Mathematics Curriculum

Topic B: The Structure of Expressions

A-SSE.A.1, A-SSE.A.2

Focus Standard:	A-SSE.A.2	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
	A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
Instructional Days:	4	
Lesson 6:	Algebraic Expressions—The Distributive Property (P) ¹	
Lesson 7:	Algebraic Expressions—The Commutative and Associative Properties (S)	
Lesson 8:	Adding and Subtracting Polynomials (S)	
Lesson 9:	Multiplying Polynomials (P)	

In Lessons 6 and 7 of this topic, students develop a precise understanding of what it means for expressions to be algebraically equivalent. By exploring geometric representations of the distributive, associative, and commutative properties for positive whole numbers and variable expressions assumed to represent positive whole numbers, students confirm their understanding of these properties and expand them to apply to all real numbers. Students use the properties to generate equivalent expressions and formalize that two algebraic expressions are equivalent if we can convert one expression into the other by repeatedly applying the commutative, associative, and distributive properties and the properties of rational exponents to components of the first expression. A goal of this topic is to address a fundamental, underlying question: *Why are the commutative, associative, and distributive properties so important in mathematics*?² The answer to the question is, of course, because these three properties help to generate all equivalent algebraic expressions discussed in Algebra I.

² The *welcomed* removal of parentheses from expressions through the use of order of operations and other tricks greatly simplifies how we write expressions, but those same tricks also make it appear as if the associative property is never used when rewriting expressions.





Topic B:

Date:

¹ Lesson Structure Key: **P**-Problem Set Lesson, **M**-Modeling Cycle Lesson, **E**-Exploration Lesson, **S**-Socratic Lesson



Lessons 6 and 7 also engage students in their first experience using a recursive definition for building algebraic expressions. Recursive definitions are sometimes confused with being circular in nature because the definition of the term uses the very term one is defining. However, a recursive definition or process is not circular because it has what is referred to as a base case. For example, a definition for algebraic expression is presented as follows:

An algebraic expression is either:

- 1. A numerical symbol or a variable symbol or
- 2. The result of placing previously generated algebraic expressions into the two blanks of one of the four operators (() + (), () - (), () \times (), () \div ()) or into the base blank of an exponentiation with an exponent that is a rational number.

Part (1) of this definition serves as a base case, stating that any numerical or variable symbol is in itself an algebraic expression. The recursive portion of the definition is in part (2) where one can use any previously generated algebraic expression to form new ones using the given operands. Recursive definitions are an important part of the study of sequences in Module 3. Giving students this early experience lays a nice foundation for the work to come.

Having a clear understanding of how algebraic expressions are built and what makes them equivalent provides a foundation for the study of polynomials and polynomial expressions.

In Lessons 8 and 9, students learn to relate polynomials to integers written in base x, rather than our traditional base of 10. The analogies between the system of integers and the system of polynomials continue as they learn to add, subtract, and multiply polynomials and to find that the polynomials for a system that is closed under those operations (e.g., a polynomial added to, subtracted from, or multiplied by another polynomial) always produces another polynomial.

We use the terms polynomial and polynomial expression in much the same way as we use the terms number and numerical expression. Where we would not call 27(3 + 8) a number, we would call it a numerical expression. Similarly, we reserve the word polynomial for polynomial expressions that are written as a sum of monomials.



The Structure of Expressions 10/22/14



Date: