Lesson 16: Population Problems

Student Outcomes

- Students write and use algebraic expressions and equations to solve percent word problems related to populations of people and compilations.

Lesson Notes

In this module, students have continued to deepen their understanding of ratios and proportional relationships by solving a variety of multi-step percent problems using algebraic equations, expressions, and visual models. The concept relating 100% as "a whole" is a foundation that students applied in problems including percent increase and decrease, percent error, markups, markdowns, commission, and scale drawings.

Lessons 16–18 provide students with further applications related to percents—specifically, problems involving populations, mixtures, and counting. Students will apply their knowledge of algebra from Module 3 to solve multi-step percent word problems. In Lessons 16 and 17, students will use the equation $\text{Quantity} = \text{Percent} \times \text{Whole}$ to solve mixture and population problems. Lesson 18 concludes Topic D with counting problems involving percents, which prepare students for probability.

Classwork

Opening Exercise (4 minutes)

Students will work with partners to fill in the information in the table. Remind students that a vowel is a, e, i, o, or u.

Opening Exercise

Number of girls in classroom:	**Number of boys in classroom:**	**Total number of students in classroom:**
Percent of the total number of students that are girls:	**Percent of the total number of students that are boys:**	**Percent of boys and girls in the classroom:**
Number of girls whose names start with a vowel:	**Number of boys whose names start with a vowel:**	**Number of students whose names start with a vowel:**
Percent of girls whose names start with a vowel:	**Percent of boys whose names start with a vowel:**	
Percent of the total number of students that are girls whose names start with a vowel:	**Percent of the total number of students that are boys whose names start with a vowel:**	**Percent of students whose names start with a vowel:**

Discussion (5 minutes)

- How did you calculate the percent of boys in the class? How did you calculate the percent of girls in the class?
  - *Take the number of each gender group, divide by the total number of students in the class, then multiply by* 100%*.*
- What is the difference between the percent of girls whose names begin with a vowel and the percent of students who are girls whose names begin with a vowel?
  - *The first is the number of girls whose names begin with a vowel divided by the total number of girls, as opposed to the number of girls whose names begin with a vowel divided by the total number of students.*
- Is there a relationship between the two?
  - *Yes, if you multiply the percent of students who are girls and the percent of girls whose names begin with a vowel, it equals the percent of students who are girls and whose names begin with a vowel.*
- If the percent of boys whose names start with a vowel and percent of girls whose names start with a vowel were given and you were to find out the percent of all students whose names start with a vowel, what other information would be necessary?
  - *You would need to know the percent of the total number of students that are boys or the percent of the total number of students that are girls.*

Example 1 (5 minutes)

Individually, students will read and make sense of the word problem. Class will reconvene to work out the problem together.

Scaffolding:

Consider offering premade tape diagrams for students. Also consider starting with tasks that are both simpler and more concrete, such as, "Out of 100 people, 60% are girls, and 20% of the girls wear glasses. How many of the total are girls that wear glasses?" Relating the visual models to simpler examples will lead towards success with more complex problems.

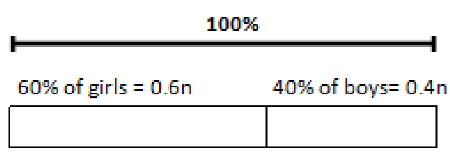
MP.1

Example 1

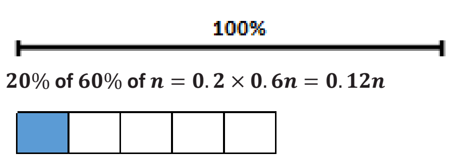
A school has 60% girls and 40% boys. If 20% of the girls wear glasses and 40% of the boys wear glasses, what percent of all students wears glasses?

Let n represent the number of students in the school.

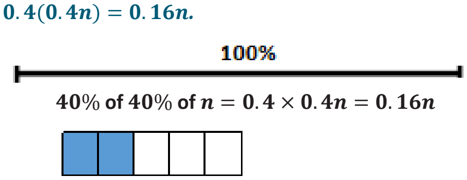
The number of girls is $0.6n$. The number of boys is $0.4n$.



