

Linear Equations: A Guide to High School Algebra I Standards

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A-CED.A[★] | Create equations that describe numbers or relationships.

N-Q.A[★] | Reason quantitatively and use units to solve problems.

■ A.REI.A | Understand solving equations as a process of reasoning and explain the reasoning.

■ A.REI.B | Solve equations and inequalities in one variable.

○ A.REI.C | Solve systems of equations.

■ A.REI.D | Represent and solve equations and inequalities graphically.

Welcome to the UnboundEd Mathematics Guide series! These guides are designed to explain what new, high standards for mathematics say about what students should learn in each grade, and what they mean for curriculum and instruction. This guide, the first for Algebra I, includes three parts. The first part gives a “tour” of the standards involving linear equations using freely available online resources that you can use or adapt for your class. The second part shows how linear equations relate to other concepts in Algebra I. And the third part explains where linear equations are situated in the progression of learning in Grades K-8.

Part 1: What do the standards say?

Algebra I is full of interesting and useful ideas, many of which are important for success in higher math and in a range of careers. So why begin this series with linear equations? First off, the standards about linear equations represent the “peak” of a long progression of learning, which, for many students, began with formally writing and solving simple equations in Grade 6. So linear equations are a good way for new high school students to “ease into” the challenges of their first year, with material that relates strongly to what they already know. At the same time, many of the ideas they’ll come to understand through linear equations—solving equations as a process of reasoning, modeling with equations, graphing equations—will be used again and again in Algebra I and II, so it’s good that students encounter these early.

Another reason to start with linear equations is that four of the six clusters covered in this guide (A-CED.A, A-REI.A, A-REI.B, and A-REI.D) are also recognized as part of the “major work” of Algebra I by the PARCC Model Content Frameworks, meaning they deserve a significant amount of attention over the course of the school year.¹ Prioritizing major work within the year ensures that those standards are sure to get the attention they deserve.

The high school standards are organized into five “categories,” and within each category are a number of “domains.” The standards involving linear equations are spread across two domains in the Algebra category—“Creating Equations” (A-CED) and “Reasoning with Equations and Inequalities” (A-REI). We also have a domain from the Number & Quantity category (“Quantities,” or N-Q), which will impact work throughout the year. Before we get started with the content in these standards, though, let’s pause and take a look at the standards themselves. As you read, think about:

- Where do these standards emphasize conceptual understanding of important ideas?
- How are students expected to use equations to represent and model real-world situations?

A-CED.A★ | Create equations that describe numbers or relationships.

A-CED.A.1

Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

A-CED.A.2

Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A-CED.A.3

Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

A-CED.A.4

Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm’s law $V = IR$ to highlight resistance R .*

N-Q.A★ | Reason quantitatively and use units to solve problems.

N-Q.A.1

Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

N-Q.A.2

Define appropriate quantities for the purpose of descriptive modeling.

N-Q.A.3

Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

A.REI.A | Understand solving equations as a process of reasoning and explain the reasoning.**A-REI.A.1**

Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.REI.B | Solve equations and inequalities in one variable.**A-REI.B.3**

Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

A.REI.C | Solve systems of equations.**A-REI.C.5**

Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A-REI.C.6

Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

A.REI.D | Represent and solve equations and inequalities graphically.

A-REI.D.10

Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A-REI.D.12

Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

★ Indicates modeling standards. You can read more about modeling standards and their importance for high school work [here](#).

The order of the standards doesn't indicate the order in which they have to be taught. Standards are only a set of expectations for what students should know and be able to do by the end of each year; they don't prescribe an exact sequence or curriculum. You could choose to teach these standards one by one, or in an integrated fashion. For example, you might choose to begin your year with solving one-variable equations and inequalities, focusing first on conceptual understanding of the process (A-REI.A.1), and then developing skill with more complicated equations (A-REI.B.3). From there, you might focus on modeling with equations in one variable (A-CED.A.1 and standards from N-Q.A). After that, you could move on to two-variable equations (A-CED.A.2, A-CED.A.3, plus the N-Q.A standards again), graphing (A-REI.D.10, and A-REI.D.12), and systems of equations (A-REI.C.5, A-REI.C.6). Standard A-CED.A.4, which involves rearranging formulas, might be taught throughout the year whenever multivariable equations are presented. (This is just one example, however; the high school standards can be sequenced in a variety of ways that result in a coherent experience for students.²)

Modeling with linear equations

As you may have noticed while reading the standards above, the algebra standards are concerned not just with mathematical concepts and procedures, but with applications of those ideas in real-world contexts. Students shouldn't just be able to solve equations; they should be modeling with those equations in order to solve problems.³ At the same time, not all modeling is quite the same. Are students solving word problems with relatively "thin" contexts, where quantities and variables are already defined for them, and which are always described by equations of the same form? ("Billy makes \$15 per hour and has already saved \$200. How many hours does he need to work in order to save \$650 total?") Or are they engaging with richer problems that require a greater degree of interpretation, involve a sophisticated understanding of units and quantities, and where students have to devise and test various models in order to find the one that works best? The Standards are pushing us to consider problems that match what students will find in college and career contexts—where, very often, they'll have to define the parameters of a problem before they can devise a mathematical way to solve it. Below, we'll see a few examples that you might use to help students reach this higher bar.

Understanding quantities

One way the Standards set expectations for modeling is through the use of quantities and units; this is focus of the N-Q domain. (In general, the N-Q standards will not be addressed or assessed in isolation, but represent the set of high school-level skills involved in all work with modeling.) For our purposes, a **quantity** is a measurement of an attribute. Quantities therefore have both a numerical part and a unit—5 cm, for example, or 30 mph.⁴ Basic units can be informal (a desk might be about five pencil lengths wide) or standard (the same desk might also be 24 inches wide). Basic units can be divided into smaller units (as 1 cm can be divided into 10 mm) or compiled into larger units (as 3 feet forms 1 yard). Basic units can be also be derived from other, simpler units. For example, area can be measured in cm^2 , a unit derived from two lengths, and speed can be measured in km/h , a unit derived from two different attributes.

Along with having a general awareness of units, students should also understand how units can “point the way” to a solution, and determine whether a given solution is reasonable or not. For example, in a problem about fuel efficiency, students might realize that the solution will be in miles per gallon (mpg), leading to an insight about how the equation that models the situation must be structured (proportionally). They should also be able to use units appropriately in graphs—knowing, for example, that the best scale for an axis representing average annual income is \$1,000 (rather than \$1).N-Q.A.1

Students should also be able to identify and highlight quantities of interest. In a profit equation, for example, they should realize they’re trying to find gross cost and assign this quantity a variable, such as g . This means they’ll need to see problems that don’t specify a variable or direct them how to solve. On occasion, they should also be able to introduce improvised units. A business application might require them to create a unit called “manager-hours” and distinguish this from another unit for “technician-hours.”N-Q.A.2

Lastly, students should understand that most real-world problems require some judgment as to the level of accuracy appropriate to a particular context.N-Q.A.3 The dosage of a certain drug might need to be reported to the milligram (or an even finer unit of mass), while the population of a country might only need to be accurate to the nearest hundred thousand people. During a lesson, this might mean frequently asking students to choose and explain, in a practical way, an appropriate unit for the situation at hand.

Types of units in high school

It’s important that high school students are exposed to a variety of units, and that the units move beyond what was studied in middle school. The introduction to the Number & Quantity standards sum up what students have learned, and where they should be going: “In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area and volume. In high school, students encounter a wider variety of units in modeling, e.g. acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages.”⁵

Creating one-variable equations

As students approach creating equations in one variable,A-CED.A.1 it’s important for them to consider a range of real-world contexts. In the past, it may have been sufficient for high school students to model simple situations by an equation $ax + b = c$, where a , b , and c are integers. (The problem about Billy’s wages in the introduction to this section is one such example.) Now, however, this type of work takes place in middle school, where students should be writing and solving equations involving rational numbers in Grades 7 and 8. (

■ **7.EE.B.4**, ■ **8.EE.C.7**) This doesn’t mean that simple equations are a bad starting point for high school work—just that they can’t be the end goal. Students should move past these simple cases and into more authentic contexts as quickly as possible. This task is a fair example of the type of situation students should encounter in Algebra I.

