Operations & Algebraic Thinking: A Guide to Grade 2 Mathematics Standards

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2.OA.A | Represent and solve problems involving addition and subtraction.
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Welcome to the UnboundEd Mathematics Guide series!These guides are designed to explain what new, high standards for mathematics say about what students should learn in each grade, and what they mean for curriculum and instruction.This guide, the first for Grade 2, includes three parts. The first part gives a "tour" of the standards for the first two clusters of **Operations & Algebraic Thinking** (**2.OA.A** and **2.OA.B**) using freely available online resources that you can use or adapt for your class. The second part shows how Operations & Algebraic Thinking relate to representing and interpreting data in Grade 2 (**2.MD.D**). And the third part explains where the skills and understandings within **2.OA** and **2.OA.B** are situated in the progression of learning in Grades K-2.Throughout all of our guides, we include a large number of sample math problems. We strongly suggest tackling these problems yourself to help best understand the methods and strategies we're covering, and the potential challenges your students might face.

Part 1: What do these standards say?

Much of students' early learning in mathematics revolves around the operations of addition, subtraction, multiplication and division. But as we know from experience, being able to calculate an answer isn't enough; students need to have a complete understanding of each of these operations in order to become flexible problem solvers and achieve success in later grades. The standards in the Operations & Algebraic Thinking (OA) domain, which runs from Kindergarten through Grade 5, are designed to meet this need. The OA standards specify the meanings and properties of each operation, fluencies that students should achieve at each grade level, and the types of problems they should be able to solve. In Grade 2, where the emphasis is on addition and subtraction, the standards focus on using mental strategies to add and subtract within 20 and using addition and subtraction to problem solve within 100. (**2.OA.A.1**, **2.OA.B.2**)

In Grade 2, it makes sense to discuss addition and subtraction by taking an in-depth look at the first two clusters in the OA domain, 2.OA.A and **2.OA.B**. (The third cluster, **2.OA.C**, prepares students for multiplication and will be addressed in another guide.) Clusters **2.OA.A** and **2.OA.B** are both part of the **major work** of the grade, as indicated by the green square above.[1] And a large majority of time should be spent teaching the major work of the grade. Since these major clusters contain standards that require quite a bit of time to master, beginning the school year with a focus on these clusters is a good idea. With respect to problem-solving, students should engage in solving addition and subtraction problems throughout the entire year, integrating new strategies (from the Number & Operations in Base Ten standards) and new contexts (from the Measurement & Data standards) as the year progresses. Also, fluency with addition and subtraction requires careful conceptual development and significant practice, so beginning the year with addition and subtraction makes sense.

The other cluster discussed in this guide comes from the Measurement & Data (MD) domain. This is a supporting cluster, and the standards in this cluster can help students deepen their understanding of the standards in the OA domain. We will talk about these standards in detail in Part 2.

> It's important to note that the **clusters, and the standards within the clusters, are not necessarily sequenced in the order in which they have to be taught.** (Standards are only a set of expectations of what students should know and be able to do by the end of each year; they don't prescribe an exact sequence or curriculum.) So planning your instruction sequence carefully can ensure your students continue to build on previous understandings. As we go, think about the connections you see between standards, and how you can use these connections to help students build on their previous understandings.
>
> Also, manipulatives are frequently a large part of the tasks and lessons that follow. They are most effective when connected to written methods, rather than taught as a method for computing or solving unto themselves. Used this way, manipulatives can be an important way to build conceptual understanding of numbers and operations.[2]

Throughout the guide, we'll look at examples of tasks and lessons that focus on students' abilities to make sense of problems and persevere in solving them (**MP.1**). As students are exposed to varied contexts and problem types, they will need to think carefully to understand each problem and develop an appropriate solution method.

Let's begin our discussion of the OA standards in Grade 2 by taking a deep look at the second cluster, **2.OA.B**. As we dig into this cluster, we will also examine some of the Grade 1 standards and a few of the Kindergarten standards. This is necessary because the strategies required for addition and subtraction in Grade 2 are defined in the Grade 1 standards. In particular, the mental strategies mentioned in **2.OA.B.2** are described in **1.OA.C.6**. And the strategies in **1.OA.C.6** are directly supported by important understandings in the Kindergarten standards.

K.CC.A.2 "Count forward beginning from a given number…"

K.OA.A.3 "Decompose numbers less than or equal to 10 into pairs in more than one way…"

K.OA.A.4 "For any number from 1 to 9, find the number that makes 10 when added to the given number…"

1.OA.C.6 "Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten…decomposing a number leading to a ten…using the relationship between addition and subtraction…and creating equivalent but easier or known sums…"

2.OA.B.2 "Fluently add and subtract within 20 using mental strategies." (See standard **1.OA.C.6** for a list of mental strategies.)

The progression for addition and subtraction is thoughtfully articulated in the language of the standards. The Grade 1 standards, in particular, shed a lot of light on the culminating expectations set by the Grade 2 standards. In Grade 2 we can best support students along the progression by helping them deepen their understanding of addition and subtraction and by increasing their flexibility with strategies. But we want to be sure to do all of this without sacrificing their conceptual understanding.

Let's begin by reading the standard in cluster **2.OA.B**, and then we'll think through what it means and how it looks in practice.

2.OA.B | Add and subtract within 20.

2.OA.B.2
Fluently add and subtract within 20 using mental strategies.
By end of Grade 2, know from memory all sums of two one-digit numbers.*

**See standard 1.OA.6 for a list of mental strategies.*

Fluency with adding and subtracting within 20 is an extension of the fluency students developed in previous years—adding and subtracting within 5 in Kindergarten, (**K.OA.A.5**) then adding and subtracting within 10 in Grade 1 (**1.OA.C.6**) As we read in the second sentence of this standard, students "know from memory all sums of two one-digit numbers" by the end of Grade 2. This "memory" aspect is something that is built in time; the key to fluency progression is that it's firmly rooted in conceptual understanding, not rote memorization. This is accomplished as students learn and practice strategies in Grade 1 (**1.OA.C.6**) that make addition and subtraction tangible to them; once comfortable and confident with them, they can begin internalizing these as mental strategies in Grade 2. So what are these strategies from Grade 1? Let's briefly discuss them, along with their roots in Kindergarten.[3]

Methods for single-digit addition and subtraction

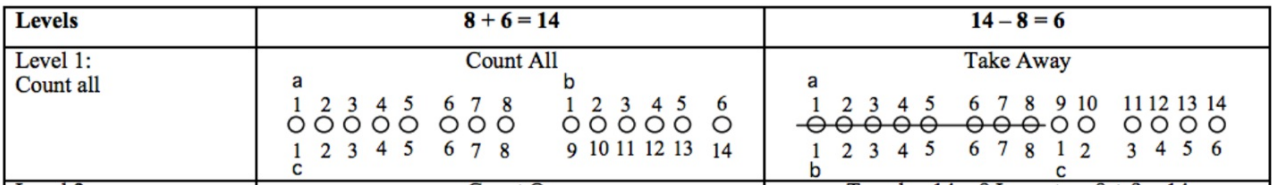
The Grade 1 standard, 1.OA.A.6, states that students should use strategies such as "counting on," "making ten," "decomposing a number leading to a ten," and "creating equivalent but easier or known sums" as a means of achieving fluency. These strategies are part of a hierarchy of computation methods for single-digit addition and subtraction. These methods can be thought of in terms of levels.

- Level 1: Counting all
- Level 2: Counting on
- Level 3: Convert to an easier problem

It's important to understand these levels, and the appropriate prerequisites, so that we can support students with using increasingly advanced methods. Students primarily work with the Level 1 method in Kindergarten, then move on to the more efficient and widely applicable strategies of Levels 2 and 3 in Grade 1. Our goal in Grade 2 is to build students' fluency and flexibility with Level 2 and 3 strategies so that they can perform addition and subtraction computations within 20 mentally. (**2.OA.B.2**) Let's take a closer look at what these levels mean and how they're tied to the standards.