The number of girls wearing glasses is as follows:
$0.2(0.6n) = 0.12n$.



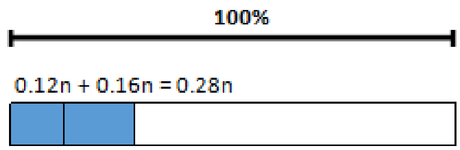
The number of boys wearing glasses is as follows:
$0.4(0.4n) = 0.16n$.



The total number of students wearing glasses is $0.12n + 0.16n = 0.28n$.

$0.28 = 28\%$, so 28% of the students wear glasses.

- Can you explain the reasonableness of the answer?
  - *Yes, if we assume there are* 100 *students,* 20% *of* 60 *girls is* 12 *girls, and* 40% *of* 40 *boys is* 16 *boys. The number of students who wear glasses would be* 28 *out of* 100 *or* 28%.



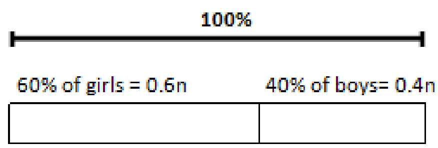
Exercises 1–2 (5 minutes)

Exercise 1

How does the percent of students who wear glasses change if the percent of girls and boys remains the same (that is, 60% **girls and** 40% **boys), but** 20% **of the boys wear glasses and** 40% **of the girls wear glasses?**

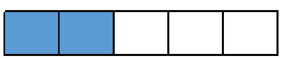
Let n *represent the number of students in the school.*

The number of girls is $0.6n$*. The number of boys is* $0.4n$*.*



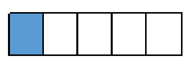
Girls who wear glasses:

40% of 60% of $n = 0.4 \times 0.6n = 0.24n$

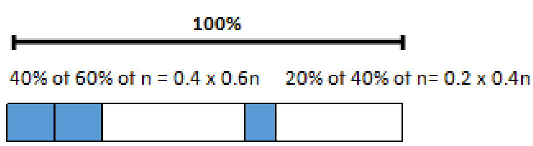


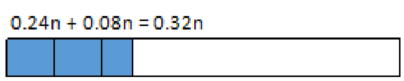
Boys who wear glasses:

20% of 40% of $n = 0.2 \times 0.4n = 0.08n$



Students who wear glasses:





32% *of students wear glasses.*

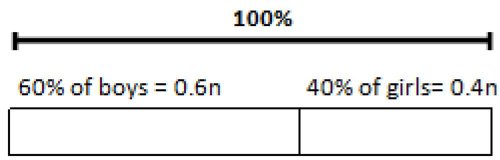
Exercise 2

How would the percent of students who wear glasses change if the percent of girls is 40% of the school and the percent of boys is 60% of the school, and 40% of the girls wear glasses and 20% of the boys wear glasses? Why?

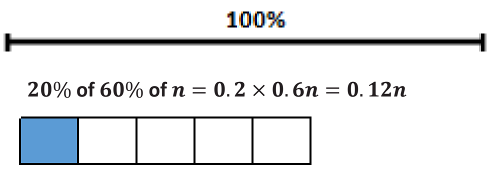
The number of students wearing glasses would be equal to the answer for Example 1 because all of the percents remain the same except that a swap is made between the boys and girls. So, the number of boys wearing glasses is swapped with the number of girls, and the number of girls wearing glasses is swapped with the number of boys, but the total number of students wearing glasses is the same.

Let n represent the number of students in the school.

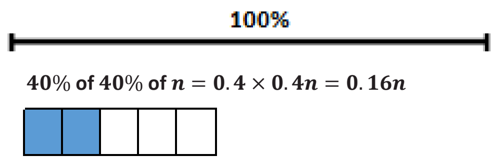
The number of boys is $0.6n$. The number of girls is $0.4n$.



