Lesson 17: Mixture Problems

Student Outcomes

- Students write and use algebraic expressions and equations to solve percent word problems related to mixtures.

Classwork

Opening Exercise (10 minutes)

In pairs, students will use their knowledge of percent to complete the charts and answer mixture problems. To highlight MP.1, consider asking students to attempt to make sense of and solve the Opening Exercise without the chart, then have students explain the solution methods they developed.

> *Scaffolding:*
>
> Doing an actual, physical demonstration with containers of water and juice to illustrate the Opening Exercise will aid in understanding. Additionally, using visuals to show examples of customary measurement units will help students that may be unfamiliar with these terms (ounce, cup, pint, quart, gallon).

MP.1

Opening Exercise

Imagine you have two equally sized containers. One is pure water, and the other is 50% water and 50% juice. If you combined them, what percent of juice would be the result?

	1st liquid	2nd liquid	Resulting liquid
Amount of liquid (gallons)	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$
Amount of pure juice (gallons)	$\mathbf{0 = 1 \times 0}$ Quantity = Percent × Whole	$\mathbf{0.5 = 0.5 \times 1}$ Quantity = Percent × Whole	$\mathbf{0.5 = x \times 2}$ Quantity = Percent × Whole

25% *of the resulting mixture is juice because* $\frac{0.5}{2} = \frac{1}{4}$.

If a 2-gallon container of pure juice is added to 3 gallons of water, what percent of the mixture is pure juice?

Let x represent the percent of pure juice in the resulting juice mixture.

	1st liquid	2nd liquid	Resulting liquid
Amount of liquid (gallons)	$\mathbf{2}$	$\mathbf{3}$	$\mathbf{5}$
Amount of pure juice (gallons)	$\mathbf{2.0 = 1.0 \times 2}$ Quantity = Percent × Whole	$\mathbf{0 = 0 \times 3}$ Quantity = Percent × Whole	$\mathbf{2.0 = x \times 5}$ Quantity = Percent × Whole

- What is the percent of pure juice in water?
  - *Zero percent.*
- How much pure juice will be in the resulting mixture?
  - 2 *gallons because the only pure juice to be added is the first liquid.*
- What percent is pure juice out of the resulting mixture?
  - 40%

If a 2-gallon container of juice mixture that is 40% pure juice is added to 3 gallons of water, what percent of the mixture is pure juice?

	1st liquid	2nd liquid	Resulting liquid
Amount of liquid (gallons)	2	3	5
Amount of pure juice (gallons)	$0.8 = 0.4 \times 2$ Quantity = Percent × Whole	$0 = 0 \times 3$ Quantity = Percent × Whole	$0.8 = x \times 5$ Quantity = Percent × Whole

- How many gallons of the juice mixture is pure juice?
  - $2(0.40) = 0.8$ *gallons*
- What percent is pure juice out of the resulting mixture?
  - 16%
- Does this make sense relative to the prior problem?
  - *Yes, because the mixture should have less juice than the prior problem.*

If a 2-gallon juice cocktail that is 40% pure juice is added to 3 gallons of pure juice, what percent of the resulting mixture is pure juice?

	1st liquid	2nd liquid	Resulting liquid
Amount of liquid (gallons)	2	3	5
Amount of pure juice (gallons)	$0.8 = 0.4 \times 2$ Quantity = Percent × Whole	$3 = 1.00 \times 3$ Quantity = Percent × Whole	$3.8 = x \times 5$ Quantity = Percent × Whole

- What is the difference between this problem and the previous one?
  - *Instead of adding water to the two gallons of juice mixture, pure juice is added, so the resulting liquid contains* 3.8 *gallons of pure juice.*
- What percent is pure juice out of the resulting mixture?
  - *Let* x *represent the percent of pure juice in the resulting mixture.*

$$x(5) = 40\%(2) + 100\%(3)$$
$$5x = 0.8 + 3$$
$$5x = 3.8$$
$$x = 0.76$$

The mixture is 76% *pure juice.*

Discussion (5 minutes)

MP.7

- What pattern do you see in setting up the equations?
  - $\text{Quantity} = \text{Percent} \times \text{Whole}$. *The sum of parts or mixtures is equal to the resulting mixture. For each juice mixture, you multiply the percent of pure juice by the total amount of juice.*
- How is the form of the expressions and equations in the mixture problems similar to population problems from the previous lesson (e.g., finding out how many boys and girls wear glasses)?
  - *Just as you would multiply the sub-populations (such as girls or boys) by the given category (students wearing glasses) to find the percent in the whole population, mixture problems parallel the structure of population problems. In mixture problems, the sub-populations are the different mixtures, and the category is the potency of a given element. In this problem the element is pure juice.*

Example 1 (5 minutes)

Allow students to answer the problems independently and reconvene as a class to discuss the example.