Level 1: Counting all

Using the **Level 1** method (i.e., direct modeling by counting all or taking away), students use manipulatives or drawings to model putting together each addend (a number that is added to another number) and then counting the total (in the case of addition). In the case of subtraction, students model the total, then take away an addend, and then count the remaining objects to determine the result. The diagram below illustrates count-all and take-away (and count-remaining) strategies.

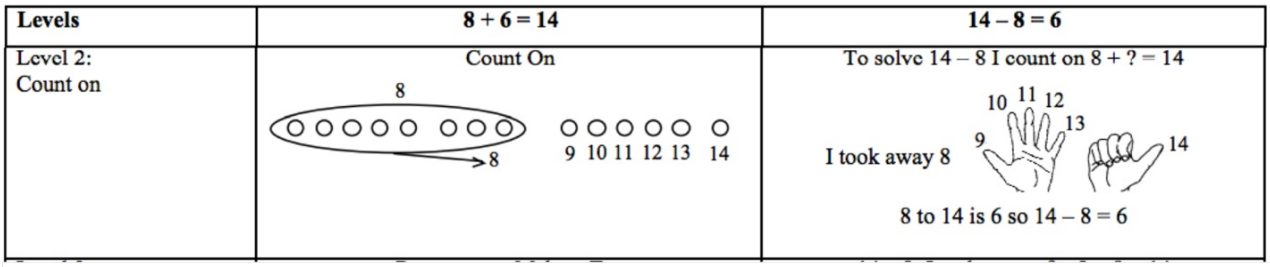


Source: Draft K-5 Progression on Counting and Cardinality and Operations and Algebraic Thinking p. 36

This strategy is tied closely to the foundational work with counting that students do in Kindergarten. While this is not one of the mental strategies students should be using in Grade 2, understanding this strategy will help us to coherently introduce new strategies and support students that may not be on grade level.

Level 2: Counting on

The **Level 2** method (i.e., counting on) is a more efficient method than counting all and is listed as a strategy that students should do mentally in Grade 2; that is, students should have experience with this method from Grade 1, and we want to further their comfort and fluency with it in Grade 2. With counting on, students understand that the total is composed of addends, and counting on from one addend by the other addend can be used to find the total (i.e., addition). Additionally, counting on from a known addend, to a given total, can be used to determine an unknown addend (i.e., subtraction). The diagram below illustrates count-on strategies for addition and subtraction.



Source: Draft K-5 Progression on Counting and Cardinality and Operations and Algebraic Thinking p. 36

This thinking, of course, can be applied to word problems as well. The example below is an "Addend Unknown" problem (more on what that means in a moment) and could be solved by counting on:

EngageNY Grade 2, Module 1, Lesson 7: Homework

Robert has 16 cups. Some are red. Nine are blue. How many cups are red?

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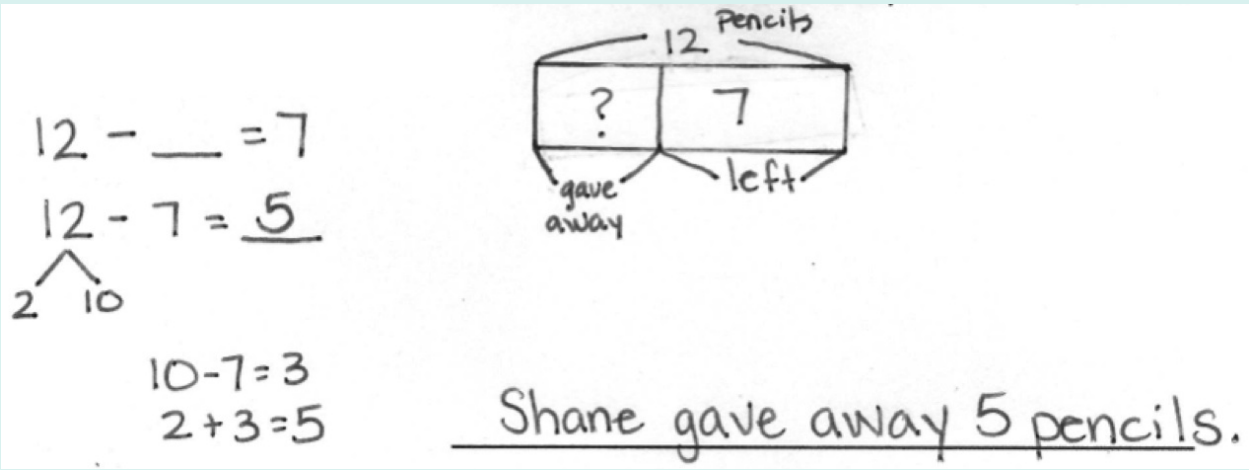
Using counting on, students would start with 9, count to 16 (e.g., "nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen) while tracking the count of numbers on their hands, to determine the number of red cups to be 7.

Level 3 methods (i.e., convert to an easier problem) are the most sophisticated strategies for adding and subtracting single-digit numbers, and also the most efficient. These strategies include "making 10," "decomposing a number leading to a10" and "creating equivalent but easier or known sums (e.g., doubling plus or minus 1, or plus or minus 2)" and are listed as strategies that students should do mentally in Grade 2. Extensive work with Level 3 methods is important in Grade 2, as these make up most of the strategies referenced from Grade 1. (**1.OA.C.6**). Remember: our goal in Grade 2 is to build students' fluency and flexibility with Level 2 and 3 strategies so that they can perform addition and subtraction computations within 20 mentally. (**2.OA.B.2**) In the example below, students decompose a number leading to a ten in order to subtract.

EngageNY Grade 2, Module 1, Lesson 7: Problem Set

Shane has 12 pencils. He gives some pencils to his friends. Now, he has 7 left. How many pencils did he give away?



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Extensive work in K-1 supports knowing decomposition of numbers within 20(**K.OA.A.3**, **K.OA.A.4**, **1.NBT.B.2**) Since students are fluent with pairs of addends of 10, subtracting 7 from 10 is easier than subtracting 7 from 12.

Our goal in Grade 2 is to give students enough opportunity and support to engage with these computation strategies so that they can do them mentally. These strategies support fluency with single-digit sums (and related subtraction), but they also support addition and subtraction strategies with larger numbers. For example, imagine if the word problem above was changed to: Robert has 40 cups. Some are red. Eighteen are blue. How many cups are red? Students might solve this problem by counting on with ones and tens (e.g., 18... 28, 38, 39, 40), since students learned to mentally find 10 more or 10 less than any number in Grade 1. (**1.NBT.C.5**)That said, when adding and subtracting two- and three-digit numbers, Level 3 methods and strategies based on place value (which are supported by Level 3 methods) become much more efficient than counting on. For example, using the example of Robert and his cups, students might quickly add 2 to make 20 (not make a 10, but make the *next* 10) and then add 20 to get 40. Then they would add 2 + 20 to determine there are 22 red cups. When we discuss problem solving within 100, we will see more examples of how Level 3 methods support addition and subtraction with two- and three-digit numbers.

Properties of addition and the relationship between addition and subtraction

The mental strategies referred to in standard **2.OA.B.2** also include applying properties of addition and using the relationship between addition and subtraction to fluently add and subtract within 20. The properties of addition include the commutative property of addition and the associative property of addition; these are two properties that allow us to create "equivalent but easier or known sums."(**1.OA.C.6**) For example, understanding the commutative property means that we could think of 1 + 8 = ? as 8 + 1 = ? (the order of the addends does not change the sum). In this way, despite the way an equation is written, students can add in the way that is most efficient for them. The associative property of addition supports Level 3 methods such as make a 10. For example, to find the total for 2 + 6 + 4 as written, a student would add first add 2 + 6, which is 8, and then add 4, to get 12. Formally, this is written as (2 + 6) + 4; the 2 + 6 is grouped together and added first. By the associative property, *any grouping* of addends will yield the same sum, so a student could add 6 + 4 first, which is 10, and then add 2, to also get 12. Formally, this is written as 2 + (6 + 4).