Buying a Car

Suppose a friend tells you she paid a total of \$16,368 for a car, and you'd like to know the car's list price (the price before taxes) so that you can compare prices at various dealers. Find the list price of the car if your friend bought the car in:

- Arizona, where the sales tax is 6.6%.
- New York, where the sales tax is 8.25%.
- A state where the sales tax is r .

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This task isn't overly convoluted, and the solution process isn't even that difficult. But it's not a “textbook” word problem, either; students are working with realistic numbers (not just integers) and real-world ideas like price and tax. Note also the way the task presumes fluency with percents and rational number arithmetic, both understandings from Grade 7. (■ **7.RP.A.3**, ■ **7.NS.A.3**) Just as the modeling process suggests, the task also requires students to define a quantity (list price) as a variable as part of the solution process. N-Q.A.2 For example, in Part (a), students could write the equation $16,368 = 1.066p$, where p is the list price. This aspect of the problem—having to define a variable, rather than having it defined already by the problem—is a small but significant detail in helping students become proficient problem solvers.

Creating two-variable equations

As your students progress from equations in one variable to those in two variables, A-CED.A.2 defining quantities and making assumptions continue to be important skills. Take, for example, this task, which requires students to do both.

Clea on an Escalator

It takes Clea 60 seconds to walk down an escalator when it is not operating, and only 24 seconds to walk down the escalator when it is operating. How many seconds does it take Clea to ride down the operating escalator when she just stands on it?

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This is an interesting and challenging example of modeling a real life situation with mathematics. As they read and interpret this task, students need to assume that both Clea and the elevator are traveling at constant rates; they’ll also need to introduce variables to represent these rates—one for Clea and one for the elevator. This mirrors the decision-making process that students will often use in real life, as they have to parse and identify the meaningful elements of various situations. During the initial stages of solving this problem, you might help them through this process by asking questions like, “What assumptions do we need to make?”, “What variables matter here, and what are those variables?”, and “What does that mean?” With enough practice, questions like these should become second-nature when students first encounter a problem.

Representing constraints

Students should also be able to represent constraints in situations by equations. [A-CED.A.3](#) It’s often the case in the real world that there is only one constraint given on a situation (as opposed to two or more, which might be modeled with a system of equations). Creating and using a single constraint is therefore a valuable skill for students to have in order to be ready for college and career. This task is an example of a situation with a single constraint involving more sophisticated, derived units (kilograms per hectare). As you read, notice how this task isn’t about solving systems of equations—no solution is required here, and the equations that students write aren’t explicitly identified as a system. Rather, the emphasis is solely on writing the two constraints using unit reasoning (another connection to the N-Q standards). By avoiding any mention of systems, the task allows students to focus on the often challenging task of modeling.

Growing Coffee

The coffee variety Arabica yields about 750 kg of coffee beans per hectare, while Robusta yields about 1200 kg per hectare (reference). Suppose that a plantation has a hectares of Arabica and r hectares of Robusta.

- Write an equation relating a and r if the plantation yields 1,000,000 kg of coffee.
- On August 14, 2003, the world market price of coffee was \$1.42 per kg of Arabica and \$0.73 per kg of Robusta. Write an equation relating a and r if the plantation produces coffee worth \$1,000,000.

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In this task, students use “unit reasoning” to recognize that, in Part (a), the total is given in kilograms, yet the variables are given in terms of hectares. Since the rates are also provided as kilogram/hectare, students should see that they must multiply the rate (e.g., 750 kg/hectare) by the variable in order to yield an answer in kilograms. From a teaching perspective, this makes Part (a) quite substantial. If you think your students might struggle to write the first equation, consider asking questions about the units involved, helping students to notice that they’re different. You might also have students make a table with three columns—hectares, kilograms of Arabica and kilograms of Robusta—to see the relationships and start forming expressions and equations.

Solving equations and inequalities

Solving equations and inequalities has been, and continues to be, a staple of Algebra I. This continues the type of thinking students began using to solve equations in Grade 6-8. In high school, this thinking culminates by viewing solving an equation as a series of logical arguments leading to a solution. Rather than focusing solely on the procedures involved, the Standards want students to understand *why* the process of solving an equation works, and what each step in that process means. A-REI.A.1 If students only learn the steps, there is a much greater risk that they will eventually forget these or even make up invalid steps.⁶ What’s more, when students do later work with equations that may have extraneous solutions (radical and rational equations), understanding the reasoning involved will be necessary to correctly solve these. Let’s take a look at a few lessons that introduce students to the idea of “solving as reasoning.”

Reasoning with equations and inequalities

One of the primary design features of the Standards is that students to develop conceptual understanding prior to procedure. Though both are important, procedure is grounded, more memorable, and thus more replicable when constructed on a foundation of conceptual understanding. This is particularly true in the solving of equations and inequalities. When students are first able to recognize and communicate why various approaches are taken to solving, their procedural skill builds more quickly and lasts longer than learning procedures alone. For example, it is all too easy to forget that, when solving $\frac{3}{2}x - 5 = 3x + 4$ we first multiply everything by 2, then move the 8 over, then move the $3x$ over, and finally divide by 3. Such instruction ensures that students see solving as unique and different each time they do it. However, when we reason that we need to isolate the variable on one side of the equation and gather constants on the other side of the equation, and that we do so using properties of equality—including additive and multiplicative inverses in this case—we develop a reliable and replicable method for solving that will yield to fluency shortly, and which will also be extensible to other types of equations and inequalities later on.

Equations can be true or false, so a naked equation like “ $2x - 3 = 5$ ” doesn’t tell us much. Asking a student to solve this equation means, “Find the value of x that will make this equation true.” Solving an equation is a process of reasoning that leads from one equation to another equation with the same solutions—and on to another with still the same solutions, until the solution to every equation in the

chain becomes clear. Students should be able to articulate the notion of equations with the same solution, and should be able to justify each step in a solution process using the properties of operations. The two exercises below are examples.

Algebra I, Module 1, Lesson 12: Exercise 1

- a. Use the commutative property to write an equation that has the same solution set as

$$\begin{aligned}x^2 - 3x + 4 &= (x + 7)(x - 12)(5) \\ -3x + x^2 + 4 &= (x + 7)(5)(x - 12)\end{aligned}$$

- b. Use the associative property to write an equation that has the same solution set as

$$\begin{aligned}x^2 - 3x + 4 &= (x + 7)(x - 12)(5) \\ (x^2 - 3x) + 4 &= ((x + 7)(x - 12))(5)\end{aligned}$$

- c. Does this reasoning apply to the distributive property as well?

Yes, it does apply to the distributive property.

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Algebra I, Module 1, Lesson 12: Exercise 4

- Does it matter which step happens first? Let's see what happens with the following example.

Do a quick count-off, or separate the class into quadrants. Give groups their starting points. Have each group designate a presenter so the whole class can see the results.

Exercise 4

Consider the equation $3x + 4 = 8x - 16$. Solve for x using the given starting point.

Group 1	Group 2	Group 3	Group 4
Subtract $3x$ from both sides	Subtract 4 from both sides	Subtract $8x$ from both sides	Add 16 to both sides
$3x + 4 - 3x = 8x - 16 - 3x$ $4 = 5x - 16$ $4 + 16 = 5x - 16 + 16$ $20 = 5x$ $\frac{20}{5} = \frac{5x}{5}$ $4 = x$ (4)	$3x + 4 - 4 = 8x - 16 - 4$ $3x = 8x - 20$ $3x - 8x = 8x - 20 - 8x$ $-5x = -20$ $\frac{-5x}{-5} = \frac{-20}{-5}$ $x = 4$ (4)	$3x + 4 - 8x = 8x - 16 - 8x$ $-5x + 4 = -16$ $-5x + 4 - 4 = -16 - 4$ $-5x = -20$ $\frac{-5x}{-5} = \frac{-20}{-5}$ $x = 4$ (4)	$3x + 4 + 16 = 8x - 16 + 16$ $3x + 20 = 8x$ $3x + 20 - 3x = 8x - 3x$ $20 = 5x$ $\frac{20}{5} = \frac{5x}{5}$ $4 = x$ (4)

- Therefore, according to this exercise, does it matter which step happens first? Explain why or why not.
No, because the properties of equality produce equivalent expressions, no matter the order in which they happen.
- How does one know “how much” to add/subtract/multiply/divide? What is the goal of using the properties, and how do they allow equations to be solved?

