multistep ratio or percent problem, you can give them the tools to carefully interpret a variety of situations. Here are a few tips:

- If necessary, have students try some simpler percent problems from Grade 6. In solving these, remind students that percents are just a type of rate, and the same ideas and strategies that work for other ratio and rates problems apply.
- Spend some time on all of the various problems named in the standard: interest, tax, markups/markdowns, gratuities, commissions, fees, percent increase/decrease, error.
- After introducing each situation, give students plenty of mixed practice. In time, they should learn to discern the key differences between scenarios.
- Give students plenty of experience with helpful representations, such as ratio tables, double number lines and tape diagrams.

The role of Mathematical Practices

The standards don't just include knowledge and skills; they also recognize the need for students to engage in certain important practices of mathematical thinking and communication. These "Mathematical Practices" have their own set of standards, which contain the same basic objectives for Grades K-12.[3] (The idea is that students should cultivate the same habits of mind in increasingly sophisticated ways over the years.) But rather than being "just another thing" for teachers to incorporate into their classes, the practices are ways to help students arrive at the deep conceptual understandings required in each grade. In other words, the Practices help students get the content. The table below contains a few examples of how the Mathematical Practices might help students understand and work with ratios, proportional relationships, and percents in Grade 7.

Opportunities for Mathematical Practices:	**Teacher actions:**
Students are able to look for and make use of structureMP.7 in the context of multistep percent problems when they explain why the structure of percents often leads to multiple solution methods. For example, students should realize and articulate why they can find the final cost of an item after 5% tax by either adding 5% of the item's price to its original price, or simply by calculating 105% of the original price.	You can encourage students to see structure by having them model their thinking with various conceptual representations—tables, tape diagrams and double number lines. And you can have students present multiple solutions to the same problem to the rest of the class, and ask other students to explain how these methods are similar and different. (This lesson involving markup, markdown, and similar types of problems offers a number of opportunities to compare and contrast different methods.)
Students can model with mathematics (**MP.4**) in order to overcome common challenges in working with multistep percent problems, such as understanding the nuances of percent increase and decrease.	Present students with situations where conceptual representations can lend insight into a problem. For example, if an account contains $200 to start, its value decreases by 10% one month, and then increases by 10% the next month, why is the final value of the account not $200? Students might work through each part of this problem using a double number line diagram, and notice that in the first part, the "whole" is $200, while in the second part, the "whole" is only $180.
Students can make sense of problems and persevere when solving them (**MP.1**) when they solve a variety of problems involving proportional relationships—particularly problems that require using several representations of a relationship.	Consider problems that require students to gather information about a situation from one particular representation. For example, a graph of someone's earnings over an 8-hour shift might reveal a unit rate of $25 per hour. How much will this person earn in a week of full-time work? In a month? The answers to these questions likely require students to switch to a different representation—perhaps a table or an equation. With practice, students will become more skilled at analyzing given information and planning an efficient solution process before getting to work.

Podcast clip: Importance of the Mathematical Practices with Andrew Chen and Peter Coe (start 30:33, end 43:39)

Part 2: How do Ratios & Proportional Relationships relate to other parts of Grade 7?

There are lots of connections among standards in Grade 7; if you think about the standards long enough, you'll probably start to see these relationships everywhere.[4] A few are so important, though, that they deserve special attention. In this section, we'll talk about the connections between the Ratio & Proportional Relationships domain and the Geometry and Expressions & Equations domains in Grade 7.