Students will apply and often combine these properties of addition to support making an easier problem; for this reason they are sometimes referred to as a single property ("any order, any grouping") at this grade level. This leads to an important note about properties at this grade level: The Standards don't require that students know the formal names of the properties, as this distracts from the conceptual understanding of these properties. The focus is on applying properties for the purposes of solving problems.

In addition to applying the properties of addition, understanding the relationship between addition and subtraction is described as a mental strategy. Specifically, students come to understand that a subtraction problem can be thought of as an unknown addend problem. For example, in the problem above about Shane's pencils, students could have solved the problem 12 - ? = 5 by thinking 5 + ? = 12.

Fluency activities

There are many activities that can build fluency from a conceptual foundation; a step in this direction is often accomplished by paying attention to the numbers we use in our examples. Rather than asking students to recite the answers to a more or less "random" collection of addition and subtraction facts, we can choose facts that illuminate certain strategically advantageous structures. The example below focuses on the relationship between pairs that add to 10:

EngageNY Grade 2, Module 1, Lesson 6: Fluency

Take from Ten (2 minutes)

Note: This activity develops the automaticity necessary to subtract fluently from the ten when using the take from ten strategy in Lessons 6, 7, and 8.

T: When I say 10 – 9, you say 10 – 9 = 1. Ready? 10 – 9.
S: 10 – 9 = 1.
T: 10 – 5.
S: 10 – 5 = 5.

Continue with the following sequence: 10 – 2, 10 – 4, 10 – 6, 10 – 7, 10 – 3, and 10 – 8.

T: When I say 1, you say 9. Ready? 1.
S: 9.
T: 2.
S: 8.

Continue with the following sequence: 5, 0, 4, 7, 3, 8, 6, and 10.

EngageNY Grade 2, Module 1, Lesson 6 from EngageNY.org of the New York State Education Department is licensed under CC BY-NC-SA 3.0.

This example supports mental strategies by highlighting the different decompositions of 10, which are advantageous in determining other sums and differences within 20.

So far we've looked at some of the Grade 2 fluencies plus the associated computation methods, and important understandings regarding the properties of addition and the relationship between addition and subtraction. Now let's look more closely at how students apply these methods and understandings as strategies for solving addition and subtraction word problems. We'll also examine how place value understandings play a role. Let's begin by reading the standard in cluster **2.OA.A**.

2.OA.A | Represent and solve problems involving addition and subtraction.

2.OA.A.1
Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.*

*See Glossary, Table 1.

Problem-solving with addition and subtraction

Extensive work with problem-solving is really important in Grades K-2. Using addition and subtraction to solve real-world problems helps students develop meaning for addition and subtraction, fosters growth with computation strategies, and helps them develop fluency with addition and subtraction facts. The types of word problems referred to in **2.OA.A.1** are further delineated into 12 subtypes as demonstrated in Table 1 of the standards.[4] And the table below, adapted from the Standards, highlights the specific subtypesthat students should master at each grade level.[5] By the end of Grade 2, students should master all of the problem types in the table.It will be helpful to spend some time digging into the different problem situations shown in the table, as we will refer to them extensively throughout the rest of this discussion.

Many of us have learned that solving word problems involves finding "key words,"— words like "more" and "total" tell us to add, while words like "fewer" and "less" tell us to subtract. But what about a problem like this:

> Lucy has six fewer apples than Julie. Lucy has eight apples. How many apples does Julie have?

The "key word" in this problem, "fewer," actually hints at the wrong operation; subtracting will not result in the correct answer. A better way to help students with problem-solving is to help them think situationally about the varied contexts. Using word problems to create meaning for operations helps students to better understand how to apply operations.

Table 2: Addition and subtraction situations by grade level.

	Result Unknown	Change Unknown	Start Unknown
Add To	*A* bunnies sat on the grass. *B* more bunnies hopped there. How many bunnies are on the grass now? $A + B = \square$	*A* bunnies were sitting on the grass. Some more bunnies hopped there. Then there were *C* bunnies. How many bunnies hopped over to the first *A* bunnies? $A + \square = C$	Some bunnies were sitting on the grass. *B* more bunnies hopped there. Then there were *C* bunnies. How many bunnies were on the grass before? $\square + B = C$
Take From	*C* apples were on the table. I ate *B* apples. How many apples are on the table now? $C - B = \square$	*C* apples were on the table. I ate some apples. Then there were *A* apples. How many apples did I eat? $C - \square = A$	Some apples were on the table. I ate *B* apples. Then there were *A* apples. How many apples were on the table before? $\square - B = A$
	Total Unknown	**Both Addends Unknown[1]**	**Addend Unknown[2]**
Put Together /Take Apart	*A* red apples and *B* green apples are on the table. How many apples are on the table? $A + B = \square$	Grandma has *C* flowers. How many can she put in her red vase and how many in her blue vase? $C = \square + \square$	*C* apples are on the table. *A* are red and the rest are green. How many apples are green? $A + \square = C$ $C - A = \square$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	*"How many more?" version.* Lucy has *A* apples. Julie has *C* apples. How many more apples does Julie have than Lucy? *"How many fewer?" version.* Lucy has *A* apples. Julie has *C* apples. How many fewer apples does Lucy have than Julie? $A + \square = C$ $C - A = \square$	*"More" version suggests operation.* Julie has *B* more apples than Lucy. Lucy has *A* apples. How many apples does Julie have? *"Fewer" version suggests wrong operation.* Lucy has *B* fewer apples than Julie. Lucy has *A* apples. How many apples does Julie have? $A + B = \square$	*"Fewer" version suggests operation.* Lucy has *B* fewer apples than Julie. Julie has *C* apples. How many apples does Lucy have? *"More" version suggests wrong operation.* Julie has *B* more apples than Lucy. Julie has *C* apples. How many apples does Lucy have? $C - B = \square$ $\square + B = C$

Darker shading indicates the four Kindergarten problem subtypes. Grade 1 and 2 students work with all subtypes and variants. Unshaded (white) problems are the four difficult subtypes or variants that students should work with in Grade 1 but need not master until Grade 2. Adapted from CCSS, p. 88, which is based on *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, National Research Council, 2009, pp. 32–33.

[1] This can be used to show all decompositions of a given number, especially important for numbers within 10. Equations with totals on the left help children understand that = does not always mean "makes" or "results in" but always means "is the same number as." Such problems are not a problem subtype with one unknown, as is the Addend Unknown subtype to the right. These problems are a productive variation with two unknowns that give experience with finding all of the decompositions of a number and reflecting on the patterns involved.

[2] Either addend can be unknown; both variations should be included.

Source: Draft K-5 Progression on Counting & Cardinality and Operations & Algebraic Thinking, p. 9

Students engage with these types of problems beginning in Kindergarten. IrGrade 2, they extend previous work from Kindergarten and Grade 1 in three ways:[6]

- Students master all 12 subtypes shown in the table above.
- Students extend problem-solving to whole numbers within 100.
- Students solve two-step problems.