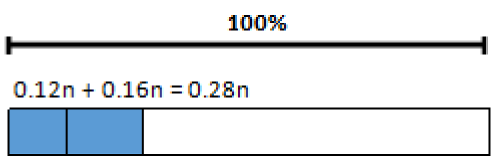
Boys who wear glasses:



Girls who wear glasses:



Students who wear glasses:



MP.2 & MP.7

- Explain why the expressions $0.12n + 0.16n$ and $0.28n$ are equivalent. Also, explain how they reveal different information about the situation.
  - *The equivalence can be shown using the distributive property; $0.12n$ represents the fact that 12% of the total are girls that wear glasses; $0.16n$ represents the fact that 16% of the total are boys that wear glasses; $0.28n$ represents the fact that 28% of the total wear glasses.*

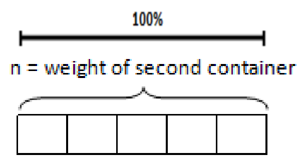
Example 2 (5 minutes)

Give students time to set up the problem using a tape diagram. Work out the example as a class.

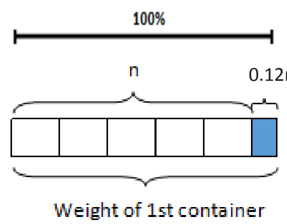
Example 2

The weight of the first of three containers is 12% more than the second, and the third container is 20% lighter than the second. By what percent is the first container heavier than the third container?

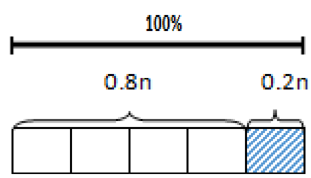
Let n represent the weight of the second container. (The tape diagram representation for the second container is divided into five equal parts to show 20%. This will be useful when drawing a representation for the third container and also when sketching a 12% portion for the first container since it will be slightly bigger than half of the 20% portion created.)



The weight of the first container is $(1.12)n$.



The weight of the third container is $(0.80)n$.



The following represents the difference in weight between the first and third container:

$$1.12n - 0.80n = 0.32n$$

Recall that the weight of the third container is $0.8n$

$0.32n \div 0.8n = 0.4$. The first container is 40% heavier than the third container.

Or $1.4 \times 100\% = 140\%$, which also shows that the first container is 40% heavier than the third container.

MP.2 & MP.7

- How can we represent the weight of the third container using another expression (besides $0.8n$)?
  - $n - 0.20n$
- Compare these two expressions and what they tell us.
  - *$n - 0.20n$ tells us that the third container is 20% less than the second container, while $0.8n$ shows that the third container is 80% of the second container. Both are equivalent.*
- After rereading the problem, can you explain the reasonableness of the answer?
  - *If the second container weighed 100 lb., then the first container weighs 112 lb., and the third container weighs 80 lb. $112 \div 80 = 1.4$. So, the first container is 40% more than the third.*

- What is the importance of the second container?
  - *It is the point of reference for both the first and third containers, and both expressions are written in terms of the second container.*

Exercise 3 (3 minutes)

Exercise 3

Matthew's pet dog is 7% heavier than Harrison's pet dog, and Janice's pet dog is 20% lighter than Harrison's. By what percent is Matthew's dog heavier than Janice's?

Let h represent the weight of Harrison's dog.

Matthew's dog is $1.07h$, and Janice's dog is $0.8h$.

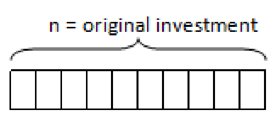
Since $1.07 \div 0.8 = \frac{107}{80} = 1.3375$, Mathew's dog is 33.75% heavier than Janice's dog.