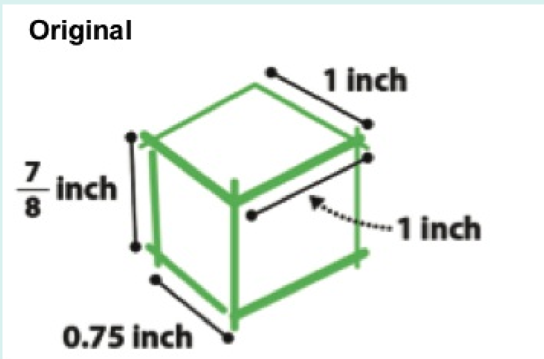
Geometry: Scale drawings

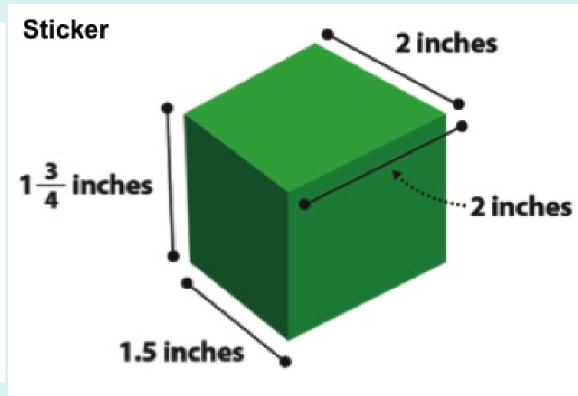
Let's start with the Geometry standards, which in Grade 7 are strongly related to proportional relationships through scale drawings. (**7.G.A.1**) This is such a natural connection that this standard may often be taught in the same unit with proportional relationships (as opposed to a stand-alone geometry unit). This lesson plan is part of just such a sequence. After being introduced to scale drawings in the previous lesson, students investigate how an object and its scaled image are related in a proportional relationship.

Grade 7, Module 1, Lesson 17: Example 1

Example 1: Jake's Icon

Jake created a simple game on his computer and shared it with his friends to play. They were instantly hooked, and the popularity of his game spread so quickly that Jake wanted to create a distinctive icon so that players could easily identify his game. He drew a simple sketch. From the sketch, he created stickers to promote his game, but Jake wasn't quite sure if the stickers were proportional to his original sketch.





Original	Sticker
1 in.	2 in.
$\frac{3}{4}$ in.	$1\frac{1}{2}$ in.
1 in.	2 in.
$\frac{7}{8}$ in.	$1\frac{3}{4}$ in.

Grade 7, Module 1, Lesson 17Available from engageny.org/resource/grade-7-mathematics-module-1-topic-d-lesson-17; accessed 2015-05-29.

The lesson plan includes a few questions following the problem that are worth highlighting here. Notice how they help students make the connection to prior learning about constant of proportionality.

- What relationship do you see between the measurements?
- Is the sticker proportional to the original sketch?
- How do you know?
- What is this [the unit rate] called?

From there, students are able to try scaling measurements for themselves. One nice thing about this sequence of problems is how it uses the language of proportional relationships to call up prior learning. Students can clearly see that this is an extension of something they already know, not a completely new topic. It also uses a familiar representation (a table) so that students can more easily identify the relationship between the two sets of measurements.

Expressions & Equations: Writing and solving equations

The story on Expressions & Equations starts with the work students do to solve problems with equations and inequalities. (**7.EE.B.4**) Recall that as students were building an understanding of proportional relationships, they had to express proportional relationships as equations. (**7.RP.A.2.C**) For example, in our gas mileage problem above, the number of miles,*m*, someone can drive on g gallons of gas when the car gets 28 miles per gallon was $g = 28m$. Students also had to solve multistep problems involving proportional and nonproportional aspects. This was the case in our taxi problem above, where the driver collected a charge for every fraction of a mile, but received a $2 tip regardless of the distance driven. Both of these ideas—modeling with equations and distinguishing aspects of problems (is it proportional or not?) come into play as students write and solve equations. A task like this might help students put them together.

Sports Equipment Set

Jonathan wants to save up enough money so that he can buy a new sports equipment set that includes a football, baseball, soccer ball, and basketball. This complete boxed set costs $50. Jonathan has $15 he saved from his birthday. In order to make more money, he plans to wash neighbors' windows. He plans to charge $3 for each window he washes, and any extra money he makes beyond $50 he can use to buy the additional accessories that go with the sports box set.

Write and solve an inequality that represents the number of windows Jonathan can wash in order to save at least the minimum amount he needs to buy the boxed set. Graph the solutions on the number line. What is a realistic number of windows for Jonathan to wash? How would that be reflected in the graph?

So what do we have here? Someone is earning $3*per window*—a proportional relationship. But there is also the $15 already saved—a quantity not proportional to the number of windows washed. So we have to deal with that, along with the fact that we need everything (savings, earnings from window washing) to be at least $50 when all's said and done. Representing situations in an equation like this can be tricky when students are first starting out, so it might help to think about the situation using a table and unevaluated expressions, just as they did when writing equations for proportional relationships.

Windows	Savings ($)
1	$(3 \times 1) + 15$
2	$(3 \times 2) + 15$
3	$(3 \times 3) + 15$
4	$(3 \times 4) + 15$
w	

Once students have thought several of these situations through, they'll begin to recognize the proportional and nonproportional "parts" of the relationship based on context, and will need to rely on tables or other scaffolds less and less. You can help them begin to distinguish the different components in a situation through questioning. For example:

- What is the constant of proportionality? How do you know?
- Which of the quantities isn't proportional? How do you know?
- What quantity are we trying to find, and how will we represent it?