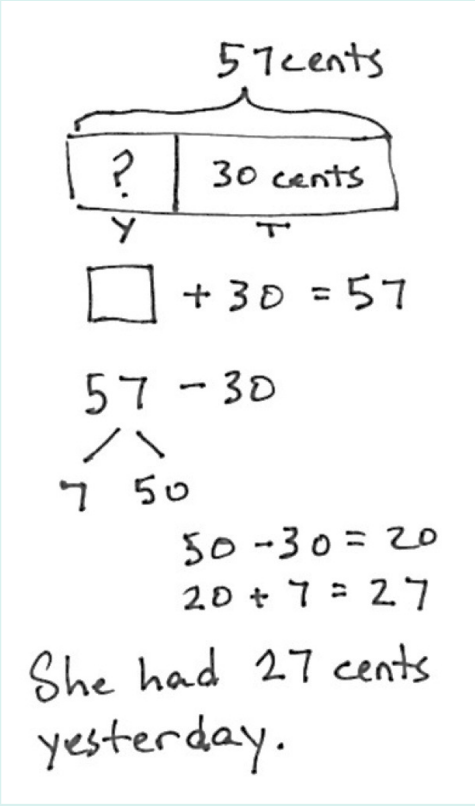
"Add to with start unknown"

This is one of the more difficultsubtypes that students engage with in Grade 2. In the example below, students are trying to determine how much money there was to begin with:

Grade 2, Module 4, Lesson 2: Application

Susan has 57 cents in her piggy bank. If she just put in 30 cents today, how much did she have yesterday?

Note: This *add to with start unknown* problem gives students a chance to apply their new learning. It also provides an opportunity to work through a common mistake; many students will add and give the answer 87 cents. Encourage students to draw a tape diagram to show what is known. This will help them identify the whole and one part, guiding them to subtract to find the missing part.



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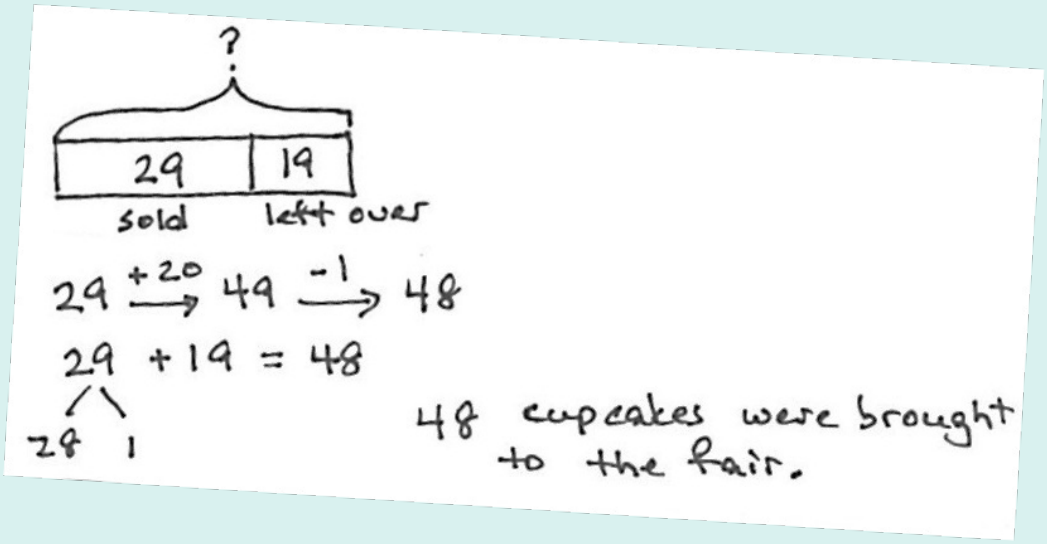
This kind of problem is challenging because it is difficult to determine what the unknown is; is it an addend or the total? Using diagrams, such as the tape diagram shown, to represent the knowns will help students identify the unknowns. Computationally, this student demonstrates making an easier problem (Level 3) using place value strategies by decomposing 57 into tens and ones and then subtracting 3 tens from 5 tens. Understanding that a two-digit number is composed of tens and ones is a first-grade understanding, (**1.NBT.B.2**) as is subtracting multiples of ten from two-digit numbers.(**1.NBT.C.6**)

"Take from with start unknown"

This is another of the more difficultsubtypes that students engage with in Grade 2.Similar to "add to with start unknown" problems, it can be challenging to determine what the unknown is. In the example below, students are trying to determine how many cupcakes there were to begin with.

Grade 2, Module 4, Lesson 8: Application

At the school fair, 29 cupcakes were sold, and 19 were left over. How many cupcakes were brought to the fair?



Note: This problem is intended for guided practice to help students gain familiarity with the *take from with start unknown* problem type. The language of these problem types can be confusing to students. Guide students to see that when both parts are known, we add to find the total.

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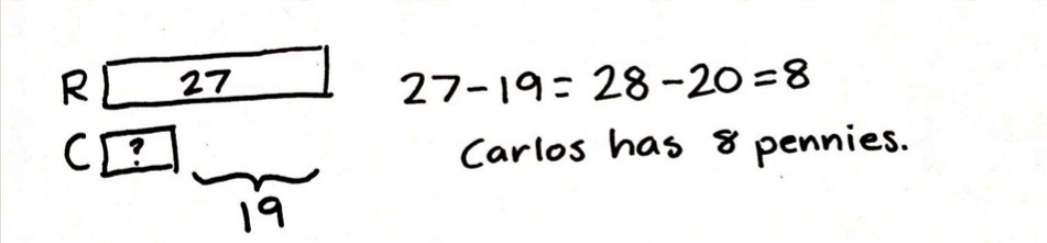
As we've discussed, in Grade 2 our focus should be on ensuring students can apply Level 3 methods—in which they convert to an easier problem—when solving any of the subtypes described in the table above. Asking students to show or explain their thinking in problems will help us understand what levels our students are using, and how we can support them in flexibly employing the advanced Level 3 strategies necessary for Grade 2 work. In the example above, both strategies demonstrate making an easier problem. In the first strategy, a student adds 1 to 19 to make a multiple of 10: adding 20 to 29 is easier than adding 19 to 29. Then the student remembers to subtract 1 from 49 since the 1 that was added to 19 was not taken from 29 (note that the arrows shown are simply a way of recording the student's mental calculations). In the second strategy, the student decomposes 29 in order to make a multiple of 10 with 19 (19 + 1 = 20). Then the student easily adds 20 to 28.

"Compare with smaller unknown"

Both this and the "compare with bigger unknown" subtype have variations that are particularly challenging because the words "more" and "less" in these kinds of problems suggest the wrong operation. In the example below, the word "more" suggests that the operation should be addition, but this is a subtraction problem (or unknown addend).

Grade 2, Module 7, Lesson 5: Application

Rita has 19 more pennies than Carlos. Rita has 27 pennies. How many pennies does Carlos have?



Note: In this problem, the context shifts to money. This leads into today's Concept Development, where students work with money data to solve word problems, and segues into problem solving with coins and bills in Topic B. The problem type is *compare with smaller unknown*, one of the more difficult problem types because *more* suggests the wrong operation. Guide students, as needed, to draw a tape diagram to solve.

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In the solution for this problem, the student uses a related problem to find the number of pennies Carlos has. The student adds 1 to 27 and adds 1 to 19; the difference of the related problem is the same.

Two-step word problems

In Grade 2, students engage with two-step word problems for all subtypes, but because they're still developing proficiency with the most challenging subtypes, it might be best to initially avoid these types in two-step problems. Also, most of the two-step problems students engage with should involve single-digit addends.[7] Since two-step problems are new to Grade 2, using smaller numbers allows students to focus on problem-solving rather than computation. Below are some variations of two-step problems.

Examples of two-step Grade 2 word problems

Two easy subtypes with the same operation, resulting in problems represented as, for example, $9 + 5 + 7 = \square$ or $16 - 8 - 5 = \square$ and perhaps by drawings showing these steps:

> Example for $9 + 5 + 7$: There were 9 blue balls and 5 red balls in the bag. Aki put in 7 more balls. How many balls are in the bag altogether?