MP.2

Example 1

A 5-gallon container of trail mix is 20% nuts. Another trail mix is added to it, resulting in a 12-gallon container of trail mix that is 40% nuts.

a. Write an equation to describe the relationships in this situation.

Let j represent the percent of nuts in the second trail mix that is added to the first trail mix to create the resulting 12-gallon container of trail mix.

$$\mathbf{0.4(12) = 0.2(5) + j(12 - 5)}$$

b. Explain in words how each part of the equation relates to the situation.

$$\textbf{Quantity} = \textbf{Percent} \times \textbf{Whole}$$

$$(\textbf{Resulting gallons of trail mix})(\textbf{Resulting } \% \textbf{ of nuts}) = (\mathbf{1^{st}} \textbf{ trail mix in gallons})(\% \textbf{ of nuts}) + (\mathbf{2^{nd}} \textbf{ trail mix in gallons})(\% \textbf{ of nuts})$$

c. What percent of the second trail mix is nuts?

$$\mathbf{4.8 = 1 + 7j}$$
$$\mathbf{4.8 - 1 = 1 - 1 + 7j}$$
$$\mathbf{3.8 = 7j}$$
$$\mathbf{j \approx 0.5429}$$

About 54% of the second trail mix is nuts.

- What information is missing from this problem?
  - *The amount of the second trail mix, but we can calculate it easily because it is the difference of the total trail mix and the first trail mix.*
- How is this problem different from the Opening Exercises?
  - *Instead of juice, the problem is about trail mix. Mathematically, this example is not asking for the percent of a certain quantity in the resulting mixture but, rather, asking for the percent composition of one of the trail mixes being added.*
- How is the problem similar to the Opening Exercises?
  - *We are still using* $\text{Quantity} = \text{Percent} \times \text{Whole}$.
- Is the answer reasonable?
  - *Yes, because the second percent of nuts in the trail mix should be a percent greater than 40% since the first trail mix is 20% nuts.*

Exercise 1 (5 minutes)

Exercise 1

Represent each situation using an equation, and show all steps in the solution process.

a. **A 6-pint mixture that is 25% oil is added to a 3-pint mixture that is 40% oil. What percent of the resulting mixture is oil?**

Let x represent the percent of oil in the resulting mixture.

$$\begin{aligned} 0.25(6) + 0.40(3) &= x(9) \\ 1.5 + 1.2 &= 9x \\ 2.7 &= 9x \\ x &= 0.3 \end{aligned}$$

The resulting 9-pint mixture is 30% oil.

b. **An 11-ounce gold chain of 24% gold was made from a melted down 4-ounce charm of 50% gold and a golden locket. What percent of the locket was pure gold?**

Let x represent the percent of pure gold in the locket.

$$\begin{aligned} 0.5(4) + (x)(7) &= 0.24(11) \\ 2 + 7x &= 2.64 \\ 2 - 2 + 7x &= 2.64 - 2 \\ \frac{7x}{7} &= \frac{0.64}{7} \\ x &\approx 0.0914 \end{aligned}$$

The locket was about 9% gold.

c. **In a science lab, two containers are filled with mixtures. The first container is filled with a mixture that is 30% acid. The second container is filled with a mixture that is 50% acid, and the second container is 50% larger than the first. The first and second containers are then emptied into a third container. What percent of acid is in the third container?**

Let m represent the total amount of mixture in the first container.

$0.3m$ is the amount of acid in the first container.

$0.5(m + 0.5m)$ is the amount of acid in the second container.

$0.3m + 0.5(m + 0.5m) = 0.3m + 0.5(1.5m) = 1.05m$ is the amount of acid in the mixture in the third container.

$m + 1.5m = 2.5m$ is the amount of mixture in the third container. So, $\frac{1.05m}{2.5m} = 0.42 = 42\%$ is the percent of acid in the third container.

Example 2 (5 minutes)

Encourage students to find the missing information and set up the equation with the help of other classmates. Review the process with the whole class by soliciting student responses.

Example 2

Soil that contains 30% clay is added to soil that contains 70% clay to create 10 gallons of soil containing 50% clay. How much of each of the soils was combined?