Expressions & Equations: Solving problems involving rational numbers

The standards for Grade 7 focus heavily on problem-solving in both the Ratio & Proportional Relationships domain, and in the Expressions & Equations domain. Two of these standards— **7.RP.A.3**, which deals with multistep ratio and percent problems, and **7.EE.B.4**, which involves solving problems with rational numbers—are remarkably similar in terms of the situations they address. The difference is really in how students approach these problems. The good news is that experience with problems in one standard can help students master the other. For example, here's a task aligned to standard **7.EE.B.3**.

Discounted Books

Katie and Margarita have \$20.00 each to spend at Students' Choice book store, where all students receive a 20% discount. They both want to purchase a copy of the same book which normally sells for \$22.50 plus 10% sales tax.

- To check if she has enough to purchase the book, Katie takes 20% of\$22.50 and subtracts that amount from the normal price. She takes 10% of the discounted selling price and adds it back to find the purchase amount.
- Margarita takes 80% of the normal purchase price and then computes 110% of the reduced price.

Is Katie correct? Is Margarita correct? Do they have enough money to purchase the book?

After studying ratio and percent problems, this situation might look somewhat familiar. Students might approach this problem using the tools of proportional reasoning—such as double number lines, tape diagrams or equations involving fractional rates—but they might also see it in terms of expressions involving rational numbers. The solution section of the task gives us two sets of calculations to think about. There's Katie's method:

$$22.50 - (0.20(22.50)) = 22.50 - 4.50 = 18.00$$
$$18.00 + (0.10(18.00)) = 18.00 + 1.80 = 19.80$$

And there's Margarita's method:

$$(0.80)22.50 = 18.00$$
$$(1.10)18.00 = 19.80$$

As a way to get students thinking about the structures involved, you might ask them to explain the similarities they see between the two solutions. They might notice, for example, that decreasing a number by 20 percent is the same as finding 80 percent of that number. You might also ask them to consider whether the two students in the problem would still calculate identical answers for any item (of any price) in the bookstore. Discussing questions like these can give students additional insight into how percents work.

Part 3: Where did Ratios & Proportional Relationships come from, and where are they going?

Where do proportional relationships come from?

Proportional relationships, while formally introduced in Grade 7, are intended to build from a careful progression of prior learning—particularly the learning that occurs around ratios in Grade 6. If your students have been following a standards-aligned program for a few years, knowing the lead-up to ratios will help you leverage content from previous grades in your lessons. And if your students haven't been exposed to the standards in a meaningful way, or they're behind for other reasons, seeing where ratios come from will allow you to adapt your curriculum and lessons to make new ideas accessible. Let's briefly look at the main threads that lead up to ratios in Grades K-5; then we'll see how work with proportional relationships in Grade 7 compares to work with ratios in Grade 6. After that, we'll examine some ways that you might use this information to meet the unique needs of your students, and preview where proportional relationships are going in the next few years after Grade 7.

Podcast clip: Importance of Coherence with Andrew Chen and Peter Coe (start 9:34, end 26:19)

Connections to the elementary grades

Starting in the elementary grades, the standards lay the groundwork for ratios and proportional relationships. Though they do this in a variety of ways, there are a few progressions of learning that contribute most directly to RP in Grades 6 and 7.

- In Grades 3-5, students learn the meanings of **multiplication and division**, (■ **3.OA.A.1**, ■ **3.OA.A.2**) the relationship between those two operations, (■ **3.OA.B.6**) and **multiplicative comparison**—the idea that one quantity is so many times larger than another. (■ **4.OA.A.1**) This prepares students to see the constant of proportionality in tables and graphs. For instance, in the table below, students should be able to see that weight and cost are related by a constant factor of 3.

Grapes (lbs)	2	4	6
Cost ($)	6	12	18

- Also in Grades 3-5, students learn about the meanings of fractions,(■ **3.NF.A.1**, ■ **3.NF.A.2**) strategies for comparing fractions, (■ **3.NF.A.3**, ■ **4.NF.A.1**) and operations with fractions. (■ **5.NF.A.1**, ■ **5.NF.B.4**) This is a very involved progression, occupying its own domain in the standards (Number & Operation--Fractions, or NF) and taking a large amount of instructional time in these grades. By the end, students should be prepared to calculate unit rates and should be able to draw parallels between equivalent ratios and equivalent fractions (realizing, however, that these are not the same things).