Two easy subtypes with opposite operations, resulting in problems represented as, for example, $9 - 5 + 7 = \square$ or $16 + 8 - 5 = \square$ and perhaps by drawings showing these steps:

> Example for $9 - 5 + 7$: There were 9 carrots on the plate. The girls ate 5 carrots. Mother put 7 more carrots on the plate. How many carrots are there now?

One easy and one middle difficulty subtype:

> For example: Maria has 9 apples. Corey has 4 fewer apples than Maria. How many apples do they have in all?
>
> For example: The zoo had 7 cows and some horses in the big pen. There were 15 animals in the big pen. Then 4 more horses ran into the big pen. How many horses are there now?

Two middle difficulty subtypes:

> For example: There were 9 boys and some girls in the park. In all, 15 children were in the park. Then some more girls came. Now there are 14 girls in the park. How many more girls came to the park?

Source: Draft K-5 Progression on Counting and Cardinality and Operations and Algebraic Thinking p. 18

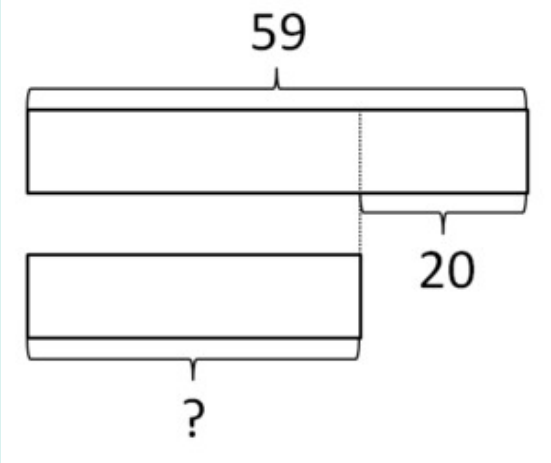
Here's another example, with sample solution:

A Pencil and a sticker

A pencil costs 59 cents, and a sticker costs 20 cents less. How much do a pencil and a sticker cost together?

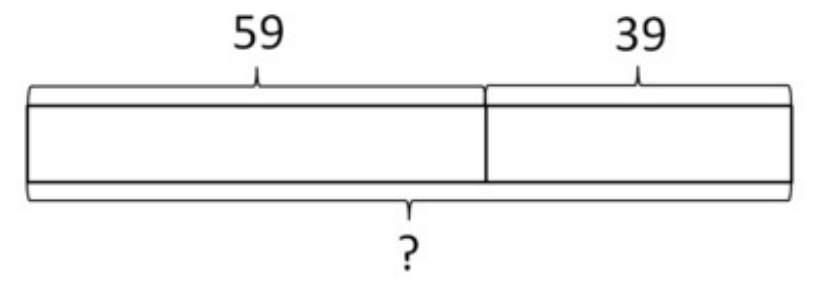
Solution

The pencil costs 59 cents, and the sticker costs 20 cents less than that:



So the sticker costs 59-20 = 39 cents.

The cost of the two together:



is 59 + 39 = 98 cents.

"A Pencil and a Sticker" by Illustrative Mathematics is licensed under CC BY 4.0.

You'll see that the numbers in this task are both two-digit. Though students do some work adding and subtracting with multiples of 10 in Grade 1, as in the first part, the second part involves Grade 2 skills and understandings.

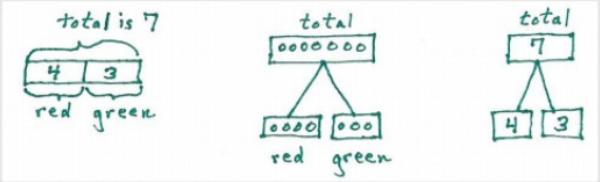
When solving two-step problems like this one, we can support students with scaffolded questions to help them create representative drawings:

- "If this section represents the cost of a pencil, how could we represent *the cost of a sticker*?"
- "If this section represents the cost of a pencil, how could we represent *the cost of a pencil and a sticker*?"

Addition and subtraction representations

Students use models (drawings and equations with a symbol for the unknown) to represent the problem situations described in Table 1. This helps them continue to develop meaning for addition and subtraction. As shown in in the two-step example above, tape diagrams are a good tool for compare problems. Tape diagrams can also be used for additive relationships; they just look a little different (See both Start Unknown examples.). Number bonds are also good tools for additive relationships.[8]

Additive relationship shown in tape, part-whole, and number-bond figures



The tape diagram shows the addends as the tapes and the total (indicated by a bracket) as a composition of those tapes. The part-whole diagram and number-bond diagram capture the composing-decomposing action to allow the representation of the total at the top and the addends at the bottom either as drawn quantities or as numbers.

Source: Draft K-5 Progression on Counting and Cardinality and Operations and Algebraic Thinking p. 16

As Grade 2 students become more advanced mathematicians, they begin replacing pictures of quantities with numbers in their diagrams (i.e., they move from drawing 7 circles to writing the number 7) They gradually increase their use of solution equations (i.e., the equation used to solve the problem) as representations over situation equations (i.e., the equation that represents the problem situation) and relate solution equations to diagrams.[9] For example, in the "add to start unknown" example from EngageNY shown above about Susan and her piggy bank (EngageNY Grade 2, Module 4, Lesson 2: Application), the student represents the situation as an unknown addend problem (e.g., ? + 30 = 57). However, the student finds the solution by subtracting: 57 – 30 = 27. Facility with writing related addition and subtraction equations supports this work. Similarly, understanding that the total can be on the right or left side of the equal sign is an important student experience; for example, students should see that ? + 30 = 57 and 57 = ? + 30 are equivalent ways of representing the situation above. Having the total on the left helps students build on the understanding introduced in Grade 1 that the equal sign does not always mean "makes" or "results in" but always means "is the same number as".[10]

Part 2: How does problem solving with addition and subtraction relate to other parts of Grade 2?

There are lots of connections among standards in Grade 2; if you think about the standards long enough, you'll probably start to see these relationships everywhere. In this section, we'll talk about the connection between the Operations & Algebraic Thinking standards (from clusters **2.OA.A** and **2.OA.B**) and the standards in the fourth cluster of the Measurement & Data (MD) domain. The standards in this MD cluster are supporting standards and can be used to support work with the OA standards.

2.MD.D | Represent and interpret data.

2.MD.D.9

Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.

2.MD.D.10

Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems* using information presented in a bar graph.

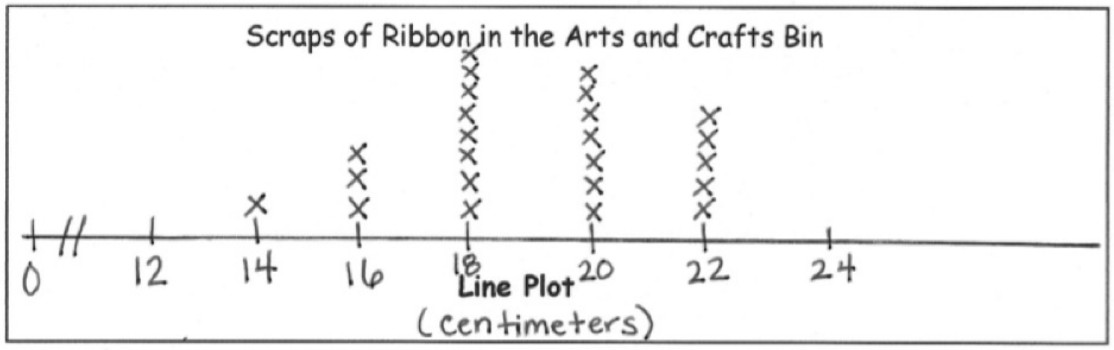
*See Glossary, Table 1

Data contexts are good places to reinforce problem-solving with addition and subtraction. In particular, they're well-suited for supporting the compare subtypes. Students should have experiences generating measurement data, recording the data on a line plot, and then problem-solving with the data. In creating a line plot, students relate the horizontal axis to a number line. Number line diagrams are important tools for addition and subtraction in Grade 2 and rely on understandings from other MD clusters. (**2.MD.A**, **2.MD.B**) In the following example, let's see how students generate measurement data, create a line plot, and make observations about the data. Note the questions about comparing data.

