## Topic A:

# Quadratic Expressions, Equations, Functions, and Their Connection to Rectangles 

A-SSE.A.1, A-SSE.A.2, A-SSE.B.3a, A-APR.A.1, A-CED.A.1, A-CED.A.2, A-REI.B.4b, A-REI.D.11, F-IF.B.4, F-IF.B.5, F-IF.B.6, F-IF.C.7a

| Focus Standards: | A-SSE.A. 1 | Interpret expressions that represent a quantity in terms of its context. ${ }^{\star}$ <br> a. Interpret parts of an expression, such as terms, factors, and coefficients. <br> b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^{n}$ as the product of $P$ and a factor not depending on $P$. |
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|  | A-SSE.A. 2 | Use the structure of an expression to identify ways to rewrite it. For example, see $x^{4}-y^{4}$ as $\left(x^{2}\right)^{2}-\left(y^{2}\right)^{2}$, thus recognizing it as a difference of squares that can be factored as $\left(x^{2}-y^{2}\right)\left(x^{2}+y^{2}\right)$. |
|  | A-SSE.B.3a | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ${ }^{\star}$ <br> a. Factor a quadratic expression to reveal the zeros of the function it defines. |
|  | A-APR.A. 1 | Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. |
|  | A-CED.A. 1 | Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. ${ }^{\star}$ |
|  | A-CED.A. 2 | Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ${ }^{\star}$ |

A-REI.B.4b Solve quadratic equations in one variable.
b. Solve quadratic equations by inspection (e.g., for $x^{2}=49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm b i$ for real numbers $a$ and $b .{ }^{1}$
A-REI.D. 11 Explain why the $x$-coordinates of the points where the graphs of the equations $y=f(x)$ and $y=g(x)$ intersect are the solutions of the equation $f(x)=g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ${ }^{\star}$
F-IF.B. $4 \quad$ For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ${ }^{\star}$
F-IF.B. $5 \quad$ Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function. *
F-IF.B. $6 \quad$ Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*
F-IF.C.7a Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
Instructional Days: 10
Lessons 1-2: Multiplying and Factoring Polynomial Expressions ( $\mathrm{P}, \mathrm{P})^{2}$
Lessons 3-4: Advanced Factoring Strategies for Quadratic Expressions (P, P)
Lesson 5: The Zero Product Property (E)
Lesson 6: Solving Basic One-Variable Quadratic Equations (P)
Lesson 7: Creating and Solving Quadratic Equations in One Variable (P)

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Lesson 8: Exploring the Symmetry in Graphs of Quadratic Functions (E)
Lesson 9: Graphing Quadratic Functions from Factored Form, f(x)=a(x-m)(x-n)(P)
Lesson 10: Interpreting Quadratic Functions from Graphs and Tables (P)
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Deep conceptual understanding of operations with polynomials is the focus of this topic. The emphasis is on using the properties of operations for multiplying and factoring quadratic trinomials, including the connections to numerical operations and rectangular geometry, rather than using common procedural gimmicks such as FOIL. In Topic A, students begin by using the distributive property to multiply monomials by polynomials. They relate binomial expressions to the side lengths of rectangles and find area by multiplying binomials, including those whose expanded form is the difference of squares and perfect squares. They analyze, interpret, and use the structure of polynomial expressions to factor, with the understanding that factoring is the reverse process of multiplication. There are two exploration lessons in Topic A. The first is Lesson 6 , in which students will explore all aspects of solving quadratic equations, including using the zero product property. The second is Lesson 8, where students explore the unique symmetric qualities of quadratic graphs. Both explorations will be revisited and extended throughout this topic and the module.

In Lesson 3, students encounter quadratic expressions for which extracting the GCF is impossible (the leading coefficient, $a$, is not 1 and is not a common factor of the terms). They discover the importance of the product of the leading coefficient and the constant ( $a c$ ) and become aware of its use when factoring expressions such as $6 x^{2}+5 x-6$. In Lesson 4, students explore other factoring strategies strongly associated with the area model, such as using the area method or a table to determine the product-sum combinations. Note that factoring by grouping will not be assessed on the New York State Regents Examination in Algebra I. Please see the standards clarification document found at the following
link: http://www.engageny.org/resource/regents-exams-mathematics-algebra-i-standards-clarifications. In Lesson 5, students discover the zero product property and solve for one variable by setting factored expressions equal to zero. In Lesson 6, they decontextualize word problems to create equations and inequalities that model authentic scenarios addressing area and perimeter.

Finally, students build on their prior experiences with linear and exponential functions and their graphs to include interpretation of quadratic functions and their graphs. Students explore and identify key features of quadratic functions and calculate and interpret the average rate of change from the graph of a function. Key features include $x$-intercepts (zeros of the function), $y$-intercepts, the vertex (minimum or maximum values of the function), end behavior, and intervals where the function is increasing or decreasing. It is important for students to use these features to understand how functions behave and to interpret a function in terms of its context.

A focus of this topic is to develop a deep understanding of the symmetric nature of a quadratic function. Students use factoring to reveal its zeros and then use these values and their understanding of quadratic function symmetry to determine the axis of symmetry and the coordinates of the vertex. Often, students are asked to use $x=-\frac{b}{2 a}$ as an efficient way of finding the axis of symmetry or the vertex. (Note: Students learn to use this formula without understanding that this is a generalization for the average of the domain values for the $x$-intercepts.) Only after students develop an understanding of symmetry will $x=-\frac{b}{2 a}$ be explored as a general means of finding the axis of symmetry.


[^0]:    ${ }^{1}$ Tasks do not require students to write solutions for quadratic equations that have roots with nonzero imaginary parts. However, tasks can require the student to recognize cases in which a quadratic equation has no real solutions.
    ${ }^{2}$ Lesson Structure Key: P-Problem Set Lesson, M-Modeling Cycle Lesson, E-Exploration Lesson, S-Socratic Lesson