- Finally, students study arithmetic patterns in tables, starting with patterns in the multiplication table in Grade 3(■ **3.OA.D.9**) (For example, they might notice that every other entry in the 5 multiplication sequence matches an entry in the 10 sequence.) This continues in Grades 4 and 5, where students continue exploring arithmetic patterns, including graphing them on the coordinate plane. (○ **4.OA.C.5**, ○ **5.OA.B.3**) By the end of Grade 5, the aim is for students to have plenty of experience looking for relationships between variables (as they will with ratio tables, graphs of proportional relationships, etc.).

Where do ratios come from, and where are they going? Part 3 of the Grade 6 guide to Ratios & Proportional Relationships contains more about each of these progressions, including problems that illustrate each one and suggestions for students who are behind in these areas.

Connections to Grade 6

So what are the differences between Grade 6 and Grade 7 in terms of Ratios & Proportional Relationships? This is an important question, because knowing where students are coming from can help you start your lessons in just the right place. Throughout this guide, we've touched upon the advances from one grade to the next, but now we'll see them illustrated with some problems.

In Grade 6, students should learn the concept of a unit rate (**6.RP.A.2**) and solve problems involving rates of whole numbers. (**6.RP.A.3**) Then, in Grade 7, they begin solving problems involving unit rates of fractions. (**7.RP.A.1**) The two problems below illustrate the difference in expectations.

Grade 6: Unit rates of whole numbers

Tickets for a baseball game cost $60 for a family of 5. Adult and youth tickets cost the same amount. Place a checkmark in either the True or False column for each of the statements below.

	T	F
2 tickets cost $24.		
For $40, you can buy 4 tickets.		
The cost is $12 per ticket.		
The cost for 10 tickets is $65.		

(Source: Ratios and Rates Mini-Assessment by Student Achievement Partners is licensed under CC 0 1.0.)

→ *In this problem, which is fairly typical of work in Grade 6, students deal with ratio and rate situations involving whole numbers only. This allows them to focus on understanding the concept of a rate and using various models, such as tape diagrams and ratio tables, to represent problems.*

Grade 7: Unit rates of fractions

During their last workout, Izzy ran 2 1/4 miles in 15 minutes, and her friend Julia ran 3 3/4 miles in 25 minutes. Each girl thought she was the faster runner. Based on their last run, which girl is correct?

(Source: Grade 7, Module 1, Lesson 11 (teacher version) from EngageNY.org of the New York State Education Department is licensed under CC BY-NC-SA 3.0)

→ *With a firm understanding of rate from the previous grade, students work on problems like this, which have a greater degree of computational difficulty. The representations, operations, and procedures that students used in Grade 6 are still useful in dealing with these problems.*

In Grade 6, students also learn the concept of a ratio (**6.RP.A.1**) and solve problems involving equivalent ratios. (**6.RP.A.3**) In Grade 7, they formalize sets of equivalent ratios as "proportional relationships" and represent these in different ways. (**7.RP.A.2**) Again, a pair of problems illustrates the advancements from one grade to the next.

Grade 6: Equivalent ratios

Javier has a new job designing websites. He is paid at a rate of $700 for every 3 pages of web content that he builds. Create a ratio table to show the total amount of money Javier has earned in ratio to the number of pages he has built.

Total Pages Built			
Total Money Earned			

Javier is saving up to purchase a used car that costs $4,200. How many web pages will Javier need to build before he can pay for the car?

(Source: Grade 6, Module 1, Lesson 9 (teacher version) from EngageNY.org of the New York State Education Department is licensed under CC BY-NC-SA 3.0)

→ *In this problem, we see students generating equivalent ratios from a context, and solving a problem by locating a particular ratio in a table. While this problem does involve a proportional relationship, students are only thinking in terms of equivalent ratios and are not yet using the language of proportionality.*

Grade 7: Proportional relationships

The table shows the amounts of tomato sauce and cheese used to make the last 4 orders at Sara's Pizza.

Number of Pizzas	Tomato Sauce (oz)	Cheese (oz)
2	10	4.5
3	15	6.75
6	30	13.5
2	10	4.5

Decide whether the relationship between number of pizzas and amount of cheese is proportional. Explain your decision.

(Source: Proportional Relationships Mini-Assessment by Student Achievement Partners is licensed under CC 0 1.0.)

