



Topic B:

Using Different Forms for Quadratic Functions

N-RN.B.3, A-SSE.A.1, A-SSE.A.2, A-SSE.B.3a, A-SSE.B.3b, A-APR.B.3, A-CED.A.1, A-CED.A.2,

A-REI.B.4, F-IF.B.4, F-IF.B.6, F-IF.C.7a, F-IC.C.8a

Focus Standards:	N-RN.B.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
	A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context.* a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. <i>For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.</i>
	A-SSE.A.2	Use the structure of an expression to identify ways to rewrite it. <i>For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.</i>
	A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
	A-APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
	A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems. <i>Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*</i>

- A-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
- A-REI.B.4 Solve quadratic equations in one variable.
- Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
 - Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b .
- F-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**
- F-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*
- F-IF.C.7a Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
- Graph linear and quadratic functions and show intercepts, maxima, and minima.
- F-IF.C.8a Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Instructional Days: 7

Lessons 11–12: Completing the Square (E, P)¹

Lesson 13: Solving Quadratic Equations by Completing the Square (P)

Lesson 14: Deriving the Quadratic Formula (P)

Lesson 15: Using the Quadratic Formula (P)

Lesson 16: Graphing Quadratic Equations From the Vertex Form, $y = a(x - h)^2 + k$ (E)

Lesson 17: Graphing Quadratic Functions From the Standard Form, $f(x) = ax^2 + bx + c$ (P)

¹ Lesson Structure Key: P-Problem Set Lesson, M-Modeling Cycle Lesson, E-Exploration Lesson, S-Socratic Lesson

In Topic A, students expanded their fluency with manipulating polynomials and deepened their understanding of the nature of quadratic functions. They rewrote polynomial expressions by factoring and used the factors to solve quadratic equations in one variable, using rectangular area as a context. They also sketched quadratic functions and learned about the key features of their graphs, with particular emphasis on relating the factors of a quadratic expression to the zeros of the function it defines.

In Lessons 11 and 12 of Topic B, students learn to manipulate quadratic expressions by completing the square. They use this knowledge to solve quadratic equations in one variable in Lesson 13 for situations where factoring is either impossible or inefficient. There is particular emphasis on quadratic functions with irrational solutions in this topic, and students use these solutions as an opportunity to explore the property of closure for rational and irrational numbers. In Lesson 14, students derive the quadratic formula by completing the square for the standard form of a quadratic equation, $y = ax^2 + bx + c$, and use it to solve quadratic equations that cannot be easily factored. They discover that some quadratic equations do not have real solutions. Students use the discriminant, in Lesson 15, to determine whether a quadratic equation has one, two, or no real solutions. In Lesson 16, students learn that the $f(x) = a(x - h)^2 + k$ form of a function reveals the vertex of its graph. They sketch the graph of a quadratic from its equation in vertex form and construct a quadratic equation in vertex form from its graph.

As students begin to work in two variables, they are introduced to business applications, which can be modeled with quadratic functions, including profit, loss, revenue, cost, etc. Then, students use all of the tools at their disposal in Lesson 17 to interpret functions and their graphs when prepared in the standard form, $f(x) = ax^2 + bx + c$. They explore the relationship between the coefficients and constants in both standard and vertex forms of the quadratic equation, and they identify the key features of their graphs.