Lesson 5: Finding One Hundred Percent Given Another Percent

Student Outcomes

- Students find 100% of a quantity (the whole) when given a quantity that is a percent of the whole by using a variety of methods including finding 1%, equations, mental math using factors of 100, and double number line models.
- Students solve word problems involving finding 100% of a given quantity with and without using equations.

Classwork

Opening Exercise (5 minutes)

Students recall factors of 100 and their multiples to complete the table below. The discussion that follows introduces students to a means of calculating whole quantities through the use of a double number line.

Opening Exercise

What are the whole number factors of 100? What are the multiples of those factors? How many multiples are there of each factor (up to 100)?

Factors of 100	Multiples of the Factors of 100	Number of Multiples
100	100	1
50	$50,\ 100$	2
25	$25,\ 50,\ 75,\ 100$	4
20	$20,\ 40,\ 60,\ 80,\ 100$	5
10	$10,\ 20,\ 30,\ 40,\ 50,\ 60,\ 70,\ 80,\ 90,\ 100$	10
5	$5,\ 10,\ 15,\ 20,\ 25,\ 30,\ 35,\ 40,\ 45,\ 50, \ldots,\ 75,\ 80,\ 85,\ 90,\ 95,\ 100$	20
4	$4,\ 8,\ 12,\ 16,\ 20,\ 24,\ 28,\ 32,\ 36,\ 40, \ldots,\ 80,\ 84,\ 88,\ 92,\ 96,\ 100$	25
2	$2,\ 4,\ 6,\ 8,\ 10,\ 12, 14,\ 16,\ 18,\ 20,\ 22, \ldots,\ 88,\ 90,\ 92,\ 94,\ 96,\ 98,\ 100$	50
1	$1,\ 2,\ 3,\ 4,\ 5,\ 6, \ldots,\ 98,\ 99,\ 100$	100

- How do you think we can use these whole number factors in calculating percents on a double number line?
  - *The factors represent all ways by which we could break 100% into equal-sized whole number intervals. The multiples listed would be the percents representing each cumulative interval. The number of multiples would be the number of intervals.*

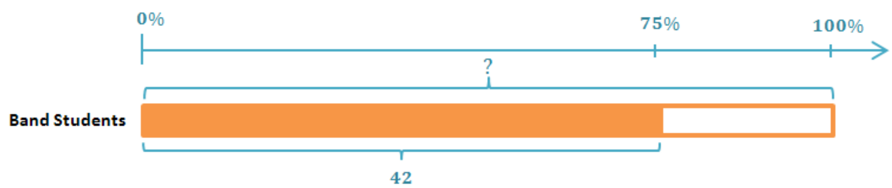
Example 1 (5 minutes): Using a Modified Double Number Line with Percents

MP.2 & MP.4

The use of visual models is a powerful strategy for organizing and solving percent problems. In this example (and others that follow), the double number line is modified so that it is made up of a percent number line and a bar model. This model provides a visual representation of how quantities compare and what percent they correspond with. We use the greatest common factor of the given percent and 100 to determine the number of equal-sized intervals to use.

> **Example 1: Using a Modified Double Number Line with Percents**
>
> **The 42 students who play wind instruments represent 75% of the students who are in band. How many students are in band?**

- Which quantity in this problem represents the whole?
  - *The total number of students in band is the whole, or* 100%.
- Draw the visual model shown with a percent number line and a tape diagram.



- Use the double number line to find the total number of students in band.
  - 100% *represents the total number of students in band, and* 75% *is* $\frac{3}{4}$ *of* 100%. *The greatest common factor of* 75 *and* 100 *is* 25.

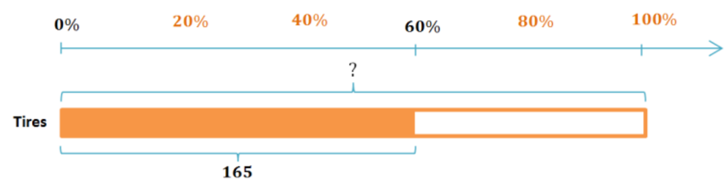
> ***The greatest common factor of 75 and 100 is 25. So, I divided the tape diagram into four equal-sized sections, each representing 25%. There are three intervals of 25% in 75%. Each of these intervals would represent 14 band students; I determined this by dividing 42 by 3. The last interval also represents 25%, or 14 students, making four groups of 14 band students in each. $14 \times 4 = 56$***
>
> ***There are 56 students in the band.***

Exercises 1–3 (10 minutes)

Solve Exercises 1–3 using a modified double number line.