Encourage students to verbalize their strategies to the class and to question each other’s reasoning and question the precision of each other’s description of their reasoning. From middle school, students recall that the goal is to isolate the variable by making s and s. Add/subtract numbers to make the zeros, and multiply/divide numbers to make the s. The properties say any numbers will work, which is true, but with the s and s goal in mind, equations can be solved very efficiently.

- The ability to pick the most efficient solution method comes with practice.

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As you think about how to use these tasks in the classroom, consider having students show multiple ways to rewrite equations in Parts (a) and (b). Then you can ask them to explain why the different equations are all equivalent in (a) and (b). From there, have students write various examples in Part (c) to demonstrate understanding, and explain the properties of equality they use in each solution process in Exercise 4.

All of the focusing on reasoning doesn’t mean that the standards don’t call for procedural skill in solving—only that concepts and procedures should work together. In fact, students should be proficient in solving all manner of equations and inequalities in one

variable, A-REI.B.3 including formulas with no numerical solution; using problems involving this sort of equation is one way that work with equations in high school is an extension of work in Grade 8. The task below is an illustration of this type of work. (Notice the connection to the A-CED domain, as students are required to highlight certain quantities from each equation. A-CED.A.4)

Rewriting Equations

In each of the equations below, rewrite the equation, solving for the indicated variable.

- a. If F denotes a temperature in degrees Fahrenheit and C is the same temperature measured in degrees Celsius, then F and C are related by the equation

$$F = \frac{9}{5}C + 32$$

Rewrite this expression to solve for C in terms of F .

- b. The surface area S of a sphere of radius r is given by

$$S = 4\pi r^2$$

Solve for r in terms of S .

- c. The height h of a diver over the water is modeled by the equation

$$h = -5t^2 + 8t + 3$$

where h denotes the height of the diver over the water (in meters) and t is time measured in seconds. Rewrite this equation, finding t in terms of h .

- d. A bacteria population P is modeled by the equation

$$P = P_0 10^{kt}$$

where time t is measured in hours, k is a positive constant, and P_0 is the bacteria population at the beginning of the experiment. Rewrite this equation to find t in terms of P .

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This is a nice example for a couple of reasons. First, there are multiple types of equations at play, allowing students to dive deeper into the various properties of equality (including the zero-product property). Second, the task is begging for a conversation about “Why would we want to do any of these?” noticing, for example, that we often have a Fahrenheit reading, so solving for C allows us to quickly find the Celsius reading. (If you use this task with students, note that it may be appropriate to omit any part which they don’t have the mathematical tools to solve. Part (d) in particular is likely out of reach for many Algebra I students.)

Systems of equations

Systems of equations are another staple of Algebra I, although just as with solving equations and inequalities, there is a focus on reasoning. Solving a system of equations can be understood as a series of propositions and conclusions, not unlike the process of solving an equation. If two “sides” of an equation are equal, and two “sides” of another equation are equal, then the sum of each pair of sides will also be equal. From this, we get what’s often called the “elimination” method of solving a system; the standards ask students to understand deeply why this method works. A-REI.C.5 More specifically, the standards prioritize the reasoning behind the method.⁷ As a result of being able to articulate why the method works, students should see that there are actually many ways to transform one or both equations in a system to make the elimination method work, not just one “right way.” The task below provides a way for students to explore this idea.

Equations in Two Unknowns

Lisa is working with the system of equations $x + 2y = 7$ and $2x - 5y = 5$. She multiplies the first equation by 2 and then subtracts the second equation to find $9y = 9$, telling her that $y = 1$. Lisa then finds that $x = 5$. Thinking about this procedure, Lisa wonders

There are lots of ways I could go about solving this problem. I could add 5 times the first equation and twice the second or I could multiply the first equation by -2 and add the second. I seem to find that there is only one solution to the two equations but I wonder if I will get the same solution if I use a different method?

- What is the answer to Lisa's question? Explain.
- Does the answer to (a) change if we have a system of two equations in two unknowns with no solutions? What if there are infinitely many solutions?

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The power of this task is that it helps highlight why the elimination method of solving systems of linear equations—a procedure often taken for granted—really works. In Part (a), students should be able to reason/show that multiplying and combining two equations works in all cases where there is a solution; and, in fact, it works to show when there are infinitely many or zero solutions, too. The key idea here is reversibility: whatever steps are taken to find the variable through multiplying and combining, can be undone to return to the original equations without extraneous solutions. Questions like, “Are there even more ways to solve than those given in the problem?” and “How could you ‘rebuild’ the original equation once you’ve transformed it?” might help them articulate this idea. Students might also benefit from comparing the graphs of an equation and several of its multiplied forms to understand why the elimination method produces a system with the same solutions as the original.

Once students have the concept down, they can begin solving using multiple methods. With practice, the goal is for them to quickly and accurately solve systems, purposefully choosing from the methods learned in Grade 8 and high school. A-REI.C.6 As they build proficiency, they should focus on more complex situations, quantities, and units, thereby strengthening their skills in the N-Q domain. They should also see problems that require them to both write and solve systems based on real-world contexts; many problems of this sort are instances of representing constraints. A-CED.A.3 The task below shows such a task.

Cash Box

Nola was selling tickets at the high school dance. At the end of the evening, she picked up the cash box and noticed a dollar lying on the floor next to it. She said,

I wonder whether the dollar belongs inside the cash box or not.

The price of tickets for the dance was 1 ticket for \$5 (for individuals) or 2 tickets for \$8 (for couples). She looked inside the cash box and found \$200 and ticket stubs for the 47 students in attendance. Does the dollar belong inside the cash box or not?

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This problem is interesting in that students aren’t just being asked to write and solve a system, but to determine which quantity (\$200 or \$201) best fits the situation. To help students get started, you might ask them to restate what the problem is asking, and how they’ll know when they’ve found the right answer. From there, you might have them start to define some of the quantities involved. An interesting discussion might come from a comparison of different ways it’s possible to represent the problem; for example, some might call the number of couples c , while others might use the same variable to represent the total number of dancers who arrived as couples, leading to two different sets of equations.

Graphing equations and inequalities

Our final pair of standards in the A-REI domain involve graphing equations and inequalities, although the emphasis isn’t on being able to fluently graph equations. Instead, the standards begin with understanding the meaning of an equation’s graph. A-REI.D.10 Notice that this standard doesn’t mention using the form of an equation (such as slope-intercept form, point-slope form, etc.) to graph, but on articulating the connection between solutions and points on a line. The goal of starting this way is to help students interpret and use the graphs they create, rather than relying on the graph as an end unto itself. The exercises below show the types of conceptual work that might precede instruction in procedures. In particular, notice the student statement at the end of the second exercise.

Algebra I, Module 1, Lesson 20: Exercise 1

1.

- a. Circle all the ordered pairs (x, y) that are solutions to the equation $4x - y = 10$.

$(3, 2)$	$(2, 3)$	$(-1, -14)$	$(0, 0)$	$(1, 6)$
$(5, 10)$	$(0, -10)$	$(3, 4)$	$(6, 0)$	$(4, -1)$

- A. How did you decide whether or not an ordered pair was a solution to the equation?

Most students will explain that they substituted and checked to see whether or not the equation was true.

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Algebra I, Module 1, Lesson 20: Exercise 2

2.

- a. Discover as many additional solutions to the equation $4x - y = 10$ as possible. Consider the best way to organize all the solutions you have found. Be prepared to share the strategies you used to find your solutions.

