Lesson 18: Counting Problems

Student Outcomes

- Students solve counting problems related to computing percents.

Lesson Notes

Students will continue to apply their understanding of percent to solve counting problems. The problems in this lesson lend themselves to the concept of probability without formal computations of combinations and permutations.

Classwork

Opening Exercise (5 minutes)

MP.1

Opening Exercise

You are about to switch out your books from your locker during passing period but forget the order of your locker combination. You know that there are the numbers 3, 16, and 21 in some order. What is the percent of locker combinations that start with 3?

Locker Combination Possibilities:

$\mathbf{3, 16, 21}$

$\mathbf{21, 16, 3}$

$\mathbf{16, 21, 3}$

$\mathbf{21, 3, 16}$

$\mathbf{16, 3, 21}$

$\mathbf{3, 21, 16}$

$$\frac{2}{6} = \frac{1}{3} = 0.33\overline{3} = 33.\overline{3}\%$$

Scaffolding:

For all problems that involve determining a number of combinations, consider using manipulatives to allow students to create different physical arrangements. Additionally, in cases where numbers or letters will cause further confusion, consider modifying tasks to involve arranging pictures or colors.

Discussion (3 minutes)

- What amounts did you use to find the percent of locker combinations that start with 3?
  - *Since there are only 2 locker combinations that start with a 3 and a total of 6 locker combinations, we used 2 and 6.*
- What amounts would you use to find the percent of locker combinations that end with a 3?
  - *There are only 2 locker combinations that end with a 3 and a total of 6 locker combinations; we would use 2 and 6.*

Allow the opportunity for students to share other solution methods and reflections with one another.

Example 1 (5 minutes)

Have students answer questions in this example independently. Reconvene as a class to share out and model solutions.

Example 1

All of the 3-letter passwords that can be formed using the letters A and B are as follows: AAA, AAB, ABA, ABB, BAA, BAB, BBA, BBB.

a. What percent of passwords contain at least two B's?

There are four passwords that contain at least two B's: ABB, BAB, BBA, and BBB. There are eight passwords total.

$\frac{4}{8} = \frac{1}{2} = 50\%$*, so* 50% *of the passwords contain at least two B's.*

b. What percent of passwords contain no A's?

There is one password that contains no A's. There are eight passwords total.

$\frac{1}{8} = 0.125 = 12.5\%$*, so* 12.5% *of the passwords contain no A's.*

- What is another way of saying, "passwords containing at least two B's"?
  - *Passwords that have one or no A's. Passwords that have two or more B's.*
- Would the percent of passwords containing one or no A's be equal to the percent of passwords containing at least two B's?
  - *Yes, because they represent the same group of passwords.*
- What is another way of saying, "passwords containing no A's"?
  - *A password that contains all B's.*

Exercises 1–2 (5 minutes)

Students may work individually or in pairs to complete Exercises 1–2.

Exercises 1–2

1. How many 4-letter passwords can be formed using the letters A and B?

16: *AAAA, AAAB, AABB, ABBB, AABA, ABAA, ABAB, ABBA,*

BBBB, BBBA, BBAA, BAAA, BBAB, BABB, BABA, BAAB

2. What percent of the 4-letter passwords contain

a. No A's?

$\frac{1}{16} = 0.0625 = 6.25\%$

b. Exactly one A?

$\frac{4}{16} = \frac{1}{4} = 25\%$

c. **Exactly two A's?**

$\frac{6}{16} = 0.375 = 37.5\%$

d. **Exactly three A's?**

$\frac{4}{16} = \frac{1}{4} = 25\%$

e. **Four A's?**

$\frac{1}{16} = 0.0625 = 6.25\%$

f. **The same number of A's and B's?**

$\frac{6}{16} = 0.375 = 37.5\%$

- Which categories have percents that are equal?
  - *No A's and four A's have the same percent.*
  - *Exactly one A and exactly three A's have the same percents.*
  - *Exactly two A's and the same number of A's and B's also have the same percents.*
- Why do you think they are equal?
  - *Four A's is the same as saying no B's, and since there are only two letters, no B's is the same as no A's.*
  - *The same reasoning can be used for exactly one A and exactly three A's. If there are exactly three A's, then this would mean that there is exactly one B, and since there are only two letters, exactly one B is the same as exactly one A.*
  - *Finally, exactly two A's and the same number of A's and B's are the same because the same amount of A's and B's would be two of each.*

Example 2 (5 minutes)

Example 2

In a set of 3-letter passwords, 40% of the passwords contain the letter B and two of another letter. Which of the two sets below meets the criteria? Explain how you arrived at your answer.