Let x be the amount of soil with 30% clay.

$$(1^{\text{st}} \text{ soil amount})(\% \text{ of clay}) + (2^{\text{nd}} \text{ soil amount})(\% \text{ of clay}) = (\text{resulting amount})(\text{resulting } \% \text{ of clay})$$
$$(0.3)(x) + (0.7)(10 - x) = (0.5)(10)$$
$$0.3x + 7 - 0.7x = 5$$
$$-0.4x + 7 - 7 = 5 - 7$$
$$-0.4x = -2$$
$$x = 5$$

5 gallons of the 30% clay soil and $10 - 5 = 5$, so 5 gallons of the 70% clay soil must be mixed to make 10 gallons of 50% clay soil.

Exercise 2 (5 minutes)

Exercise 2

The equation $(0.2)(x) + (0.8)(6 - x) = (0.4)(6)$ is used to model a mixture problem.

a. **How many units are in the total mixture?**

   6 units

b. **What percents relate to the two solutions that are combined to make the final mixture?**

   20% and 80%

c. **The two solutions combine to make 6 units of what percent solution?**

   40%

d. **When the amount of a resulting solution is given (for instance, 4 gallons) but the amounts of the mixing solutions are unknown, how are the amounts of the mixing solutions represented?**

   If the amount of gallons of the first mixing solution is represented by the variable x, then the amount of gallons of the second mixing solution is $4 - x$.

Closing (5 minutes)

MP.7

- What is the general structure of the expressions for mixture problems?
  - *The general equation looks like the following:*

$$\text{Whole Quantity} = \text{Part} + \text{Part}.$$

  - *Utilizing this structure makes an equation that looks like the following:*

$$(\text{\% of resulting quantity})(\text{amount of the resulting quantity}) =$$
$$(\text{\% of }1^{\text{st}}\text{ quantity})(\text{amount of }1^{\text{st}}\text{ quantity}) + (\text{\% of }2^{\text{nd}}\text{ quantity})(\text{amount of }2^{\text{nd}}\text{ quantity}).$$

- How do mixture and population problems compare?
  - *These problems both utilize the equation* $\text{Quantity} = \text{Percent} \times \text{Whole}$. *Mixture problems deal with quantities of solutions and mixtures as well as potencies while population problems deal with sub-groups and categories.*

Lesson Summary

- **Mixture problems deal with quantities of solutions and mixtures.**
- **The general structure of the expressions for mixture problems are**

$$\textbf{Whole Quantity} = \textbf{Part} + \textbf{Part.}$$

- **Using this structure makes the equation resemble the following:**

$$(\textbf{\% of resulting quantity})(\textbf{amount of resulting quantity}) =$$
$$(\textbf{\% of }\mathbf{1^{st}}\textbf{ quantity})(\textbf{amount of }\mathbf{1^{st}}\textbf{ quantity}) + (\textbf{\% of }\mathbf{2^{nd}}\textbf{ quantity})(\textbf{amount of }\mathbf{2^{nd}}\textbf{ quantity}).$$

Exit Ticket (5 minutes)

Name ____ Date ____________________

Lesson 17: Mixture Problems

Exit Ticket

A 25% vinegar solution is combined with triple the amount of a 45% vinegar solution and a 5% vinegar solution resulting in 20 milliliters of a 30% vinegar solution.

1. Determine an equation that models this situation, and explain what each part represents in the situation.

2. Solve the equation and find the amount of each of the solutions that were combined.

Exit Ticket Sample Solutions

A 25% vinegar solution is combined with triple the amount of a 45% vinegar solution and a 5% vinegar solution resulting in 20 milliliters of a 30% vinegar solution.

1. Determine an equation that models this situation, and explain what each part represents in the situation.

Let s represent the number of milliliters of the first vinegar solution.

$$(0.25)(s) + (0.45)(3s) + (0.05)(20 - 4s) = (0.3)(20)$$

2. Solve the equation, and find the amount of each of the solutions that were combined.

$$0.25s + 1.35s + 1 - 0.2s = 6$$
$$1.6s - 0.2s + 1 = 6$$
$$1.4s + 1 - 1 = 6 - 1$$
$$1.4s \div 1.4 = 5 \div 1.4$$
$$s \approx 3.57$$
$$3s \approx 3(3.57) = 10.71$$

$$20 - 4s \approx 20 - 4(3.57) = 5.72$$

Around 3.57 mL of the 25% vinegar solution, 10.71 mL of the 45% vinegar solution and 5.72 mL of the 5% vinegar solution were combined to make 20 mL of the 30% vinegar solution.

Problem Set Sample Solutions

1. A 5-liter cleaning solution contains 30% bleach. A 3-liter cleaning solution contains 50% bleach. What percent of bleach is obtained by putting the two mixtures together?