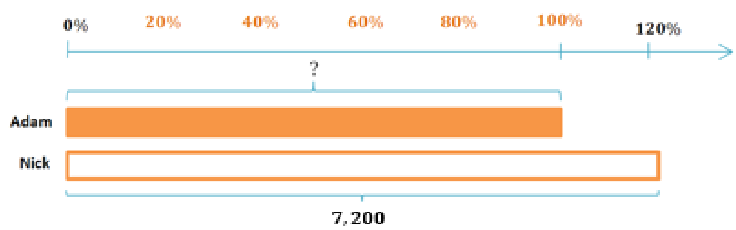
Exercises 1–3

1. **Bob's Tire Outlet sold a record number of tires last month. One salesman sold 165 tires, which was 60% of the tires sold in the month. What was the record number of tires sold?**



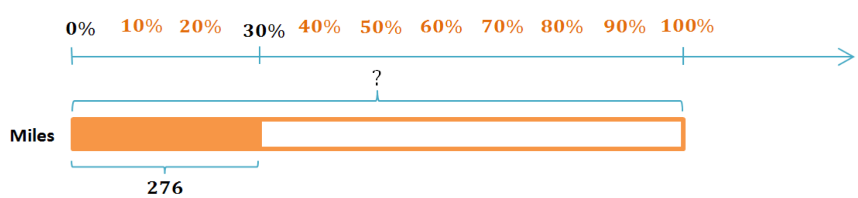
The salesman's total is being compared to the total number of tires sold by the store, so the total number of tires sold is the whole quantity. The greatest common factor of 60 and 100 is 20, so I divided the percent line into five equal-sized intervals of 20%. 60% is three of the 20% intervals, so I divided the salesman's 165 tires by 3 and found that 55 tires corresponds with each 20% interval. 100% consists of five 20% intervals, which corresponds to five groups of 55 tires. Since $5 \cdot 55 = 275$, the record number of tires sold was 275 tires.

2. **Nick currently has $7,200$ points in his fantasy baseball league, which is 20% more points than Adam. How many points does Adam have?**



Nick's points are being compared to Adam's points, so Adam's points are the whole quantity. Nick has 20% more points than Adam, so Nick really has 120% of Adam's points. The greatest common factor of 120 and 100 is 20, so I divided the 120% on the percent line into six equal-sized intervals. I divided Nick's $7,200$ points by 6 and found that $1,200$ points corresponds to each 20% interval. Five intervals of 20% make 100%, and five intervals of $1,200$ points totals $6,000$ points. Adam has $6,000$ points in the fantasy baseball league.

3. **Kurt has driven 276 miles of his road trip but has 70% of the trip left to go. How many more miles does Kurt have to drive to get to his destination?**



With 70% of his trip left to go, Kurt has only driven 30% of the way to his destination. The greatest common factor of 30 and 100 is 10, so I divided the percent line into ten equal-sized intervals. 30% is three of the 10% intervals, so I divided 276 miles by 3 and found that 92 miles corresponds to each 10% interval. Ten intervals of 10% make 100%, and ten intervals of 92 miles totals 920 miles. Kurt has already driven 276 miles, and $920 - 276 = 644$, so Kurt has 644 miles left to get to his destination.

Example 2 (10 minutes): Mental Math Using Factors of 100

Students use mental math and factors of 100 to determine the whole quantity when given a quantity that is a percent of that whole.

Example 2: Mental Math Using Factors of 100

Answer each part below using only mental math, and describe your method.

a. **If 39 is 1% of a number, what is that number? How did you find your answer?**

39 is 1% of $3,900$. I found my answer by multiplying $39 \cdot 100$ because 39 corresponds with each 1% in 100%, and $1\% \cdot 100 = 100\%$, so $39 \cdot 100 = 3,900$.

b. **If 39 is 10% of a number, what is that number? How did you find your answer?**

39 is 10% of 390. 10 is a factor of 100, and there are ten 10% intervals in 100%. The quantity 39 corresponds to 10%, so there are $39 \cdot 10$ in the whole quantity, and $39 \cdot 10 = 390$.

c. **If 39 is 5% of a number, what is that number? How did you find your answer?**

39 is 5% of 780. 5 is a factor of 100, and there are twenty 5% intervals in 100%. The quantity 39 corresponds to 5%, so there are twenty intervals of 39 in the whole quantity.