Grade 2, Module 7, Lesson 24: Problem Set

2.

Length of Ribbon Scraps (centimeters)	Number of Ribbon Scraps
14	I
16	III
18	⧸⧸⧸⧸ III
20	⧸⧸⧸⧸ II
22	⧸⧸⧸⧸



a. Describe the pattern you see in the line plot.

b. How many ribbons are 18 centimeters or longer?

c. How many ribbons are 16 centimeters or shorter?

d. Create your own comparison question related to the data.

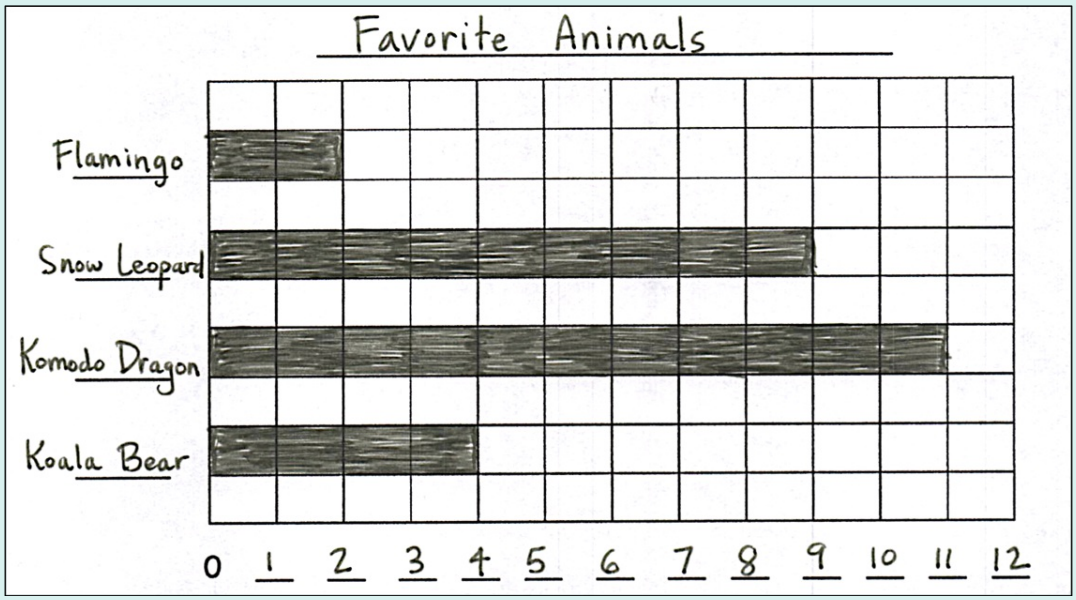
Grade 2, Module 7, Lesson 24 Available from engageny.org/resource/grade-2-mathematics-module-7-topic-f-lesson-24; accessed 2015-05-29. Copyright © 2015 Great Minds. UnboundEd is not affiliated with the copyright holder of this work.

Picture graphs and bar graphs are other opportunities to reinforce problem-solving. In fact, the language of the standard suggests as much: Solve simple put-together, take-apart, and compare problems using information presented in a bar graph(❑ **2.MD.D.10**) This standard even references Table 1. Though part of the standard addresses organizing and representing data, the focus of the standard is on interpreting data for the purposes of solving addition and subtraction problems. In the following example, students are asked to solve a simple compare problem.

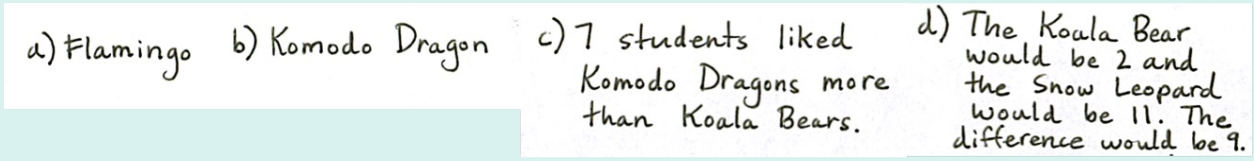
Grade 2, Module 7, Lesson 4: Application

Materials: (T) Favorite animals bar graph (Template)

After a trip to the zoo, Ms. Anderson's students voted on their favorite animals. Use the bar graph to answer the following questions.



a. Which animal got the fewest votes?
b. Which animal got the most votes?
c. How many more students liked komodo dragons than koala bears?
d. Later, two students changed their votes from koala bear to snow leopard. What was the difference between koala bears and snow leopards then?



Note: Project or draw this graph on the board. This problem reviews yesterday's Concept Development, where students learned to read and interpret a bar graph. It leads into today's lesson, where students create bar graphs to represent new data sets.

Grade 2, Module 7, Lesson 4Available from engageny.org/resource/grade-2-mathematics-module-7-topic-lesson-4; accessed 2015-05-29. Copyright © 2015 Great Minds. UnboundEd is not affiliated with the copyright holder of this work.

Another meaningful across-grade connection is to the second cluster of the MD domain: Relate addition and subtraction to length(■ **2.MD.B**) Experience with measuring and length provide access to measuring contexts for the different types of word problems; it also provides a conceptual basis for understanding the operations of addition and subtraction. In the example below, students engage with add to and compare situations with centimeters.

High Jump Competition

Presley jumped 36 cm on her first jump in the high jump competition. On her second jump, she jumped 45 cm.

a. How many total cm did Presley jump?

b. How many fewer cm did Presley jump on her first jump than her second jump?

c. Logan also jumped twice. The total of Logan's two jumps was 95 cm. How many more total cm did Logan jump than Presley?

"High Jump Competition" by Illustrative Mathematics is licensed under CC BY 4.0.

Part 3: Where do Operations & Algebraic Thinking come from, and where are they going?

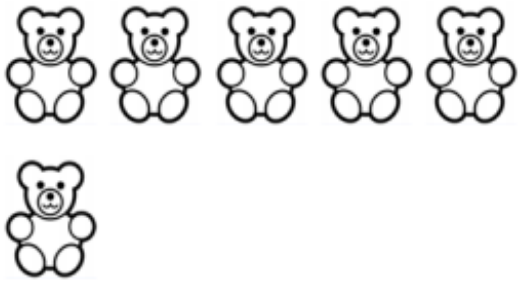
As you've no doubt surmised, the big story in K-2 is about addition and subtraction. Standards from both the OA and the Counting & Cardinality (CC) domains are significant in defining these skills and understandings. Let's take a closer look at this progression, specifically considering the strategies students learn, the fluencies they develop and problems they solve in each grade. This will allow us to think about how to explicitly connect new learning to previous understanding (and toward future learning), and about how to support students who may lack some prerequisite understandings.

Podcast clip: Importance of Coherence with Andrew Chen and Peter Coe (start 9:34, end 26:19)

Grades K-2: Developing meaning and strategies for addition and subtraction

As we've discussed, the Progressions describe three levels of problem representation and solution. Let's see how the Standards show a progression of these strategies across these three grades.

Kindergarten: "Counting all" within 10



6 – 1 = 5

(EngageNY Kindergarten Module 4 Lesson 20 from EngageNY.org of the New York State Education Department is licensed under CC BY-NC-SA 3.0.)

→ *In Kindergarten, students connect counting to cardinality (i.e., the number of objects in a group)*[11] *and count to answer "How many?" questions. (■ K.CC.B.5) In keeping with the emphasis on counting, students use the Level 1 strategy of "counting all" to add and subtract, often using drawings or physical objects. (■ K.OA.A.2) In this example, students are directed to cross out bears to represent the problem. Students would "count all" of the remaining bears to determine that there are 5.*

Grade 1: Introducing "Counting on" within 20

T: While we were cleaning up, some of the beans fell on the carpet. I picked most of them up, but I think I am still missing some. We had 7 beans in total, right?