Set 1

BBB	AAA	CAC
CBC	ABA	CCC
BBC	CCB	CAB
AAB	AAC	BAA
ACB	BAC	BCC

Set 2

CEB	BBB
EBE	CCC
CCC	EEE
EEB	CBC
CCB	ECE

For each set, I counted how many passwords have the letter B and two of another letter. Then, I checked to see if that quantity equaled 40% of the total number of passwords in the set.

In Set 1, CBC, AAB, ABA, CCB, BAA, and BCC are the passwords that contain a B and two of another letter. Set 1 meets the criteria since there are 15 passwords total and 40% of 15 is 6.

$$\text{Quantity} = \text{Percent} \times \text{Whole}$$
$$6 = 0.4(15)$$
$$6 = 6 \rightarrow \text{True}$$

In Set 2, EBE, EEB, CCB, and CBC are the only passwords that contain a B and two others of the same letter. Set 2 meets the criteria since there are 10 passwords total and 40% of 10 is 4.

$$\text{Quantity} = \text{Percent} \times \text{Whole}$$
$$4 = 0.4(10)$$
$$4 = 4 \rightarrow \text{True}$$

So, both Sets 1 and 2 meet the criteria.

Exercises 3–4 (5 minutes)

Exercises 3–4

3. **Shana read the following problem:**

> **"How many letter arrangements can be formed from the word *triangle* that have two vowels and two consonants (order does not matter)?"**

She answered that there are 30 letter arrangements.

Twenty percent of the letter arrangements that began with a vowel actually had an English definition. How many letter arrangements that begin with a vowel have an English definition?

$$0.20 \times 30 = 6$$

Six have a formal English definition.

4. **Using three different keys on a piano, a songwriter makes the beginning of his melody with three notes, C, E, and G: CCE, EEE, EGC, GCE, CEG, GEE, CGE, GGE, EGG, EGE, GCG, EEC, ECC, ECG, GGG, GEC, CCG, CEE, CCC, GEG, CGC.**

a. **From the list above, what is the percent of melodies with all three notes that are different?**

$$\frac{6}{21} \approx 28.6\%$$

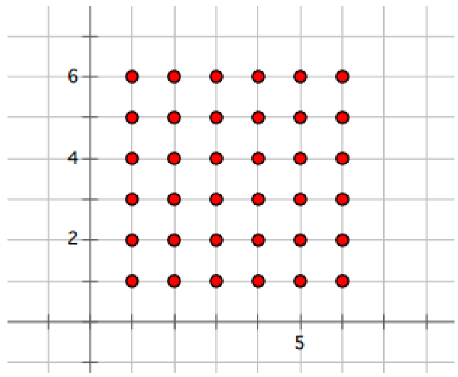
b. **From the list above, what is the percent of melodies that have three of the same notes?**

$$\frac{3}{21} \approx 14.3\%$$

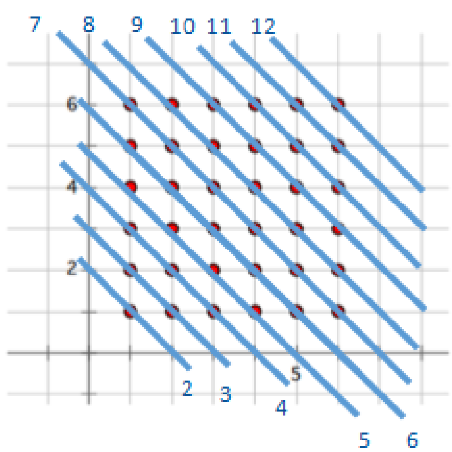
Example 3 (10 minutes)