Sample answers: $(-5, -30); (-2, -18); (2, -2); (4, 6)$

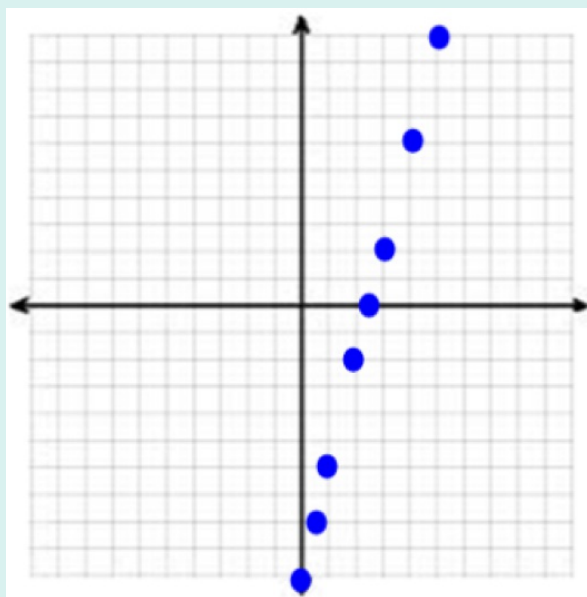
- b. Now, find five more solutions where one or more variables are negative numbers or non-integer values. Be prepared to share the strategies you used to find your solutions.

Sample answers: $(-4, -26); (-3, -22); (\frac{1}{2}, -8); (\frac{3}{2}, -4); (\frac{5}{2}, 0)$

- c. How many ordered pairs will be in the solution set of the equation $4x - y = 10$?

Infinitely many.

- d. Create a visual representation of the solution set by plotting each solution as a point (x, y) in the coordinate plane.



- e. Why does it make sense to represent the solution to the equation $4x - y = 10$ as a line in the coordinate plane?

Drawing the line is the only way to include all possible solutions. It would be impossible to plot every point that is a solution to the equation since there are infinitely many solutions.

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The same idea applies to solving systems of inequalities. A-REI.D.12 There are plenty of conventions of graphing that deserve explicit teaching (types of lines, shading, etc.) but students should be able to explain what these features represent. Again, when students learn rules or procedures without understanding why they exist or what they represent, they may forget or misapply these conventions. The exercise below and follow-up questions show how this might occur in an introductory lesson.

Algebra I, Module 1, Lesson 21: Exercise 2

2.

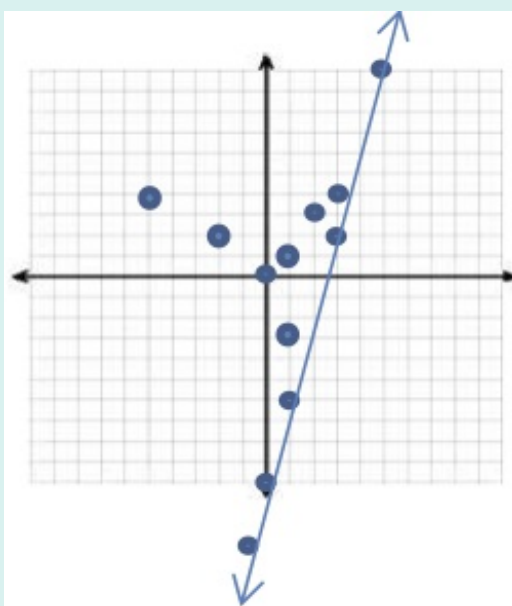
- A. Discover as many additional solutions to the inequality $4x - y \leq 10$ as possible. Organize solutions by plotting each as a point (x, y) in the coordinate plane. Be prepared to share the strategies used to find the solutions.

(There are an infinite number of correct answers, as well as an infinite number of incorrect answers. Some sample correct answers are shown.)

$(1, 1), (1, -3), (-2, 2), (-5, 4)$

- B. Graph the line $y = 4x - 10$. What do you notice about the solutions to the inequality $4x - y \leq 10$ and the graph of the line $y = 4x - 10$?

All of the points are either on the line, to the left of the line, or above the line.



- C. Solve the inequality for y .

$$y \geq 4x - 10$$

- D. Complete the following sentence.

If an ordered pair is a solution to $4x - y \leq 10$, then it will be located ___ on, above, or to the left of ___ the line $y = 4x - 10$

Explain how you arrived at your conclusion.

I observed that all the points were on one side of the line, and then I tested some points on the other side of the line and found that none of the points I tested from that side of the line were solutions to the inequality.

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In both of these tasks, students are asked to explain the meaning of points on the graph and to make conjectures about how the graph will begin to look if the manual process of plotting solution points continues. Approaching graphs in this way helps students see them for what they really are—a representations of the solutions of an equation or inequality—and ensures that they’re prepared to work with the graphs of functions later on.

The role of Mathematical Practices

The standards don’t just include knowledge and skills; they also recognize the need for students to engage in certain important practices of mathematical thinking and communication. These “mathematical practices” have their own set of standards, which contain the same basic objectives for Grades K-12.⁸ (The idea is that students should cultivate the same habits of mind in increasingly sophisticated ways over the years.) But rather than being “just another thing” for teachers to incorporate into their classes, the practices are ways to help students arrive at the deep conceptual understandings required in each grade. The table below contains a few examples of how the practices might help students understand and work with linear equations in Algebra I.

Opportunities for Mathematical Practices:	Teacher actions:
Linear equations offer rich opportunities for modeling. In particular, the use of linear equations to model a variety of situations, such as business applications involving profit, can help students understand the nature of linear equations themselves, namely their constant rate of change. (MP.4)	As students model real-world situations, encourage them to use many of the same tools employed for understanding unit rate situations in the middle grades: tables, tape diagrams, and other representations. Ask them to identify the rate of change in these situations. Then, as they use these representations to write equations, ask them how the rate of change is represented in the equation.
Linear equations have their own unique structure for students to expand and understand. (MP.7) In particular, $y = mx + b$ is not the only structure students will know for the linear equations. As they come to know others (such as $ax + by = c$), they will learn to make use of the various types depending on the situation.	For example, students can view the standard form of a linear equation as a great tool in quickly finding both the x and the y -intercepts of its graph. You might begin a lesson on rewriting linear equations by linking to what students learned earlier when slope was the focus. This would allow for rich discussions about the appropriateness for each form depending on the situation and needs.
Solving linear equations and systems linear equations requires attending to precision. (MP.6) Students should use precise reasoning in their solutions, understanding the assumptions and limits involved in that reasoning.	For example, students should realize that the process of solving an equation isn’t just a series of algebraic moves, but represents a chain of reasoning that sheds light on the original equation.A-REI.A.1 They also discover situations in which the solution process reveals no solution at all, or an infinite number of solutions.

Podcast clip: Importance of the Mathematical Practices with Andrew Chen and Peter Coe (start 30:33, end 43:39)

Part 2: How do linear equations relate to linear functions?

So far, we've been talking exclusively about equations in one and two variables. Equations are related to (yet distinct from) another important idea in Algebra I—functions. Functions have roots in Grade 8, where students should begin with an informal understanding of functions as input-output relationships. (■ **8.F.A.1**) Then, in Algebra I, they should expand on this idea further, seeing a function as an assignment of elements from one set (the domain) to the elements of another set (the range) and using function notation (i.e., " $f(x)$ "). F-IF.A.1 From there, they interpret the graphs of functions in terms of situations they represent, F-IF.B.5 and graph various types of functions. F-IF.C.7 Ultimately, students should be able to build functions that model real-world phenomena (the focus of the F-BF domain) and contrast linear, quadratic and exponential growth patterns in modeling contexts (the focus of the F-LE domain).

While functions are distinct from equations in many respects, learning about functions represents an extension of the thinking they did with two-variable equations. So without diving too deeply into functions (which will need another guide to explain thoroughly), let's take a look at how students might use some of their background in equations as a point of launch for the study of functions.

As we saw above, students should understand that the solutions to an equation in two variables are the pairs of values that make the equation true; in the graph of an equation, they can see all of these solutions on the coordinate plane. A-REI.D.10 As they explored this idea, they might have worked with simple equations like $x + y = 5$ and used substitution to find solutions. In these cases, they might have substituted a value for x and solved to find y , or substituted a value for y and solved to find x . This type of work is valuable for understanding functions, because it gives students an understanding that a graph represents pairs of related values, and a set of skills related to graphing.

Fast forward to a unit on functions: After exposure to the definition of a function, students might begin understanding the concept further by graphing an equation like $y = 5 - x$ using a table of values. In this case, one variable (x) can be seen as the input to a function, and the other as the output. This is one sense of a function: a rule that takes each x and assigns a value, $f(x)$. From there, they can generalize the concept of a function to any arbitrary set, ultimately coming to understand a function more broadly as a correspondence between two sets. They can also learn the conventions of function notation, work with the ideas of domain and range, and can begin to quickly and accurately graph linear functions.