Example 3 (5 minutes)

Example 3

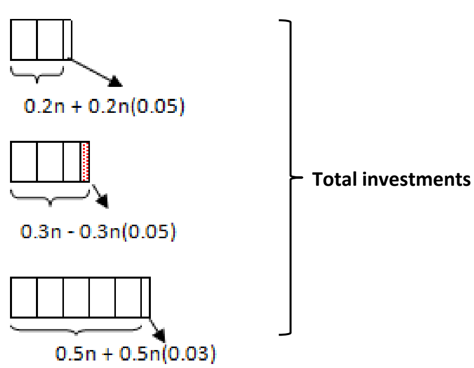
In one year's time, 20% of Ms. McElroy's investments increased by 5%, 30% of her investments decreased by 5%, and 50% of her investments increased by 3%. By what percent did the total of her investments increase?

Let n represent the dollar amount of Ms. McElroy's investments before the changes occurred during the year.



After the changes, the following represents the dollar amount of her investments:

$$\begin{aligned} &0.2n(1.05) + 0.3n(0.95) + 0.5n(1.03) \\ &= 0.21n + 0.285n + 0.515n \\ &= 1.01n. \end{aligned}$$



Since $1.01 = 101\%$, Ms. McElroy's total investments increased by 1%.

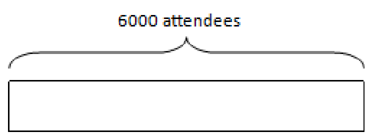
- How is an increase of 5% denoted in the equation?
  - *The result of a* 5% *increase is the whole* $(100\% = 1)$ *plus another* 5%*, which is five hundredths, and* $1 + 0.05 = 1.05$*, which is multiplied by* n*, Ms. McElroy's original investments.*
- How else can the increase of 5% be written in the equation?
  - *It can be written as the sum of the original amount and the original amount multiplied by* 0.05*.*
- Why is the 5% decrease denoted as 0.95 and an increase of 5% denoted as 1.05?
  - *The decrease is* 5% *less than* 100%*, so* $100\% - 5\% = 95\%$*. In decimal form it is* 0.95*. An increase is* 5% *more than* 100%*. The decimal form is* 1.05*.*

Exercise 4 (5 minutes)

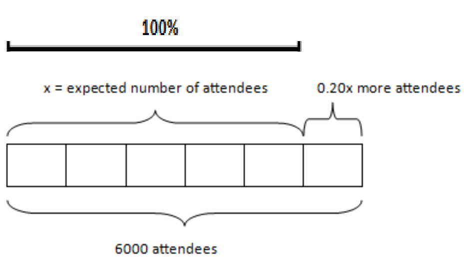
Exercise 4

A concert had $6,000$ audience members in attendance on the first night and the same on the second night. On the first night, the concert exceeded expected attendance by 20%, while the second night was below the expected attendance by 20%. What was the difference in percent of concert attendees and expected attendees for both nights combined?

Let x represent the expected number of attendees on the first night and y represent the number expected on the second night.



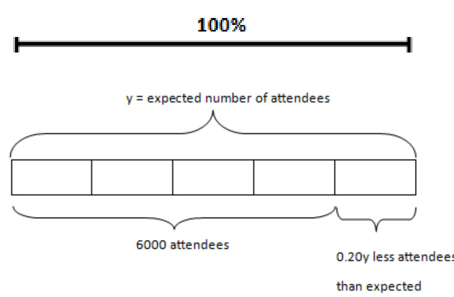
First night: $x + 0.2x = 6,000$



$$1.2x = 6,000$$
$$x = 5,000$$
$$6,000 - 5,000 = 1,000$$

The first night was attended by $1,000$ more people than expected.

Second night: $y - 0.2y = 6,000$



$$0.8y = 6,000$$
$$y = 7,500$$
$$7,500 - 6,000 = 1,500$$

The second night was attended by $1,500$ less people than expected.

$$5,000 + 7,500 = 12,500$$

$12,500$ ***people were expected in total on both nights.***

$1,500 - 1,000 = 500.$ $\frac{500}{12{,}500} \times 100\% = 4\%$. ***The concert missed its expected attendance by 4%.***

Closing (3 minutes)