Example 3

Look at the 36 points on the coordinate plane with whole number coordinates between 1 and 6, inclusive.



a. **Draw a line through each of the points which have an x-coordinate and y-coordinate sum of 7.**
Draw a line through each of the points which have an x-coordinate and y-coordinate sum of 6.
Draw a line through each of the points which have an x-coordinate and y-coordinate sum of 5.
Draw a line through each of the points which have an x-coordinate and y-coordinate sum of 4.
Draw a line through each of the points which have an x-coordinate and y-coordinate sum of 3.
Draw a line through each of the points which have an x-coordinate and y-coordinate sum of 2.
Draw a line through each of the points which have an x-coordinate and y-coordinate sum of 8.
Draw a line through each of the points which have an x-coordinate and y-coordinate sum of 9.
Draw a line through each of the points which have an x-coordinate and y-coordinate sum of 10.
Draw a line through each of the points which have an x-coordinate and y-coordinate sum of 11.
Draw a line through each of the points which have an x-coordinate and y-coordinate sum of 12.



b. **What percent of the 36 points have a coordinate sum of 7?**

$\frac{6}{36} = \frac{1}{6} = 16\frac{2}{3}\%$

c. **Write a numerical expression that could be used to determine the percent of the 36 points that have a coordinate sum of 7.**

There are six coordinate points in which the sum of the x-coordinate and the y-coordinate is 7. So,

$$\frac{6}{36} \times 100.$$

d. **What percent of the 36 points have a coordinate sum of 5 or less?**

$$\frac{10}{36} \times 100 = 27\frac{7}{9}\%$$

e. **What percent of the 36 points have a coordinate sum of 4 or 10?**

$$\frac{6}{36} \times 100 = 16\frac{2}{3}\%$$

Closing (3 minutes)

- What information must be known to find the percent of possible outcomes for a counting problem?
  - *To decipher percents, the total number of possible outcomes needs to be known as well as the different outcomes.*

Lesson Summary

To find the percent of possible outcomes for a counting problem you need to determine the total number of possible outcomes and the different favorable outcomes. The representation

$$\text{Quantity} = \text{Percent} \times \text{Whole}$$

can be used where the quantity is the number of different favorable outcomes, and the whole is the total number of possible outcomes.

Exit Ticket (5 minutes)

Name ____ Date ____________________

Lesson 18: Counting Problems

Exit Ticket

There are a van and a bus transporting students on a student camping trip. Arriving at the site, there are 3 parking spots. Let v represent the van and b represent the bus. The chart shows the different ways the vehicles can park.

	Parking Space 1	Parking Space2	Parking Space 3
Option 1	V	B	
Option 2	V		B
Option 3	B	V	
Option 4	B		V
Option 5		V	B
Option 6		B	V

a. In what percent of the arrangements are the vehicles separated by an empty parking space?

b. In what percent of the arrangements are the vehicles parked next to each other?

c. In what percent of the arrangements does the left or right parking space remain vacant?

Exit Ticket Sample Solutions

There are a van and a bus transporting students on a student camping trip. Arriving at the site, there are 3 parking spots. Let v represent the van and b represent the bus. The chart shows the different ways the vehicles can park.

a. In what percent of the arrangements are the vehicles separated by an empty parking space?

$$\frac{2}{6} = 33\frac{1}{3}\%$$

b. In what percent of the arrangements are the vehicles parked next to each other?

$$\frac{4}{6} = 66\frac{2}{3}\%$$

c. In what percent of the arrangements does the left or right parking space remain vacant?

$$\frac{4}{6} = 66\frac{2}{3}\%$$

	Parking Space 1	Parking Space2	Parking Space 3
Option 1	V	B	
Option 2	V		B
Option 3	B	V	
Option 4	B		V
Option 5		V	B
Option 6		B	V