$39 \cdot 20$

$39 \cdot 2 \cdot 10$ *Factored 20 for easier mental math.*

$78 \cdot 10 = 780$

d. **If 39 is 15% of a number, what is that number? How did you find your answer?**

39 is 15% of 260. 15 is not a factor of 100, but 15 and 100 have a common factor of 5. If 15% is 39, then because $5 = \frac{15}{3}$, 5% is $13 = \frac{39}{3}$. There are twenty 5% intervals in 100%, so there are twenty intervals of 13 in the whole.

$13 \cdot 20$

$13 \cdot 2 \cdot 10$ *Factored 20 for easier mental math.*

$26 \cdot 10 = 260$

e. **If 39 is 25% of a number, what is that number? How did you find your answer?**

39 is 25% of 156. 25 is a factor of 100, and there are four intervals of 25% in 100%. The quantity 39 corresponds with 25%, so there are $39 \cdot 4$ in the whole quantity.

$39 \cdot 4$

$39 \cdot 2 \cdot 2$ *Factored 4 for easier mental math.*

$78 \cdot 2 = 156$

Exercises 4–5 (8 minutes)

Solve Exercises 4 and 5 using mental math and factors of 100. Describe your method with each exercise.

Exercises 4–5

4. **Derrick had a 0.250 batting average at the end of his last baseball season, which means that he got a hit 25% of the times he was up to bat. If Derrick had 47 hits last season, how many times did he bat?**

   The decimal 0.250 is 25%, which means that Derrick had a hit 25% of the times that he batted. His number of hits is being compared to the total number of times he was up to bat. The 47 hits corresponds with 25%, and since 25 is a factor of 100, $100 = 25 \cdot 4$. I used mental math to multiply the following:

   $47 \cdot 4$

   $(50 - 3) \cdot 4$ *Used the distributive property for easier mental math*

   $200 - 12 = 188$

   Derrick was up to bat 188 times last season.

5. **Nelson used 35% of his savings account for his class trip in May. If he used $\$140$ from his savings account while on his class trip, how much money was in his savings account before the trip?**

   35% of Nelson's account was spent on the trip, which was $\$140$. The amount that he spent is being compared to the total amount of savings, so the total savings represents the whole. The greatest common factor of 35 and 100 is 5. 35% is seven intervals of 5%, so I divided $\$140$ by 7 to find that $\$20$ corresponds to 5%.
   $100\% = 5\% \cdot 20$, so the whole quantity is $\$20 \cdot 20 = \$400$. Nelson's savings account had $\$400$ in it before his class trip.

Closing (2 minutes)

- What does the modified double number line method and the factors of 100 method have in common?
  - *Both methods involve breaking 100% into equal-sized intervals using the greatest common factor of 100 and the percent corresponding to the part.*
- Can you describe a situation where you would prefer using the modified double number line?
  - *Answers will vary.*
- Can you describe a situation where you would prefer using the factors of 100?
  - *Answers will vary.*

Lesson Summary

To find 100% of the whole, you can use a variety of methods, including factors of 100 ($1, 2, 4, 5, 10, 20, 25, 50$, and 100) and double number lines. Both methods will require breaking 100% into equal-sized intervals. Use the greatest common factor of 100 and the percent corresponding to the part.

Exit Ticket (5 minutes)

Name ____ Date ____________________

Lesson 5: Finding One Hundred Percent Given Another Percent

Exit Ticket

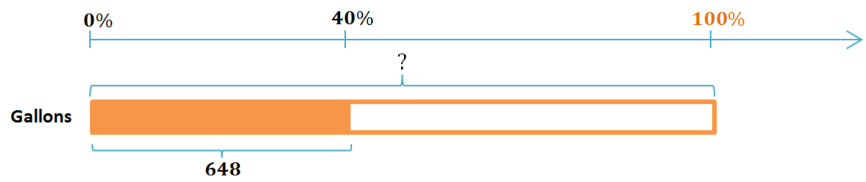
1. A tank that is 40% full contains 648 gallons of water. Use a double number line to find the maximum capacity of the water tank.

2. Loretta picks apples for her grandfather to make apple cider. She brings him her cart with 420 apples. Her grandfather smiles at her and says, "Thank you, Loretta. That is 35% of the apples that we need."