- What is the importance of defining the variable for percent population problems?
    - *We solve for and set up expressions and equations around the variable. The variable gives us a reference of what the whole* (100%) *is to help us figure out the parts or percents that are unknown.*
- How do tape diagrams help to solve for percent population problems?
    - *It is a visual or manipulative, which helps us understand the problem and set up an equation. Coupled with the* 100% *bar, it tells us whether or not our answers are reasonable.*
- Give examples of equivalent expressions from this lesson, and explain how they reveal different information about the situation.
    - *Answers may vary. For example, in Exercise 3, the first night's attendance is expressed as* $x + 0.2x$*. This expression shows that there were* 20% *more attendees than expected. The equivalent expression would be* $1.2x$*.*

Lesson Summary

When solving a percent population problem, you must first define the variable. This gives a reference of what the whole is. Then, multiply the sub-populations (such as girls and boys) by the given category (total students wearing glasses) to find the percent in the whole population.

Exit Ticket (5 minutes)

Name ____ Date ____________________

Lesson 16: Population Problems

Exit Ticket

1. Jodie spent 25% less buying her English reading book than Claudia. Gianna spent 9% less than Claudia. Gianna spent more than Jodie by what percent?

2. Mr. Ellis is a teacher who tutors students after school. Of the students he tutors, 30% need help in computer science and the rest need assistance in math. Of the students who need help in computer science, 40% are enrolled in Mr. Ellis's class during the school day. Of the students who need help in math, 25% are enrolled in his class during the school day. What percent of the after-school students are enrolled in Mr. Ellis's classes?

Exit Ticket Sample Solutions

1. Jodie spent 25% less buying her English reading book than Claudia. Gianna spent 9% less than Claudia. Gianna spent more than Jodie by what percent?

   Let c represent the amount Claudia spent, in dollars. The number of dollars Jodie spent was $0.75c$, and the number of dollars Gianna spent was $0.91c$. $0.91c \div 0.75c = \frac{91}{75} \times 100\% = 121\frac{1}{3}\%$. Gianna spent $21\frac{1}{3}\%$ more than Jodie.

2. Mr. Ellis is a teacher who tutors students after school. Of the students he tutors, 30% need help in computer science and the rest need assistance in math. Of the students who need help in computer science, 40% are enrolled in Mr. Ellis's class during the school day. Of the students who need help in math, 25% are enrolled in his class during the school day. What percent of the after-school students are enrolled in Mr. Ellis's classes?

   Let t represent the after-school students tutored by Mr. Ellis.

   Computer science after-school students: $0.3t$

   Math after-school students: $0.7t$

   After-school computer science students who are also Mr. Ellis's students: $0.4 \times 0.3t = 0.12t$

   After-school math students who are also Mr. Ellis's students: $0.25 \times 0.7t = 0.175t$

   Number of after-school students who are enrolled in Mr. Ellis's classes: $0.12t + 0.175t = 0.295t$

   Out of all the students Mr. Ellis tutors, 29.5% of the tutees are enrolled in his classes.

Problem Set Sample Solutions

1. One container is filled with a mixture that is 30% acid. A second container is filled with a mixture that is 50% acid. The second container is 50% larger than the first, and the two containers are emptied into a third container. What percent of acid is the third container?

   Let t be the amount of mixture in the first container. Then the second container has $1.5t$, and the third container has $2.5t$.

   The amount of acid in the first container is $0.3t$, the amount of acid in the second container is $0.5(1.5t) = 0.75t$, and the amount of acid in the third container is $1.05t$. The percent of acid in the third container is $\frac{1.05}{2.5} \times 100\% = 42\%$.

2. The store's markup on a wholesale item is 40%. The store is currently having a sale, and the item sells for 25% off the retail price. What is the percent of profit made by the store?

   Let w represent the wholesale price of an item.

   Retail price: $1.4w$

   Sale price: $1.4w - (1.4w \times 0.25) = 1.05w$

   The store still makes a 5% profit on a retail item that is on sale.

3. **During lunch hour at a local restaurant, 90% of the customers order a meat entrée and 10% order a vegetarian entrée. Of the customers who order a meat entrée, 80% order a drink. Of the customers who order a vegetarian entrée, 40% order a drink. What is the percent of customers who order a drink with their entrée?**

   Let e represent lunch entrées.