S: Right!

T: Now, I have 5 beans. (Show beans to the class.)

T: How many am I missing? Talk with your partner to solve this.

S: (Discuss.)

T: Let's try to count on to check how many I'm missing.

S/T: Fivvvve (gesture to beans in hand), 6, 7. (Track on fingers.)

T: How many did we count on to get up to 7? (Keep fingers out to show the two that were used to track.)

S: Two!

T: So, how many beans am I missing?

S: Two beans!

(EngageNY Grade 1 Module 1 Lesson 16 from EngageNY.org of the New York State Education Department is licensed under CC BY-NC-SA 3.0.)

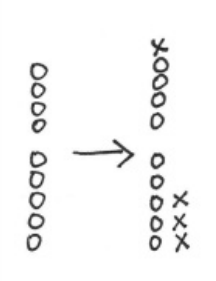
→ *In Grade 1, students learn to "count on" to solve addition and subtraction problems, in addition to Level 3 strategies. (1.OA.C.6) In this lesson plan, the teacher works with students to count on to subtract 7 - 5; doing this also requires understanding subtraction as an unknown-addend problem (1.OA.B.4) (solving 7 - 5 = ? is the same as solving 5 + ? = 7).*

Grade 2: Mental strategies

T: (Write 9 + 4 on the board.)

T: Let's draw to solve 9 + 4 using circles and Xs.

T: (Quickly draw and count aloud 9 circles in a 5-group column as seen in the first image.)



T: How many Xs will we add?

S: 4 Xs.

T: (Using the X symbol, complete the ten and draw the other 3 Xs to the right as seen in the second image.)

T: Did we make a ten?

S: Yes!

T: Our 9 + 4 is now a ten-plus fact. What fact can you see in the drawing?

S: 10 + 3 = 13.

T: 10 + 3 equals?

S: 13.

T: So, 9 + 4 equals?

S: 13. (Write the solution.)

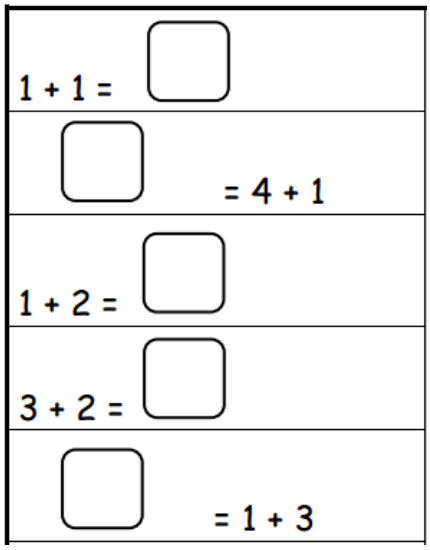
(EngageNY Grade 2 Module 1 Lesson 4 from EngageNY.org of the New York State Education Department is licensed under CC BY-NC-SA 3.0.)

→ *Students work to develop fluency with addition and subtraction within 20 in Grade 2. Students mentally apply the strategies developed in Grade 1 and know single-digit addition and subtraction facts from memory. (2.OA.C.3) Incorporating strategies based on place value (2.NBT.B.5) allows students to add and subtract larger numbers.*

Grades K-2: Fluency with addition and subtraction

From a conceptual bases of counting and strategies, students develop fluency with addition and subtraction across these three grades. The expected fluency in Kindergarten is to add and subtract within 5. (**K.OA.A.5**) In Grade 1, the fluency expectation for adding and subtracting is within 10, (**1.OA.C.6**) and in Grade 2, the fluency expectation is for adding single-digit numbers within 20 (and the associated subtractions). (**2.OA.B.2**) As with all fluencies, the expectation is that students are fluent by the end of the year. Also, fluency results from extensive work making meaning of addition and subtraction as opposed to rote memorization. The following are examples of fluency practice exercises from these grades.

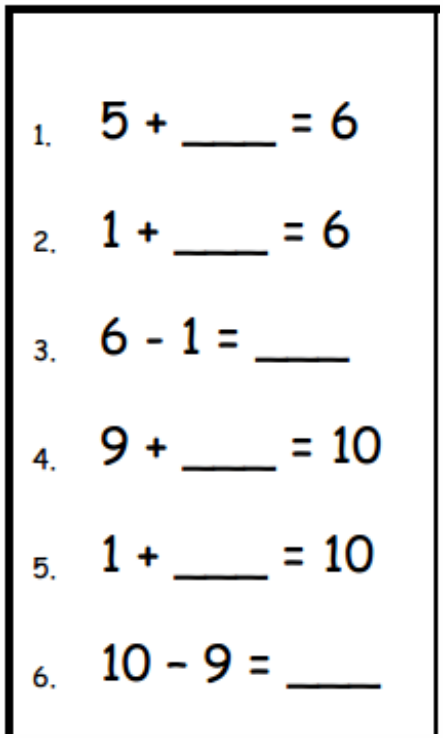
Kindergarten: Addition and subtraction within 5



(Fluency Practice, Kindergarten, Module 4, Lesson 29 from EngageNY.org of the New York State Education Department is licensed under CC BY-NC-SA 3.0.)

→ *Students work to develop fluency with addition and subtraction within 5 in Kindergarten. (K.OA.A.5) Note the varied placement of the missing sums, which supports flexible thinking about the equal sign.*

Grade 1: Addition and subtraction within 10



(Core Fluency Practice, Grade 1, Module 4, Lesson 23 from EngageNY.org of the New York State Education Department is licensed under CC BY-NC-SA 3.0.)

→ *Students work to develop fluency with addition and subtraction within 10 in Grade 1. Note the proximity of pairs like 5 + ____ = 6 and 1 + ____ = 6, which highlights the commutative property. (1.OA.B.3) The proximity of pairs like 1 + ____ = 6 and 6 - 1 = ____ highlights subtraction as an unknown addend problem (1.OA.B.4) and the relationship between addition and subtraction. (1.OA.C.6)*

Grade 2: Addition and subtraction within 20

1.	12 + 2 =
2.	14 + 5 =
3.	18 + 2 =
4.	11 + 7 =
5.	9 + 6 =
6.	7 + 8 =
7.	4 + 7 =
8.	13 - 6 =

(Core Fluency Practice, Grade 2, Module 5, Lesson 14 from EngageNY.org of the New York State Education Department is licensed under CC BY-NC-SA 3.0.)

→ *Students work to develop fluency with addition and subtraction within 20 in Grade 2. Students mentally apply the strategies developed in Grade 1 and know single-digit addition and subtraction facts from memory. (2.OA.C.3)*

Grades K-2: Application of addition and subtraction

The adapted table from the standards, as shown in Part 1 of this guide, outlines the progression of addition and subtraction word problems across Grades K-2. Let's take a closer look at these.

Kindergarten: Four simple subtypes within 10

Julia went to the beach and found 3 seashells. Her sister Megan found 2 seashells. Draw the seashells the girls found. How many did they find in all? Talk to your partner about how you know.

(Kindergarten Module 4 Lesson 1 from EngageNY.org of the New York State Education Department is licensed under CC BY-NC-SA 3.0.)

→ *In Kindergarten, students solve "add to" and "take from" problems with result unknown and "put together/take apart" problems with total unknown and both addends unknown within 10. This is a "put together" problem (Julia's shells and Megan's shells are put together) with the total unknown (we want to know how many there are in all).*

Grade 1: An initial look at all subtypes within 20 with concentration on 4 new subtypes

Toby collects shells. On Monday, he finds 6 shells. On Tuesday, he finds some more. Toby finds a total of 9 shells. How many shells does Toby find on Tuesday?