Let x represent the percent of bleach in the resulting mixture.

$$0.3(5) + 0.5(3) = x(8)$$
$$1.5 + 1.5 = 8x$$
$$3 \div 8 = 8x \div 8$$
$$x = 0.375$$

The percent of bleach in the resulting cleaning solution is 37.5%.

2. A container is filled with 100 grams of bird feed that is 80% seed. How many grams of bird feed containing 5% seed must be added to get bird feed that is 40% seed?

Let x represent the amount of bird feed, in grams, to be added.

$$0.8(100) + 0.05x = 0.4(100 + x)$$
$$80 + 0.05x = 40 + 0.4x$$
$$80 - 40 + 0.05x = 40 - 40 + 0.4x$$
$$40 + 0.05x = 0.4x$$
$$40 + 0.05x - 0.05x = 0.4x - 0.05x$$
$$40 \div 0.35 = 0.35x \div 0.35$$
$$x \approx 114.3$$

About 114.3 grams of the bird seed containing 5% seed must be added.

3. A container is filled with 100 grams of bird feed that is 80% seed. Tom and Sally want to mix the 100 grams with bird feed that is 5% seed to get a mixture that is 40% seed. Tom wants to add 114 grams of the 5% seed, and Sally wants to add 115 grams of the 5% seed mix. What will be the percent of seed if Tom adds 114 grams? What will be the percent of seed if Sally adds 115 grams? How much do you think should be added to get 40% seed?

If Tom adds 114 grams, then let x be the percent of seed in his new mixture. $214x = 0.8(100) + 0.05(114)$. Solving, we get the following:

$$x = \frac{80 + 5.7}{214} = \frac{85.7}{214} \approx 0.4005 = 40.05\%.$$

If Sally adds 115 grams, then let y be the percent of seed in her new mixture. $215y = 0.8(100) + 0.05(115)$. Solving, we get the following:

$$y = \frac{80 + 5.75}{215} = \frac{85.75}{215} \approx 0.3988 = 39.88\%.$$

The amount to be added should be between 114 and 115 grams. It should probably be closer to 114 because 40.05% is closer to 40% than 39.88%.

4. Jeanie likes mixing leftover salad dressings together to make new dressings. She combined 0.55 L of a 90% vinegar salad dressing with 0.45 L of another dressing to make 1 L of salad dressing that is 60% vinegar. What percent of the second salad dressing was vinegar?

Let c represent the percent of vinegar in the second salad dressing.

$$0.55(0.9) + (0.45)(c) = 1(0.6)$$
$$0.495 + 0.45c = 0.6$$
$$0.495 - 0.495 + 0.45c = 0.6 - 0.495$$
$$0.45c = 0.105$$
$$0.45c \div 0.45 = 0.105 \div 0.45$$
$$c \approx 0.233$$

The second salad dressing was around 23% vinegar.

5. Anna wants to make 30 mL of a 60% salt solution by mixing together a 72% salt solution and a 54% salt solution. How much of each solution must she use?

Let s represent the amount, in milliliters, of the first salt solution.

$$0.72(s) + 0.54(30 - s) = 0.60(30)$$
$$0.72s + 16.2 - 0.54s = 18$$
$$0.18s + 16.2 = 18$$
$$0.18s + 16.2 - 16.2 = 18 - 16.2$$
$$0.18s = 1.8$$
$$s = 10$$

Anna needs 10 mL of the 72% solution and 20 mL of the 54% solution.

6. **A mixed bag of candy is 25% chocolate bars and 75% other filler candy. Of the chocolate bars, 50% of them contain caramel. Of the other filler candy, 10% of them contain caramel. What percent of candy contains caramel?**

   Let c represent the percent of candy containing caramel in the mixed bag of candy.

$$\begin{aligned} 0.25(0.50) + (0.75)(0.10) &= 1(c) \\ 0.125 + 0.075 &= c \\ 0.2 &= c \end{aligned}$$

   In the mixed bag of candy, 20% of the candy contains caramel.

7. **A local fish market receives the daily catch of two local fishermen. The first fisherman's catch was 84% fish while the rest was other non-fish items. The second fisherman's catch was 76% fish while the rest was other non-fish items. If the fish market receives 75% of its catch from the first fisherman and 25% from the second, what was the percent of other non-fish items the local fish market bought from the fishermen altogether?**

   Let n represent the percent of non-fish items of the total market items.

$$\begin{aligned} 0.75(0.16) + 0.25(0.24) &= n \\ 0.12 + 0.06 &= n \\ 0.18 &= n \end{aligned}$$

   The percent of non-fish items in the local fish market is 18%.