→ *This Grade 7 problem requires students to have a formal understanding of proportional relationships, and to distinguish proportional from non-proportional relationships using, for example, a constant of proportionality.*

Lastly, in Grade 6, students solve various ratio and rate problems, many of which were single-step. (**6.RP.A.3**) In Grade 7, the focus is on more complicated "multistep" problems. (**7.RP.A.3**) Once again, we have two problems that show how expectations advance year-over-year.

Grade 6: Simple ratio and rate problems	**Grade 7: multistep ratio and rate problems**
Mr. Yoshi has 75 papers. He graded 60 papers, and he had a student teacher grade the rest. What percent of the papers did each person grade?	On Black Friday, a $300 mountain bike is discounted by 30% and then discounted an additional 10% for shoppers who arrive before 5:00 a.m. What is the cost of the bike for an early-morning shopper on Black Friday?
(Source: Grade 6, Module 1, Lesson 27 (teacher version) from EngageNY.org of the New York State Education Department is licensed under CC BY-NC-SA 3.0.)	(Source: Grade 7, Module 4, Lesson 7 (teacher version) from EngageNY.org of the New York State Education Department is licensed under CC BY-NC-SA 3.0.)
→ *This is a typical example of Grade 6 work with percents. Students are learning about percent as a specialized rate, and focus on solving simple problems in order to better understand the concept.*	→ *Presuming that students have a firm grasp of percent as a rate "per 100," work in Grade 7 asks students to apply their knowledge in new and more complicated ways.*

Suggestions for students who are behind

If, going into a unit on proportional relationships, you know your students don't have a solid grasp of the ideas named above, what can you do? It's not practical (or even desirable) to reteach Grade 6 material "from scratch"; there's plenty of new material in Grade 7, so the focus needs to be on grade-level standards. At the same time, there are strategic ways of wrapping up "unfinished learning" from prior grades within a unit on proportional relationships. Here are a few ideas for adapting your instruction to bridge the gaps.

- If a significant number of students don't understand the **concept of a unit rate**, or don't have experience with **unit rates of whole numbers**, you could plan a lesson or two on that idea before starting work with unit rates of fractions. (This Grade 6 lesson is an introduction to the idea of unit rate for students who haven't seen it before, and this one contains plenty of problems that might be useful for review.) And if you think an entire lesson is too much, you could use 2-3 unit rate problems from these lessons as "warm-ups" to start your first few lessons involving unit rates of fractions.
- If a significant number of students don't understand the **concept of a ratio**, or haven't worked much with **equivalent ratios**, you could likewise plan a lesson or two on that before introducing rates. (This Grade 6 lesson, which examines two structures within tables of equivalent ratios, is one possible starting point.) Again, if you think that an entire lesson is too much, you could use some problems involving this idea as warm-ups for your first few rate lessons, or use them as simpler examples when introducing newer content.
- If a significant number of students don't understand the **percents**, you could plan a lesson or series of warm-ups on this idea. (This Grade 6 lesson introduces percents for the first time, and this one includes several basic problems which might be helpful for students who just have some brushing up to do.)

Where are proportional relationships going?

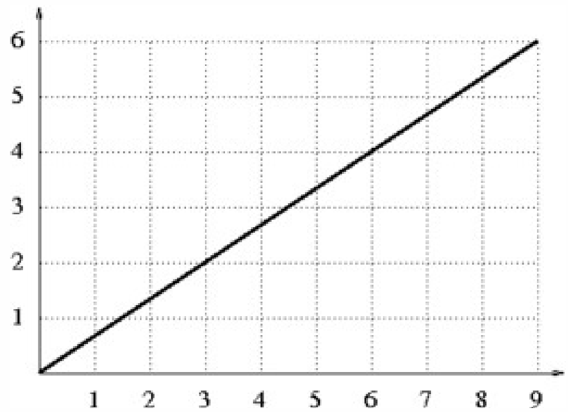
Lastly, how will students use their knowledge of proportional relationships after Grade 7? This is an important question, because the answer defines the limits of instruction in Grade 7 and explains why it's important that students thoroughly elaborate their understanding of proportional relationships through a variety of representations.

Let's recap, then: By the end of Grade 7, students should have a firm understanding of the characteristics of proportional relationships in tables and graphs, and should be able to represent proportional relationships with equations. Moreover, they should have experience computing unit rates involving fractions and decimals, and solving all kinds of ratio and percent problems.