   Use mental math to find how many apples Loretta's grandfather needs. Describe your method.

Exit Ticket Sample Solutions

1. A tank that is 40% full contains 648 gallons of water. Use a double number line to find the maximum capacity of the water tank.



I divided the percent line into intervals of 20% making five intervals of 20% in 100%. I know that I have to divide $\frac{40}{2}$ to get 20, so I divided $\frac{648}{2}$ to get 324 that corresponds with 20%. Since there are five 20% intervals in 100%, there are five 324 gallon intervals in the whole quantity, and $324 \cdot 5 = 1,620$. The capacity of the tank is $1,620$ gallons.

2. Loretta picks apples for her grandfather to make apple cider. She brings him her cart with 420 apples. Her grandfather smiles at her and says "Thank you, Loretta. That is 35% of the apples that we need."
   Use mental math to find how many apples Loretta's grandfather needs. Describe your method.

420 is 35% of $1,200$. 35 is not a factor of 100, but 35 and 100 have a common factor of 5. There are seven intervals of 5% in 35%, so I divided 420 apples into seven intervals; $\frac{420}{7} = 60$. There are 20 intervals of 5% in 100%, so I multiplied as follows:

$60 \cdot 20$

$60 \cdot 2 \cdot 10$

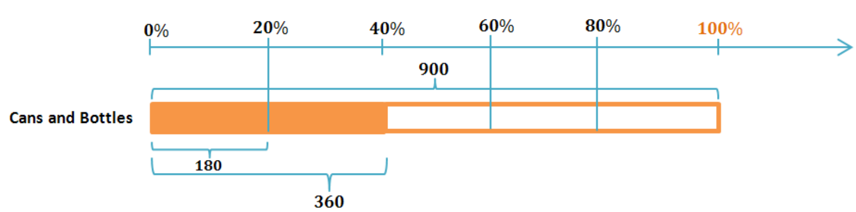
$120 \cdot 10 = 1,200$

Loretta's grandfather needs a total of $1,200$ apples to make apple cider.

Problem Set Sample Solutions

Use a double number line to answer Problems 1–5.

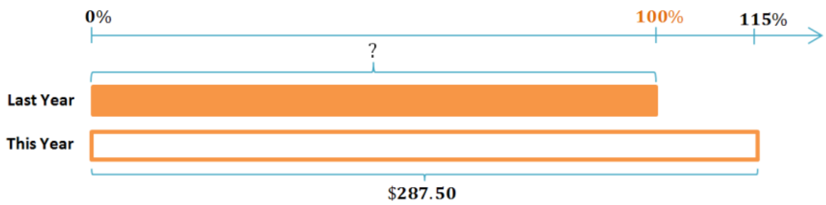
1. Tanner collected 360 cans and bottles while fundraising for his baseball team. This was 40% of what Reggie collected. How many cans and bottles did Reggie collect?



The greatest common factor of 40 and 100 is 20.

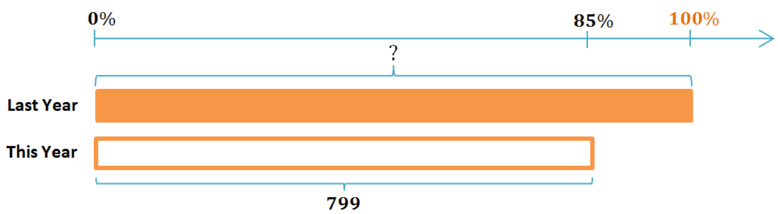
$\frac{1}{2}(40\%) = 20\%$, and $\frac{1}{2}(360) = 180$, so 180 corresponds with 20%. There are five intervals of 20% in 100%, and $5(180) = 900$, so Reggie collected 900 cans and bottles.

2. Emilio paid $\$287.50$ in taxes to the school district that he lives in this year. This year's taxes were a 15% increase from last year. What did Emilio pay in school taxes last year?