   Meat entrées: $0.9e$

   Vegetarian entrées: $0.1e$

   Meat entrées with drinks: $0.9e \times 0.8 = 0.72e$

   Vegetarian entrées with drinks: $0.1e \times 0.4 = 0.04e$

   Entrées with drinks: $0.72e + 0.04e = 0.76e$. *Therefore,* 76% *of lunch entrées are ordered with a drink.*

4. **Last year's spell-a-thon spelling test for a first grade class had 15% more words with four or more letters than this year's spelling test. Next year, there will be 5% less than this year. What percent more words have four or more letters in last year's test than next year's?**

   Let t represent this year's amount of spell-a-thon words with four letters or more.

   Last year: $1.15t$

   Next year: $0.95t$

   $1.15\ t \div 0.95t \times 100\% \approx 121\%$. *There were about* 21% *more words with four or more letters last year than there will be next year.*

5. **An ice cream shop sells 75% less ice cream in December than in June. Twenty percent more ice cream is sold in July than in June. By what percent did ice cream sales increase from December to July?**

   Let j represent sales in June.

   December: $0.25j$

   July: $1.20j$

   $1.20 \div 0.25 = 4.8 \times 100\% = 480\%$. *Ice cream sales in July increase by* 380% *from ice cream sales in December.*

6. **The livestock on a small farm the prior year consisted of 40% goats, 10% cows, and 50% chickens. This year, there is a 5% decrease in goats, 9% increase in cows, and 15% increase in chickens. What is the percent increase or decrease of livestock this year?**

   Let l *represent the number of livestock the prior year.*

   Goats decrease: $0.4l - (0.4l \times 0.05) = 0.38l$ *or* $0.95(0.4l) = 0.38l$

   Cows increase: $0.1\ l + (0.1l \times 0.09\ ) = 0.109l$ *or* $1.09(0.1l) = 0.109l$

   Chickens increase: $0.5k + (\ 0.5k \times 0.15) = 0.575l$ *or* $1.15(0.5l) = 0.575l$

   $0.38l + 0.109l + 0.575l = 1.064l$. *There is an increase of* 6.4% *in livestock.*

7. In a pet shelter that is occupied by 55% dogs and 45% cats, 60% of the animals are brought in by concerned people who found these animals in the streets. If 90% of the dogs are brought in by concerned people, what is the percent of cats that are brought in by concerned people?

Let c represent the percent of cats brought in by concerned people.

$$0.55(0.9) + (0.45)(c) = 1(0.6)$$
$$0.495 + 0.45c = 0.6$$
$$0.495 - 0.495 + 0.45c = 0.6 - 0.495$$
$$0.45c = 0.105$$
$$0.45c \div 0.45 = 0.105 \div 0.45$$
$$c \approx 0.233$$

About 23% of the cats brought into the shelter are brought in by concerned people.

8. An artist wants to make a particular teal color paint by mixing a 75% blue hue and 25% yellow hue. He mixes a blue hue that has 85% pure blue pigment and a yellow hue that has 60% of pure yellow pigment. What is the percent of pure pigment that is in the resulting teal color paint?

Let p represent the teal color paint.

$$(0.75 \times 0.85p) + (0.25 \times 0.6p) = 0.7875p$$

78.75% of pure pigment is in the resulting teal color paint.

9. On Mina's block, 65% of her neighbors do not have any pets, and 35% of her neighbors own at least one pet. If 25% of the neighbors have children but no pets, and 60% of the neighbors who have pets also have children, what percent of the neighbors have children?

Let n represent the number of Mina's neighbors.

Neighbors who do not have pets: $0.65n$

Neighbors who own at least one pet: $0.35n$

Neighbors who have children but no pets: $0.25 \times 0.65n = 0.1625n$

Neighbors who have children and pets: $0.6 \times 0.35n = 0.21n$

Percent of neighbors who have children: $0.1625n + 0.21n = 0.3725n$

37.25% of Mina's neighbors have children.