This task is an example of the type of introductory work with functions that help students advance from thinking in terms of equations and arithmetic rules to thinking about functions more generally.

The Customers

A certain business keeps a database of information about its customers.

- A. Let C be the rule which assigns to each customer shown in the table his or her home phone number. Is C a function? Explain your reasoning.

Customer Name	Home Phone Number
Heather Baker	3105100091
Mike London	3105200256
Sue Green	3234132598
Bruce Swift	3234132598
Michelle Metz	2138061124

- B. Let P be the rule which assigns to each phone number in the table above, the customer name(s) associated with it. Is P a function? Explain your reasoning.
- C. Explain why a business would want to use a person's social security number as a way to identify a particular customer instead of their phone number.

"The Customers" by Illustrative Mathematics is licensed under CC BY 4.0.

Tasks like these are nice because they encourage students to think of a function as relationship between two sets, the elements of which aren't necessarily numerical. If you're thinking about how you might introduce functions, one way might be to begin with a situation students already know, like graphing the two-variable equation mentioned above; framing the pieces of that problem in terms of function language (for example, explaining how the equation can be seen as a function rule, and how the two columns in the table of values represent the domain and range of the function); and then moving on to a problem like this one. ([This sequence of lessons from Engage NY](#) does something similar, although the functions involved aren't all linear. If you're wondering about other ways to make the jump from equations to functions, it's worth a look.)

Using function thinking to reason with equations

After some work with functions and their graphs, students can return to equations in one variable and examine them through a graphical lens. A-REI.D.11 While often taught quickly, the graphical approach to solving equations is actually very nuanced, and deserves close attention. The first thing for students to understand is that an equation consists of two expressions, one on either side of the equal sign, and each of these can be thought of as defining a function. For every given input, the expressions can be evaluated to obtain an output. (For example, the equation $2x + 5 = x + 9$ contains two expressions, and each can be evaluated at many values of x .) Solving an equation, then, can be viewed as finding a single input value that yields the same output value for the functions defined by either side of the

equation.⁹ In our earlier example, if $f(x) = 2x + 5$ and $g(x) = x + 9$, then $f(4) = g(4)$ because both $f(4) = 13$ and $g(4) = 13$. They can then relate these to the appearance of the graph of these functions: The point where the two lines intersect represents the solution to the equation. Students now have a new way to approximate solutions to equations. This sequence shows how this idea might be introduced in a lesson.

Algebra I, Module 3, Lesson 16: Opening Exercise 1

Opening Exercises

1. Solve for in the following equation: $|x + 2| - 3 = 0.5x + 1$.

$$|x + 2| = 0.5x + 4$$

$$\begin{aligned}x + 2 &= 0.5x + 4 \\0.5x &= 2 \\x &= 4\end{aligned}$$

or

$$\begin{aligned}x + 2 &= -(0.5x + 4) \\1.5x &= -6 \\x &= -4\end{aligned}$$

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Algebra I, Module 3, Lesson 16: Opening Exercise 2

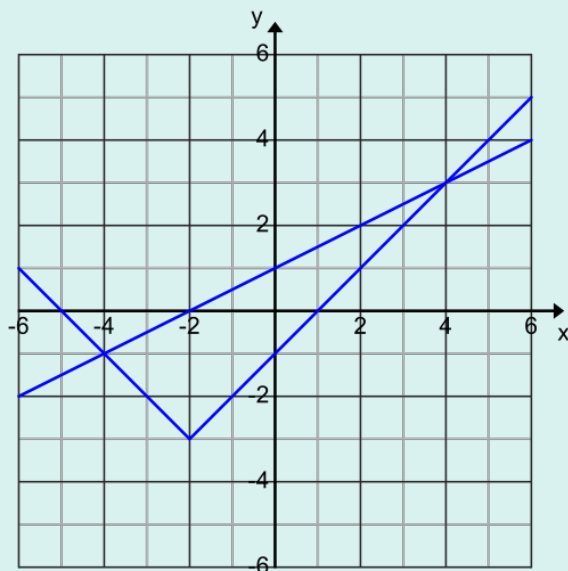
2. Now, let $f(x) = |x + 2| - 3$ and $g(x) = 0.5x + 1$. When does $f(x) = g(x)$?

A. Graph $y = f(x)$ and $y = g(x)$ on the same set of axes.

B. When does $f(x) = g(x)$?

What is the visual significance of the points where $f(x) = g(x)$?

$f(x) = g(x)$ when $x = 4$ and when $x = -4$: $(4, 3)$ and $(-4, 1)$. The points where $f(x) = g(x)$ are the intersections of the graphs of f and g .



C. Is each intersection point (x, y) an element of the graph f and an element of the graph of g ? In other words, do the functions f and g really have the same value when $x = 4$? What about when $x = -4$?

Yes. You can determine this by substituting $x = -4$ and $x = 4$ into both f and g .

$$\begin{aligned} -1 &= |-4 + 2| - 3 \\ -1 &= 2 - 3 \\ -1 &= -1 \end{aligned}$$

$$\begin{aligned} -1 &= 0.5(-4) + 1 \\ -1 &= -1 \end{aligned}$$

$$\begin{aligned} 3 &= |4 + 2| - 3 \\ 3 &= 6 - 3 \\ 3 &= 3 \end{aligned}$$

$$\begin{aligned} 3 &= 0.5(4) + 1 \\ 3 &= 3 \end{aligned}$$

$f(x)$ and $g(x)$ have the same value at each x .

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This sequence is a nice template for helping students think about why a graphical method of solving equations might be useful. First, have them solve an equation “the old-fashioned way,” using a series of algebraic moves. (This particular example involves an absolute value equation, which requires two algebraic solutions and is therefore ripe for a graphical solution. But a simpler example involving two linear functions, such as the one mentioned above, would work just as well. In fact, some students might benefit from starting with a simple case that allows them to focus on the underlying concepts of a new method, rather than the computational difficulty of a

complicated equation.) Then discuss how the expressions on either side of that equation could be used to define two functions, say f and g . Some questions to prompt student thinking might be:

- Could you evaluate these functions for the same input value?
- Will the same input value always yield the same output value in both functions?
- When would the output value of these functions be the same?
- If both functions had the same input value and the same output value at the same time, what would that mean?

Then have them graph f and g . Ask them to relate their solution to the graph of their two functions. What does the point where the two lines intersect mean? How can they explain the connection between their two methods of solution?

Part 3: Where do linear equations come from, and where are they going?

There's quite a bit of content packed into the standards involving linear equations, but they're intended to be part of a progression of learning that starts in the elementary years and picks up steam in Grades 6-8. If your students have been following a strong, standards-aligned program for several years, you might be reaping the benefits of their experience as you read this. But high school teachers often find themselves in a challenging position, as it's not always clear how students' prior learning aligns to high school expectations. Sometimes students also have real gaps in their learning that need to be filled before high school work can begin. In this section, we'll consider a few questions you might have as you're planning. First, what exactly were students supposed to learn in their elementary and middle school years that supports their work with linear equations in Algebra I? How do I leverage what they already know to make high school concepts accessible? And if some of my students are behind, how can I meet their needs without sacrificing focus on high school content?

Podcast clip: Importance of Coherence with Andrew Chen and Peter Coe (start 9:34, end 26:19)

Where do linear equations come from?

One of the most distinctive features of the standards is the way they begin preparing students for algebra in the elementary grades. In Grade 1, for example, students learn to solve problems involving addition and subtraction by considering the various meanings of those operations, and they write equations to represent situations involving an unknown quantity. (■ 1.OA.A.1) They also discover the relationship between addition and subtraction (■ 1.OA.B.4) and begin using properties of addition. (■ 1.OA.B.3) Another fundamental takeaway from Grade 1 is understanding the meaning of the equal sign. (■ 1.OA.D.7) A similar process occurs with multiplication and division in Grade 3: Students learn the meanings of the operations, (■ 3.OA.A.1 ■ 3.OA.A.2) write equations using a variable to represent an unknown, (■ 3.OA.A.3) and explore properties of operations. (■ 3.OA.B.5) Students should therefore begin Grade 6 with a firm understanding of operations and properties, and a wealth of experience representing problems with variables and equations.

