Lesson 11: Conditions on Measurements that Determine a Triangle

Classwork

Exploratory Challenge 1

1. Can any three side lengths form a triangle? Why or why not?
2. Draw a triangle according to these instructions:
* Draw segment $AB$ of length $10$ cm in your notebook.
* Draw segment $BC$ of length $3$ cm on one piece of patty paper.
* Draw segment $AC$ of length $5$ cm on the other piece of patty paper.
* Line up the appropriate endpoint on each piece of patty paper with the matching endpoint on $AB$.
* Use your pencil point to hold each patty paper in place, and adjust the paper to form $△ABC$.
1. What do you notice?
2. What must be true about the sum of the lengths of $AC$ and $BC$ if the two segments were to just meet? Use your patty paper to verify your answer.
3. Based on your conclusion for part (c), what if $BC=3 $cm as you originally had, but $AC=10$ cm in length. Could you form $△ABC$?
4. What must be true about the sum of the lengths of $AC$ and $BC$ if the two segments were to meet and form a triangle?

Exercise 1

Two sides of $△DEF$ have lengths of $5$ cm and $8$ cm. What are all the possible whole-number lengths for the remaining side?

Exploratory Challenge 2

1. Which of the following conditions determine a triangle? Follow the instructions to try and draw $△ABC$. Segment $AB$ has been drawn for you as a starting point in each case.
2. Choose measurements of $∠A$ and $∠B$ for $△ABC$ so that the sum of measurements is greater than $180°$. Label your diagram.

Your chosen angle measurements: $∠A=$ $∠B=$

Were you able to form a triangle? Why or why not?



1. Choose measurements of $∠A$ and $∠B$ for $△ABC$ so that the measurement of $∠A$ is supplementary to the measurement of $∠B$. Label your diagram.

Your chosen angle measurements: $∠A=$ $∠B= $

Were you able to form a triangle? Why or why not?



1. Choose measurements of $∠A$ and $∠B$ for $△ABC$ so that the sum of measurements is less than $180°$. Label your diagram.

Your chosen angle measurements: $∠A=$ $∠B=$

Were you able to form a triangle? Why or why not?



1. Which condition must be true regarding angle measurements in order to determine a triangle?
2. Measure and label the formed triangle in part (b) with all three side lengths and the angle measurement for $∠C$. Now, use a protractor, ruler, and compass to draw $△A'B'C'$ with the same angle measurements, but side lengths that are half as long.
3. Do the three angle measurements of a triangle determine a unique triangle? Why or why not?

Exercise 2

Which of the following sets of angle measurements determines a triangle?

1. $30°$, $120°$
2. $125°$,$ 55°$
3. $105°$, $80°$
4. $90°$,$ 89°$

1. $91°$,$ 89°$

Choose one example from above that does determine a triangle and one that does not. For each, explain why it does or does not determine a triangle using words and a diagram.

Problem Set

1. Decide whether each set of three given lengths determines a triangle. For any set of lengths that does determine a triangle, use a ruler and compass to draw the triangle. Label all side lengths. For sets of lengths that do not determine a triangle, write “Does not determine a triangle,” and justify your response.
2. $3 $cm, $4 $cm, $5 $cm
3. $1$ cm, $4 $cm, $5 $cm
4. $1$ cm, $5 $cm, $5 $cm
5. $8$ cm, $3 $cm, $4 $cm
6. $8 $cm, $8$ cm, $4$ cm
7. $4$ cm, $4 $cm, $4 $cm
8. For each angle measurement below, provide one angle measurement that will determine a triangle and one that will not determine a triangle. Provide a brief justification for the angle measurements that will not form a triangle. Assume that the angles are being drawn to a horizontal segment $AB$; describe the position of the non-horizontal rays of angles $∠A$ and $∠B$.

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| --- | --- | --- | --- |
| $$∠A$$ | $∠B$: A Measurement that Determines a Triangle | $∠B$: A Measurement that Doesn’t Determine a Triangle | Justification for No Triangle |
| $$40°$$ |  |  |  |
| $$100°$$ |  |  |  |
| $$90°$$ |  |  |  |
| $$135°$$ |  |  |  |

1. For the given side lengths, provide the minimum and maximum whole-number side lengths that determine a triangle.

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| --- | --- | --- |
| Given Side Lengths | Minimum Whole Number Third Side Length | Maximum Whole Number Third Side Length |
| $5$ cm, $6 $cm |  |  |
| $3$ cm, $7 $cm |  |  |
| $4 $cm, $10 $cm |  |  |
| $1 $cm, $12 $cm |  |  |