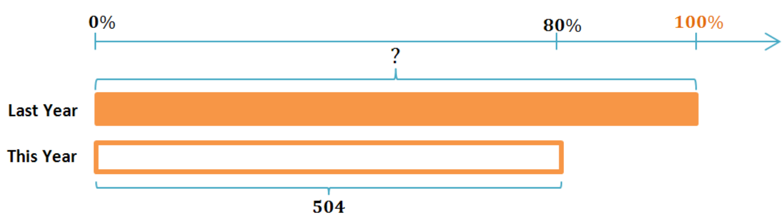
The greatest common factor of 100 and 115 is 5. There are 23 intervals of 5% in 115%, and $\frac{287.5}{23} = 12.5$, so 12.5 corresponds with 5%. There are 20 intervals of 5% in 100%, and $20(12.5) = 250$, so Emilio paid $\$250$ in school taxes last year.

3. A snowmobile manufacturer claims that its newest model is 15% lighter than last year's model. If this year's model weighs 799 lb., how much did last year's model weigh?



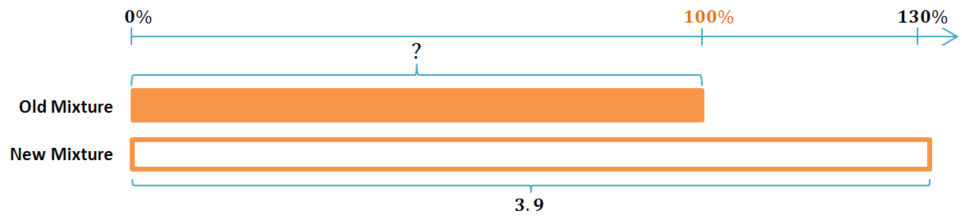
15% lighter than last year's model means 15% less than 100% of last year's model's weight, which is 85%. The greatest common factor of 85 and 100 is 5. There are 17 intervals of 5% in 85%, and $\frac{799}{17} = 47$, so 47 corresponds with 5%. There are 20 intervals of 5% in 100%, and $20(47) = 940$, so last year's model weighed 940 pounds.

4. Student enrollment at a local school is concerning the community because the number of students has dropped to 504, which is a 20% decrease from the previous year. What was the student enrollment the previous year?



A 20% decrease implies that this year's enrollment is 80% of last year's enrollment. The greatest common factor of 80 and 100 is 20. There are 4 intervals of 20% in 80%, and $\frac{504}{4} = 126$, so 126 corresponds to 20%. There are 5 intervals of 20% in 100%, and $5(126) = 630$, so the student enrollment from the previous year was 630 students.

5. The color of paint used to paint a race car includes a mixture of yellow and green paint. Scotty wants to lighten the color by increasing the amount of yellow paint 30%. If a new mixture contains 3.9 liters of yellow paint, how many liters of yellow paint did he use in the previous mixture?



The greatest common factor of 130 and 100 is 10. There are 13 intervals of 10% in 130%, and $\frac{3.9}{13} = 0.3$, so 0.3 corresponds to 10%. There are 10 intervals of 10% in 100%, and $10(0.3) = 3$, so the previous mixture included 3 liters of yellow paint.

Use factors of 100 and mental math to answer Problems 6–10. Describe the method you used.

6. Alexis and Tasha challenged each other to a typing test. Alexis typed 54 words in one minute, which was 120% of what Tasha typed. How many words did Tasha type in one minute?

The greatest common factor of 120 and 100 is 20, and there are 6 intervals of 20% in 120%, so I divided 54 into 6 equal-sized intervals to find that 9 corresponds to 20%. There are five intervals of 20% in 100%, so there are five intervals of 9 words in the whole quantity. $9 \cdot 5 = 45$, so Tasha typed 45 words in one minute.

7. Yoshi is 5% taller today than she was one year ago. Her current height is 168 cm. How tall was she one year ago?

5% taller means that Yoshi's height is 105% of her height one year ago. The greatest common factor of 105 and 100 is 5, and there are 21 intervals of 5% in 105%, so I divided 168 into 21 equal-sized intervals to find that 8 cm corresponds to 5%. There are 20 intervals of 5% in 100%, so there are 20 intervals of 8 cm in the whole quantity. $20 \cdot 8 \text{ cm} = 160 \text{ cm}$, so Yoshi was 160 cm tall one year ago.

8. Toya can run one lap of the track in 1 min. 3 sec., which is 90% of her younger sister Niki's time. What is Niki's time for one lap of the track?

$1 \text{ min. } 3 \text{ sec} = 63 \text{ sec.}$ The greatest common factor of 90 and 100 is 10, and there are nine intervals of 10 in 90, so I divided 63 sec. by 9 to find that 7 sec. corresponds to 10%. There are 10 intervals of 10% in 100%, so 10 intervals of 7 sec. represents the whole quantity, which is 70 sec. $70 \text{ sec.} = 1 \text{ min. } 10 \text{ sec.}$ Niki can run one lap of the track in 1 min. 10 sec.