Problem Set Sample Solutions

1. A six-sided die (singular for dice) is thrown twice. The different rolls are as follows:

1 and 1, 1 and 2, 1 and 3, 1 and 4, 1 and 5, 1 and 6,
2 and 1, 2 and 2, 2 and 3, 2 and 4, 2 and 5, 2 and 6,
3 and 1, 3 and 2, 3 and 3, 3 and 4, 3 and 5, 3 and 6,
4 and 1, 4 and 2, 4 and 3, 4 and 4, 4 and 5, 4 and 6,
5 and 1, 5 and 2, 5 and 3, 5 and 4, 5 and 5, 5 and 6,
6 and 1, 6 and 2, 6 and 3, 6 and 4, 6 and 5, 6 and 6.

a. What is the percent that both throws will be even numbers?

$$\frac{9}{36} = 25\%$$

b. What is the percent that the second throw is a 5?

$$\frac{6}{36} = 16\frac{2}{3}\%$$

c. What is the percent that the first throw is lower than a 6?

$$\frac{30}{36} = 83\frac{1}{3}\%$$

2. You have the ability to choose three of your own classes, art, language, and physical education. There are three art classes (A1, A2, A3), two language classes (L1, L2), and two P.E. classes (P1, P2) to choose from. The order does not matter and you must choose one from each subject.

A1, L1, P1	A2, L1, P1	A3, L1, P1
A1, L1, P2	A2, L1, P2	A3, L1, P2
A1, L2, P1	A2, L2, P1	A3, L2, P1
A1, L2, P2	A2, L2, P2	A3, L2, P2

Compare the percent of possibilities with A1 in your schedule to the percent of possibilities with L1 in your schedule.

A1: $\frac{4}{12} = 33\frac{1}{3}\%$ *L1:* $\frac{6}{12} = 50\%$

There is a greater percent with L1 in my schedule.

3. Fridays are selected to show your school pride. The colors of your school are orange, blue, and white, and you can show your spirit by wearing a top, a bottom, and an accessory with the colors of your school. During lunch, 11 students are chosen to play for a prize on stage. The table charts what the students wore.

Top	W	O	W	O	B	W	B	B	W	W	W
Bottom	B	O	B	B	O	B	B	B	O	W	B
Accessory	W	O	B	W	B	O	B	W	O	O	O

a. What is the percent of outfits that are one color?

$\frac{2}{11} = 18\frac{2}{11}\%$

b. What is the percent of outfits that include orange accessories?

$\frac{5}{11} = 45\frac{5}{11}\%$

4. Shana wears two rings (G represents gold, and S represents silver) at all times on her hand. She likes fiddling with them and places them on different fingers (pinky, ring, middle, index) when she gets restless. The chart is tracking the movement of her rings.

	Pinky Finger	Ring Finger	Middle Finger	Index Finger
Position 1		G	S	
Position 2			S	G
Position 3	G		S	
Position 4				S, G
Position 5	S	G		
Position 6	G	S		
Position 7	S		G	
Position 8	G		S	
Position 9		S, G		
Position 10		G	S	
Position 11			G	S
Position 12		S		G
Position 13	S, G			
Position 14			S, G	

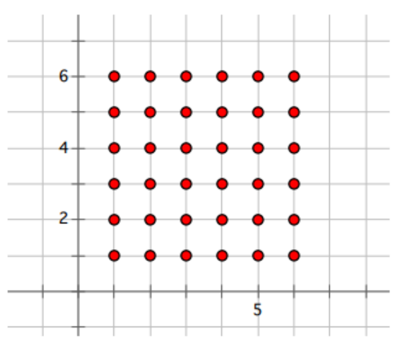
a. What percent of the positions shows the gold ring on her pinky finger?

$\frac{4}{14} \approx 28.57\%$

b. What percent of the positions shows both rings on the same finger?

$\frac{4}{14} = 28\frac{4}{7}\%$

5. Use the coordinate plane below to answer the following questions.



a. What is the percent of the 36 points whose quotient of $\frac{x\text{-coordinate}}{y\text{-coordinate}}$ is greater than one?

$\frac{15}{36} = 41\frac{2}{3}\%$

b. What is the percent of the 36 points whose coordinate quotient is equal to one?

$\frac{6}{36} = 16\frac{2}{3}\%$