Grades 6-8: Solving equations and inequalities

Following in-depth work with algebraic expressions in Grade 6, students should be ready to understand the process of solving an equation or inequality ("Which values from a specified set, if any, make the equation or inequality true?") (■ 6.EE.B.5) and solve simple problems using equations. (■ 6.EE.B.7) In Grade 7, the problems grow more complex, and so do the equations students use to solve them. (■ 7.EE.B.4) And by Grade 8, students should be solving equations that require them to transform the expressions on either side. They also see equations with no solution and infinite solutions, and learn to recognize these cases by their final form. (■ 8.EE.C.7) The problems below illustrate how expectations advance from grade to grade.

Grade 6:

Think about what these equations mean, and find their solutions. Write a sentence explaining how you know your solution is correct.

- a. $x + 6 = 10$
- b. $1000 - y = 400$
- c. $100 = m + 99$
- d. $0.99 = 1 - t$
- e. $3a = 300$
- f. $\frac{1}{2}p = 8$
- g. $10 = 0.1w$
- h. $1 = 50b$

Source: [Illustrative Mathematics, "Make Use of Structure"](#)

→ This problem, which might be used with introductory lessons on solving equations in Grade 6, has students find the solution to each equation without explicitly performing any operations. Rather, they're asked to consider the meaning of an equation (a statement of equality) and find a value that would make each equation true. These ideas will be the basis for solving equations throughout the middle grades and into high school.

Grade 7:

The taxi fare in Gotham City is \$2.40 for the first $\frac{1}{2}$ mile and additional mileage charged at the rate \$0.20 for each additional 0.1 mile. You plan to give the driver a \$2 tip. How many miles can you ride for \$10?

Source: [Illustrative Mathematics, "Gotham City Taxis"](#)

→ This is a fair example of the type of problem students might be expected to solve in Grade 7. At this point, students should be able to write and solve an equation such as $2x + 4.40 = 10$. This problem is similar to the last one in that it doesn't result in a complicated equation, but still requires students to carefully interpret the situation.

Grade 8:

What value of x would make the following linear equation true?






$$\frac{1}{2}(4x+6)-2=-(5x+9)$$






Begin by transforming both sides of the equation into a simpler form.

Source: [EngageNY Grade 8, Module 4, Lesson 6](#)

→ This problem involves rational coefficients and requires students to apply properties of operations in order to transform the equation. At this point, the order of computations isn't self-evident, either; students will have to strategize about which operations to perform first.

Grades 6-8: Graphing equations

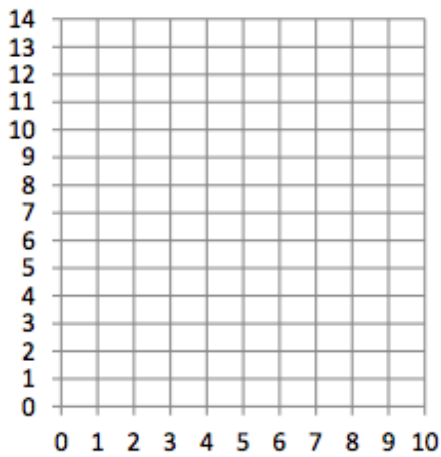
Students encounter the coordinate plane for the first time in Grade 5, where they graph simple relationships in the first quadrant. ( **5.G.A.1**,  **5.G.A.2**) In Grade 6, students learn about ratios and proportional relationships, and represent simple situations with equations in two variables. As they do, they graph these on the coordinate plane. ( **6.EE.C.9**) They also expand their knowledge of the number system to include all of the rational numbers (positive and negative), allowing them to plot ordered pairs in all four quadrants of the coordinate plane. ( **6.NS.C.6**) Work with coordinates continues in Grade 7, where students explore proportional relationships through their graphs. ( **7.RP.A.2**)

In Grade 8, students use graphs extensively: to understand the concept of slope, ( **8.EE.B.5**,  **8.EE.B.6**) to distinguish linear and nonlinear functions, ( **8.F.A.3**) and to compare functions represented in different forms. ( **8.F.A.2**) They also begin to work with systems of equations, using graphs to understand the meaning of the solution to a system. ( **8.EE.C.8**) The problems below illustrate the progression of ideas from one grade to the next.

Grades 6-7: Graphs of simple equations and proportional relationships

Using the ratio provided, create a table that shows that money received is proportional to the number of candy bars sold. Plot the points in your table on the grid.

x Candy Bars Sold	y Money Received (\$)
2	3

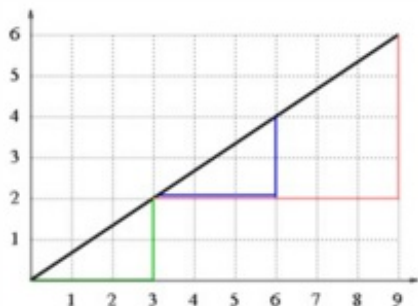


Source: [EngageNY Grade 7, Module 1, Lesson 5](#)

→ In this Grade 7 problem, students are asked to graph a proportional relationship on the coordinate plane. From this, they can determine what the unit rate in the relationship is and use the graph to solve follow-up problems.

Grade 8: Understanding slope

Eva, Carl and Maria are computing the slope between pairs of points on the line shown below. Eva finds the slope between the points (0,0) and (3,2). Carl finds the slope between the points (3,2) and (6,4). Maria finds the slope between the points (3,2) and (9,6). They have each drawn a triangle to help with their calculations (shown below).



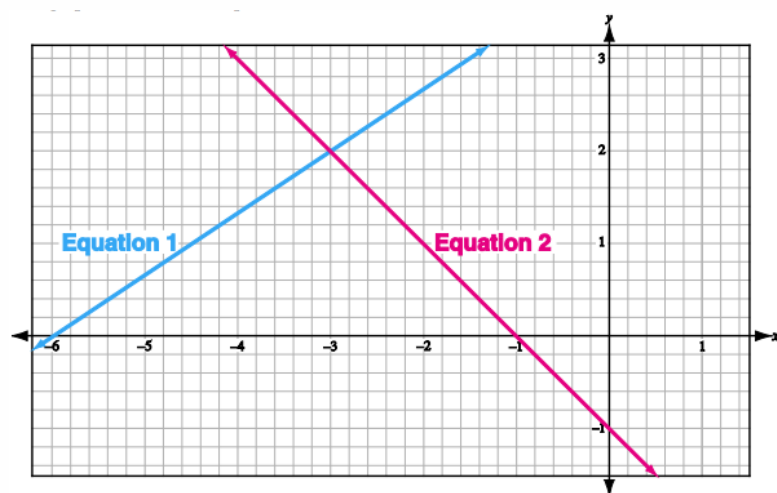
- Which student has drawn which triangle? Finish the slope calculation for each student. How can the differences in the x - and y -values be interpreted geometrically in the pictures they have drawn?
- Consider any two points (x_1, y_1) and (x_2, y_2) on the line shown above. Draw a triangle like the triangles drawn by Eva, Carl, and Maria. What is the slope between these two points? Why should this slope be the same as the slopes calculated by the three students?

Source: [Illustrative Mathematics, "Slopes Between Points on a Line"](#)

→ In Grade 8, students formalize the concept of slope, and discover that the slope between any two points on a line is constant. The purpose of this problem is to help students see the relationship between slope and similarity by examining several "slope triangles." Then they're ready to generalize the idea of constant slope from the particular line shown in this problem to any line.

Grade 8: Systems of equations

The graphs of two linear equations are shown below.



Check the box next to each correct statement.

- ☐ $(-3, 2)$ is a solution for equation 1.
- ☐ $(-2, 1)$ is a solution for equation 1.
- ☐ $(-3, 2)$ is a solution for equation 2.
- ☐ $(0, -6)$ is a solution for equation 2.
- ☐ $(-3, 2)$ is a solution to both equations.
- ☐ $(-3, 2)$ is the only solution to both equations.
- ☐ $(-3, 2)$ is the value of x for the solution to the two equations.