(Grade 1 Module 1 Lesson 30 from EngageNY.org of the New York State Education Department is licensed under CC BY-NC-SA 3.0.)

→ *In Grade 1, students are exposed to all 12 subtypes. They master solving "add to" and "take from" problems, this time with the change unknown, and "put together/take apart" problems with an unknown addend. They are also exposed to a variety of compare problems; full mastery of all compare subtypes is not expected until Grade 2. This is an "add to" problem (shells are added to Toby's initial six shells) with the change unknown (we need to figure out how many more are required to get to a total of 9 shells).*

Grade 2: All subtypes mastered within 100

Mei's frog leaped several centimeters. Then, it leaped 34 centimeters. In all, it leaped 50 centimeters. How far did Mei's frog leap at first? Draw a picture and write a number sentence to explain your thinking.

(Grade 2 Module 2 Lesson 9 from EngageNY.org of the New York State Education Department is licensed under CC BY-NC-SA 3.0.)

→ *In Grade 2, students master all subtypes, with an emphasis on work with the most challenging types: "add to" and "take from" problems again, this time with the start unknown; and certain more challenging compare problems. Students also solve two-step problems. This is an "add to with start unknown" problem (34 centimeters are added to several unknown centimeters to get 50 centimeters). Students need to figure out the number of centimeters the frog leaped initially.*

Suggestions for students who are below grade level

If, going into a unit on addition and subtraction, you know your students don't have a solid grasp of the ideas developed in Kindergarten and Grade 1, what can you do? It's not practical (or even desirable) to reteach everything students should have learned in these grades; there's plenty of new material in Grade 2, so the focus needs to be on grade-level standardsAt the same time, there are strategic ways of wrapping up "unfinished learning" from prior grades and honing essential fluencies within a unit on Grade 2 addition and subtraction. Here are a few ideas for adapting your instruction to bridge the gaps.

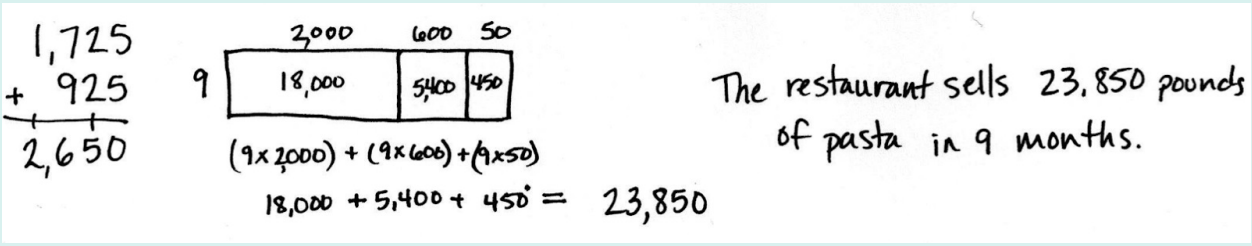
- If a significant number of students don't have **fluency with the count sequence**, you could ensure daily choral practice as a "warm up" or other activity during the day. (This activity, which includes several counting activities, might be helpful for this purpose.)
- If a significant number of students don't have a firm foundation with **the Level 1 strategy of "counting all," or the Level 2 strategy of "counting on,"** you could plan a couple of lessons on these strategies prior to lessons that focus on Level 3 strategies. (This lesson, along with others in EngageNY's Kindergarten Module 4, offers opportunities for students to add by counting concrete objects. This lesson, along with others in EngageNY's Grade 1 Module 1, Topic D, offers opportunities for students to "count on." To deliberately connect to Grade 2 standards, you might use the same examples to model or discuss other strategies, or apply the same ideas to larger numbers.)
- For students who lack **fluency with addition and subtraction within 5 or 10**, consider incorporating drills on these facts into your weekly routine. Strategy work, as named above, will be vital for fluency as well. Starting within 5 can then build naturally over time to fluency with larger numbers. (For an example of this type of activity, see the Core Fluency Practice in this lesson (Kindergarten) or this lesson (Grade 1).)
- If a significant number of students don't have a firm foundation with **the addition and subtraction problem subtypes associated with Kindergarten or Grade 1**, you could likewise plan a lesson or two on these before moving into more advanced Grade 2 subtypes. (This lesson, along with others in EngageNY's Kindergarten Module 4, offers opportunities for students to solve add to, take from, and put together/take apart problems with the result or total unknown. This lesson, and others in EngageNY's Grade 1 Module 4 Topics C and G, offer students the opportunity to solve unknown change and addend problems. To deliberately connect to Grade 2 standards, once students are comfortable with these examples you might adapt the same ones to have an unknown starting quantity. If you think that an entire lesson is too much, you could use these problems as "warm ups" each day leading into the more challenging grade level examples.)

Beyond K-2

Developing a solid understanding of addition and subtraction problem types in K-2 is important for developing understanding of multiplication and division. Students draw on their understanding of properties of addition, the relationship between addition and subtraction, and decomposition of numbers to build meaning for multiplication and division. Also, strong facility with addition and subtraction and related problem situations helps students to better distinguish multiplication and division from addition and subtraction. In the Grade 4 example below, students solve a two-step word problem involving addition and multiplication of whole numbers. By this point, students use the standard algorithm for multi-digit addition and subtraction and properties of operations (e.g., the distributive property, which is based on decomposing numbers) and area models to multiply.

EngageNY Grade 4, Module 3, Lesson 11: Problem Set

6. A restaurant sells 1,725 pounds of spaghetti and 925 pounds of linguini every month. After 9 months, how many pounds of pasta does the restaurant sell?



Grade 4, Module 3, Lesson 11 from EngageNY.org of the New York State Education Department is licensed under CC BY-NC-SA 3. 0.

Students need a firm understanding of which operations to apply and how to perform them. This comes from having many experiences developing meaning for the operations and using strategies rooted in conceptual understanding.

Though the standards in the OA domain concentrate on experience with whole numbers, the concepts, properties, methods, and representations extend beyond to other number systems. Students extend their engagement with the 12 subtypes to include addition and subtraction of fractions and decimals in Grades 3-5.[12] Ultimately students will bring a host of problem-solving strategies to bear when solving problems involving all four operations and the full scope of the rational number system.7.NS.B.3

Congratulations on making it through this Grade 2 guide for Operations & Algebraic Thinking with Connections to DataWe hope it provided a clear and detailed explanation of what these standards say, and how Operations & Algebraic Thinking relates to other mathematical concepts in Grade 2, as well as Kindergarten and Grade 1.

The content here can be helpful when writing or evaluating a scope and sequence, a unit plan or lesson plan. And you can find additional resources here:

Student Achievement Partners: Focus in Grade 2

Draft K-5 Progression on Counting and Cardinality and Operations and Algebraic Thinking

EngageNY: Grade 2 Module 1 Materials

EngageNY: Grade 2 Module 4 Materials

Illustrative Mathematics Grade 2 Tasks

Endnotes

[1] The Common Core State Standards for Mathematics (CCSSM) are organized into major, additional, and supporting clusters in the Focus by Grade Level documents from Student Achievement Partners.

[2] See the K-8 Publishers' Criteria for the Common Core State Standards for Mathematics, p. 18

[3] For a more thorough discussion of these strategies you can consult our Grade 1 guide to Operations & Algebraic Thinking.

[4] Table 1, from the Glossary of the Common Core Standards for Mathematics

[5] Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking, p. 9.

[6] ibid. p. 9.

[7] ibid. p. 18.

[8] ibid. p. 16.

[9] ibid. p. 17.

[10] ibid. p. 9.

[11] For students coming to second grade without the prerequisite understandings of the count sequence and of cardinality, please see our Content Guide on Counting & Cardinality in Kindergarten.

[12] ibid. p. 19.