In Grade 8, there is no separate group of standards for ratios and proportional relationships; these ideas merge completely with the algebra content in the Expressions & Equations domain and the Functions domain. The concept of unit rate evolves into slope, (**8.EE.B.5**) and students will discover important properties of slope. They explore the connection between proportional relationships (i.e. can be represented by an equation $y = mx$) and linear equations more generally ($y = mx + b$). (**8.EE.B.6**) And after being formally introduced to the concept of a function, students will model linear relationships with functions. (**8.F.B.4**) Students will rely on these concepts throughout high school and, in many cases, in post-secondary work as well. One last progression of problems illustrates where students are headed.

Grade 8: Understanding slope (8.EE.B.6)

Eva, Carl, and Maria are computing the slope between pairs of points on the line shown below.



Eva finds the slope between the points (0,0) and (3,2). Carl finds the slope between the points (3,2) and (6,4). Maria finds the slope between the points (3,2) and (9,6). They have each drawn a triangle to help with their calculations (shown below).

i. Which student has drawn which triangle? Finish the slope calculation for each student. How can the differences in the x- and y-values bez interpreted geometrically in the pictures they have drawn?

ii. Consider any two points ($x1$,$y1$) and ($x2$,$y2$) on the line shown above. Draw a triangle like the triangles drawn by Eva, Carl, and Maria. What is the slope between these two points? Why should this slope be the same as the slopes calculated by the three students?

→ *Having learned to recognize unit rates of proportional relationships in graphs while in Grade 7, students are poised to take on problems like this one. Here, students formalize their understanding of slope, and understand why the slope of a line is the same between any two points on the line.*

Grade 8: Comparing functions (8.F.A.2)

Maureen and Shannon decide to rent stand-up paddleboards while on vacation. Shop A rents paddleboards for $7.75 per hour. Shop B's prices are shown on the poster below. Which shop offers a cheaper hourly rental rate?

Hours	Price
0.5	$3.80
4	$30.36
7	$53.13
8	$60.72

(Source: Functions Mini-Assessment by Student Achievement Partners is licensed under CC 0 1.0.)

→ *In this problem, we can see students comparing functions (which happen to involve proportional relationships) represented in two different ways. They'll also have to compare nonproportional linear relationships, so time in Grade 7 examining the features of graphs, tables, and equations is well-spent.*

Grade 8: Modeling with functions (8.F.A.4)

You have $100 to spend on a barbeque where you want to serve chicken and steak. Chicken costs $1.29 per pound and steak costs$3.49 per pound.

a) Find a function that relates the amount of chicken and the amount of steak you can buy.

b) Graph the function. What is the meaning of each intercept in this context? What is the meaning of the slope in this context? Use this (and any other information represented by the equation or graph) to discuss what your options are for the amounts of chicken and amount of steak you can buy for the barbeque.

(Source: "Chicken and Steak, Variation 1" by Illustrative Mathematics is licensed under CC BY 4.0.)

→ *This problem requires students to put knowledge of unit rate to work in creating a function and interpreting the features of its graph. Without a good understanding of unit rate from Grades 6 and 7, a task like this becomes very difficult.*

If you've just finished this entire guide, congratulations! Hopefully it's been informative, and you can return to it as a reference when planning lessons, creating units, or evaluating instructional materials. For more guides in this series, please visit our Enhance Instruction page. For more ideas of how you might use these guides in your daily practice, please visit our Frequently Asked Questions page. And if you're interested in learning more about Ratios & Proportional Relationships in Grade 7, don't forget these resources:

Student Achievement Partners: Focus in Grade 7

Draft 6-7 Progression on Ratios and Proportional Relationships

EngageNY: Grade 7 Module 1 Materials (proportional relationships)

EngageNY: Grade 7 Module 4 Materials (percents)

Illustrative Mathematics Grade 7 Tasks

Endnotes

[1] In this series, major clusters and standards are denoted by a . For more information on the major work of Grade 7, see the Student Achievement Partners guide Focus in Grade 7.

[2] Much of the information in this section is taken from the Draft 6-7 Progression on Ratios and Proportional Relationships, one of a series of papers that describes the big ideas behind the standards and how those ideas fit together. If you're interested in learning more about ratios and proportional relationships, it's a good resource.

[3] You can read the full text of the Standards for Mathematical Practice here.

[4] The idea that standards relate strongly to one another is known as coherence, and is a distinctive feature of the Common Core State Standards for Mathematics. If you're interested in exploring more of the connections between standards, you might want to try the Student Achievement Partners Coherence Map web app