Source: [Student Achievement Partners, Simultaneous Linear Equations Mini-Assessment](#)

→ This problem shows how students in Grade 8 are using and interpreting graphs as tools for understanding systems of equations. In order to evaluate each statement, they'll need to know what the points on each line mean, and in particular what the intersection point means for the system.

Suggestions for students who are behind

If, going into a unit on linear equations, you know your students don't have a solid grasp of the ideas named above (or haven't encountered them at all), what can you do? It's not practical (or even desirable) to re-teach everything students should have learned in Grades 6-8; there's plenty of new material in Algebra I, so the focus needs to be on grade-level standards. At the same time, there are strategic ways of wrapping up "unfinished learning" from prior grades and honing essential fluencies within a unit on linear equations. Here are a few ideas for adapting your instruction to bridge the gaps.

If a significant number of students have limited experience **writing linear equations**—including interpreting the essential elements of a problem and representing unknown quantities with variables—you could plan a lesson or two on these ideas before starting high school-level work with modeling. ([This Grade 7 lesson](#) contains some problems that might be helpful in planning. You could also consult [this collection of Illustrative Mathematics tasks](#) for sample problems.) And if you think an entire lesson is too much, but your students

could still use some review, you could use 2-3 of these problems as “warm-ups” to start your first few modeling lessons, or “just-in-time” examples in the course of instruction.

If a significant number of students have limited experiences **solving equations** and/or don’t understand the **meaning of the solution to an equation**, you could likewise plan a lesson on these fundamentals before attending to high school-level equations. ([This Grade 6 lesson](#) could provide a starting point, although [this Grade 7 lesson](#) might be even better.) Again, if you think an entire lesson is too much, you could also use a few problems as warm-ups to start your lessons on reasoning, or “just-in-time” examples in the course of instruction.

If a significant number of students have limited experience with the **coordinate plane** or aren’t yet able to **accurately plot points**, you could plan a lesson or series of warm-ups on this idea. ([This Grade 8 lesson](#) on graphing linear equations, for example, could be adapted to focus on plotting points while also supporting high school work with graphing.)

The high school standards focus on solving equations as a process of reasoning (rather than the computational difficulty of problems). This presents an opportunity to review skills from the middle grades while addressing high school standards at the same time. If, for example, a significant number of students struggle **performing important computations**—including operations with rational numbers and using the distributive property—you could plan problems involving these skills into your initial lessons on solving equations. ([This Grade 7 lesson](#) offers a good example of how this might look, and [this Grade 8 lesson](#) has some more complicated equations you might use.)

For students who lack **fluency with important operations**—including operations with fractions, decimals and applications of properties—consider incorporating drills involving these operations into your weekly routine. (Drills are more common in the elementary and middle grades, but with a little convincing, even high school students will engage in these sorts of activities.) These could be as simple as a set of ten problems on a certain focus skill (page 22 of [this Grade 6 lesson](#) has an example) or as involved as a timed “sprint” exercise (see pages 26-29 of [this Grade 6 lesson](#) for how this might look).

Where are linear equations going?

So where is work with linear equations taking students? It’s an important question, because ideas that come up again and again in Algebra I and Algebra II—solving equations as a process of reasoning, modeling real-world situations with equations, systems of equations, and the features of graphs—deserve plenty of time and attention up front. That way, students won’t have to learn these ideas over again in new contexts, but will be able to extend and adapt what they already know. For example, students should see the solving quadratic equations by factoring as similar to solving linear equations, since both cases involve using properties of operations to create a sequence of equations with the same solutions. Even though the work on the page might look different, the underlying ideas are the same.

Later in Algebra I, students will encounter quadratic equations, and will first solve simple cases by factoring. (Later on, they’ll be able to use factoring to explain completing the square, and by extension, the Quadratic Formula). While solving a quadratic equation by factoring might look different from solving a linear equation, it involves the same principles. Using established properties of operations, students start from one equation and build a chain of reasoning that allows them to draw a conclusion about the solution(s) to their initial equation.^{A-REI.A.1} In Algebra II, students will apply this same reasoning to solving rational and radical equations.^{A-REI.A.2} In these cases, students need to have a firm grasp of the reasoning used in the solution process in order to understand how extraneous solutions might arise.

Algebra I: Reasoning with linear equations

Describe the property used to convert the equation from one line to the next:

$$x(1 - x) + 2x - 4 = 8x - 24 - x^2$$

$$x - x^2 + 2x - 4 = 8x - 24 - x^2$$

$$x + 2x - 4 = 8x - 24$$

$$3x - 4 = 8x - 24$$

$$3x + 20 = 8x$$

$$20 = 5x$$

- Why are we sure that the initial equation, $x(1 - x) + 2x - 4 = 8x - 24 - x^2$, and the final equation, $20 = 5x$, have the same solution set?
- What is the common solution set to all these equations?

Source: [EngageNY Algebra I, Module 1, Lesson 13](#)

→ This problem, similar to others we've seen in this guide, asks students to justify each step of solving an equation with an operation or property. Parts (a) and (b) emphasize that each step yields a related equation with the same solution set as the first.

Algebra I: Reasoning with quadratic equations

The Zero Product Property (ZPP) states that if the product of two numbers is zero, then at least one of the numbers is zero. In symbols, if $ab = 0$, then $a = 0$ or $b = 0$. We can use this property when we solve equations where a product is 0. For each equation below, use the ZPP to find all solutions. Explain each step in your reasoning.

- $x(13 - 4x) = 0$
- $7(y + 12) = 0$
- $(x - 19)(x + 3) = 0$
- $(y - 6)(3z - 4) = 0$

Source: [Illustrative Mathematics, "Zero Product Property 3"](#)

→ This problem asks students to explain how using the Zero Product Property can lead to a solution in certain equations. Notice how the focus is still on using properties to justify a conclusion. Viewed in this way, quadratics are remarkably similar to linear equations.

Algebra II: Reasoning with radical equations

- a. Solve the following two equations by isolating the radical on one side and squaring both sides:

i. $\sqrt{2x+1}-5=-2$

ii. $\sqrt{2x+1}+5=2$

Be sure to check your solutions.

- b. If we raise both sides of an equation a power, we sometimes obtain an equation that has more solutions than the original one. (Sometimes the extra solutions are called extraneous solutions.) Which of the following equations result in extraneous solutions when you raise both sides to the indicated power? Explain.

iii. $\sqrt{x}=5$, square both sides

iv. $\sqrt{x}=-5$, square both sides

v. $\sqrt[3]{x}=5$, cube both sides

vi. $\sqrt[3]{x}=-5$, cube both sides

- c. Create a square root equation similar to the one in part (a) that has an extraneous solution. Show the algebraic steps you would follow to look for a solution, and indicate where the extraneous solution arises.

Source: [Illustrative Mathematics, "Radical Equations"](#)

→ This problem requires students to think about a few simple equations that yield extraneous solutions, realizing that squaring both sides of an equation is not a "reversible" operation. By seeing how extraneous solutions are a by-product of the reasoning they use to solve, they become less of an "add-on" to the solution process.

Students also continue modeling with equations throughout Algebra I and Algebra II. While linear equations can describe a large number of situations, students will eventually encounter instances where they just won't suffice.^{A-CED.A.1} In these cases, they'll have to determine the characteristics of the situation and select the best available model, whether exponential or quadratic. At the same time, the same skills they used to model with linear equations will apply: determining appropriate units,^{N-Q.A.1} highlighting quantities of interest,^{N-Q.A.2} and deciding whether a situation is best modeled by a single equation or a system.^{A-CED.A.2}

Algebra I: Creating linear equations

A checking account is set up with an initial balance of \$4,800, and \$400 is removed from the account each month for rent (no other transactions occur on the account).

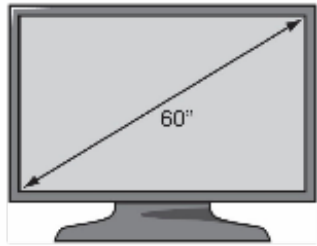
- a. Write an equation whose solution is the number of months, m , it takes for the account balance to reach \$2,000.
- b. Make a plot of the balance after m months for $m = 1, 3, 5, 7, 9, 11$ and indicate on the plot the solution to your equation in part (a).

Source: [Illustrative Mathematics](#), “Paying the Rent”

→ This problem, typical for Algebra I and similar to others in this guide, involves modeling a simple linear relationship. (Other tasks might neglect to specify a quantity of interest and a variable, requiring students to determine those for themselves.) Students will apply a similar process of modeling in situations involving other types of relationships, and should ultimately be able to determine the best model (linear, quadratic, or exponential) for a given scenario.

Algebra I: Creating quadratic equations

TV screens are measured on the diagonal. The diagram below shows a 60-inch TV screen.



For this TV, the ratio of height to width is 0.618.

- What is the area of this TV screen?
- For any TV screen with a height-to-width ratio of 0.618, write a function $A(d)$ that gives the screen area A , in square inches, when the length of the diagonal is d (measured in inches).
- The cost of making any size TV screen is $0.037A(d) + 0.5d + 10$. What is the largest screen size that can be built for \$75?

Source: Student Achievement Partners, ["Quadratic Equations Mini-Assessment"](#)

→ This challenging problem, which integrates a number of skills from algebra and geometry, results in a quadratic equation. In part (a), notice how students are required to determine quantities of interest (i.e. length and width) for themselves, and represent these with variables using the information given.

Algebra II: Creating exponential equations

The following table contains U.S. population data for the two most recent census years, 2000 and 2010.

Census Year	U.S. Population (in millions)
2000	281.4
2010	308.7

- a. Steve thinks the data should be modeled by a linear function.

What is the average rate of change in population per year according to this data?

Write a formula for a linear function, L , to estimate the population t years since the year 2000.

- b. Phillip thinks the data should be modeled by an exponential function.

What is the growth rate of the population per year according to this data?

Write a formula for an exponential function, E , to estimate the population t years since the year 2000.

- c. Who has the correct model? How do you know?

Source: [EngageNY, Algebra II, Module 3, Lesson 27](#)

→ This problem requires students to create both a linear and an exponential model and think about how well they might represent the situation. At this point, they're still using the same habits of modeling (for example, determining appropriate units for growth rates) as with linear equations.

In Algebra I, students are mostly concerned about “2 by 2” systems of linear equations. In Algebra II, they’ll move past these and see linear-quadratic systems^{A-REI.C.7} and even “3 by 3” linear systems. If students have a firm foundation with graphing and systems—they can explain the meaning of the point of intersection between two graphs, and can explain how the common algebraic methods of solving systems work—this little twist won’t be a problem. In fact, they’ll be able to apply similar methods they used with linear systems in these new situations.

Algebra I: “2 by 2” linear systems

Without graphing, construct a system of two linear equations where $(-2, 3)$ is a solution to the first equation but not to the second equation, and where $(5, -2)$ is a solution to your system.

After you have created your system of equations, graph your system. Explain how your graph shows that your system satisfies the required conditions.

Source: [Illustrative Mathematics, “Find a System”](#)

→ This problem goes along with other “2 by 2” situations we’ve seen so far and requires students to understand how the solution to an equation and the solution to a system are related but distinct. Students who can deal with this type of problem are well-prepared to take on more challenging cases in Algebra II (see below).

Algebra II: Linear-quadratic systems

Sketch the circle with equation $x^2 + y^2 = 1$ and the line with equation $y = 2x - 1$ on the same pair of axes.

- a. There is one solution to the pair of equations

$$\begin{aligned}x^2 + y^2 &= 1 \\ y &= 2x - 1\end{aligned}$$

that is clearly identifiable from the sketch. What is it? Verify that it is a solution.

- b. Find all the solutions to this pair of equations.

Source: [Illustrative Mathematics, “The Circle and the Line”](#)

→ Once students understand the meaning of graphs and the graphing approach to solving 2 by 2 linear systems, a problem like this isn’t too much of a stretch. The algebraic methods with which they might solve part (b) are the same as those used in Algebra I.

Algebra II/Precalculus: “3 by 3” linear systems

Dillon is designing a card game where different colored cards are assigned point values. Kryshna is trying to find the value of each colored card. Dillon gives him the following hints. If I have 3 green cards, 1 yellow card, and 2 blue cards in my hand, my total is 9. If I discard 1 blue card, my total changes to 7. If I have 1 card of each color (green, yellow, and blue), my cards total 1.

- a. Write a system of equations for each hand of cards if x is the value of green cards, y is the value of yellow cards, and z is the value of blue cards.
- b. Solve the system using any method you choose.

Source: [EngageNY Precalculus and Advanced Topics, Module 2, Lesson 15](#)

→ This problem results in a “3 by 3” system of linear equations, which students can choose to solve algebraically or, in precalculus, using matrices. In order to apply algebraic methods to a situation this complicated, students will need to rely on a firm understanding of why those methods work—something best achieved when working with much simpler systems in Algebra I.

What are all of these connections saying? That by taking the time to teach a few key ideas in-depth, there is a payoff for students (and teachers) down the line. Different types of equations shouldn’t mean an entirely new style of thinking and a new set of skills to learn, but should be an exercise in adapting the same concepts and habits students have used before.

If you’ve just finished this entire guide, congratulations! Hopefully it’s been informative, and you can return to it as a reference when planning lessons, creating units, or evaluating instructional materials. For more guides in this series, please visit our [Enhance Instruction page](#). For more ideas of how you might use these guides in your daily practice, please visit our [Frequently Asked Questions page](#). And if you’re interested in learning more about linear equations in Algebra I, don’t forget these resources:

[PARCC Model Content Frameworks](#)

[Draft High School Progression on Algebra](#)

[EngageNY: Algebra I Materials](#)

[Illustrative Mathematics Algebra Tasks](#)

Endnotes

[1] Though many people know PARCC as an organization dedicated to assessment, much of PARCC’s early work focused on helping educators interpret and implement the Standards. Its Model Content Frameworks were among those efforts: They describe the mathematics that students learn at each grade level and in each high school course, and the relative amount of time and attention that standards should be given in a grade or course. Many teachers, regardless of their state’s affiliation with PARCC, have found these documents helpful. You can find them [here](#). In this guide, major clusters and standards are denoted by a green square (e.g. A-REI.A.1).

[2] If you’re interested in seeing another high-quality Algebra I sequence, you might want to check out the [Illustrative Mathematics Algebra I course blueprint](#).

[3] The [Draft High School Progression on Modeling](#) one of a series of “Progressions” documents by the authors of the Standards, puts it this way: “In the standards, modeling means using mathematics or statistics to describe (i.e., model) a real-world situation and deduce additional information about the situation by mathematical or statistical computation and analysis.” If you’re interested in learning more about the entire collection of high school modeling standards, this document is a good resource.

[4] The [Draft 6-7 Progression on Ratios and Proportional Relationships](#) also mentions: “Quantities may be discrete, e.g., 4 apples, or continuous, e.g., 4 inches. They may be measurements of physical attributes such as length, area, volume, weight or other measurable attributes such as income.”

[5] Read the entire introduction to the Number & Quantity standards [here](#).

[6] For a more extensive discussion of the ideas and assumptions involved in solving equations, see p.13 of the [Draft High School Progression on Algebra](#).

[7] The [Draft High School Progression on Algebra](#) puts it this way: “An important step is realizing that a solution to a system of equations must be a solution to all of the equations in the system simultaneously. Then the process of adding one equation to another is understood as ‘if the two sides of one equation are equal, and the two sides of another equation are equal, then the sum of the left sides of the two equations is equal to the sum of the right sides.’” (p. 14).

[8] You can read the full text of the Standards for Mathematical Practice [here](#).

[9] Seen another way, this method “involves the rather sophisticated move of reversing the reduction of an equation in two variables to an equation in one variable. Rather, it seeks to convert an equation in one variable, $f(x) = g(x)$, to a system of equations in two variables, $y = f(x)$ and $y = g(x)$, by introducing a second variable y and setting it equal to each side of the equation. If x is a solution to the original equation then $f(x)$ and $g(x)$ are equal, and thus (x, y) is a solution to the new system.” For more information on this idea, see the [Draft High School Progression on Algebra](#) (p. 15).