9. An animal shelter houses only cats and dogs, and there are 25% more cats than dogs. If there are 40 cats, how many dogs are there, and how many animals are there total?

25% more cats than dogs means that the number of cats is 125% the number of dogs. The greatest common factor of 125 and 100 is 25. There are 5 intervals of 25% in 125%, so I divided the number of cats into 5 intervals to find that 8 corresponds to 25%. There are four intervals of 25% in 100%, so there are four intervals of 8 in the whole quantity. $8 \cdot 4 = 32$. There are 32 dogs in the animal shelter.

The number of animals combined is $32 + 40 = 72$, so there are 72 animals in the animal shelter.

10. Angie scored 91 points on a test but only received a 65% grade on the test. How many points were possible on the test?

The greatest common factor of 65 and 100 is 5. There are 13 intervals of 5% in 65%, so I divided 91 points into 13 intervals and found that 7 points corresponds to 5%. There are 20 intervals of 5% in 100%, so I multiplied 7 points times 20, which is 140 points. There were 140 points possible on Angie's test.

For Problems 11–17, find the answer using any appropriate method.

11. **Robbie owns 15% more movies than Rebecca, and Rebecca owns 10% more movies than Joshua. If Rebecca owns 220 movies, how many movies do Robbie and Joshua each have?**

    Robbie owns 253 movies, and Joshua owns 200 movies.

12. **20% of the seventh-grade students have math class in the morning. $16\frac{2}{3}\%$ of those students also have science class in the morning. If 30 seventh-grade students have math class in the morning but not science class, find how many seventh-grade students there are.**

    There are 180 seventh-grade students.

13. **The school bookstore ordered three-ring notebooks. They put 75% of the order in the warehouse and sold 80% of the rest in the first week of school. There are 25 notebooks left in the store to sell. How many three-ring notebooks did they originally order?**

    The store originally ordered 500 three-ring notebooks.

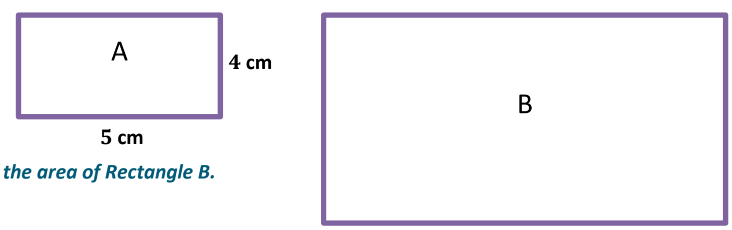
14. **In the first game of the year, the modified basketball team made 62.5% of their foul shot free throws. Matthew made all 6 of his free throws, which made up for 25% of the team's free throws. How many free throws did the team miss altogether?**

    The team attempted 24 free throws, made 15 of them, and missed 9.

15. **Aiden's mom calculated that in the previous month, their family had used 40% of their monthly income for gasoline, and 63% of that gasoline was consumed by the family's SUV. If the family's SUV used $\$261.45$ worth of gasoline last month, how much money was left after gasoline expenses?**

    The amount of money spent on gasoline was $\$415$; the monthly income was $\$1037.50$. The amount left over after gasoline expenses was $\$622.50$.

16. **Rectangle A is a scale drawing of Rectangle B and has 25% of its area. If Rectangle A has side lengths of 4 cm and 5 cm, what are the side lengths of Rectangle B?**



$\text{Area}_A = \text{length} \times \text{width}$

$\text{Area}_A = (5 \text{ cm})(4 \text{ cm})$

$\text{Area}_A = 20 \text{ cm}^2$

The area of Rectangle A is 25% of the area of Rectangle B.

$25\% \times 4 = 100\%$

$20 \times 4 = 80$

So, the area of Rectangle B is 80 cm^2.

The value of the ratio of area A to area B is the square of the scale factor of the side lengths $A\colon B$.

The value of the ratio of area $A\colon B$ is $\frac{20}{80} = \frac{1}{4}$, and $\frac{1}{4} = \left(\frac{1}{2}\right)^2$, so the scale factor of the side lengths $A\colon B$ is $\frac{1}{2}$.

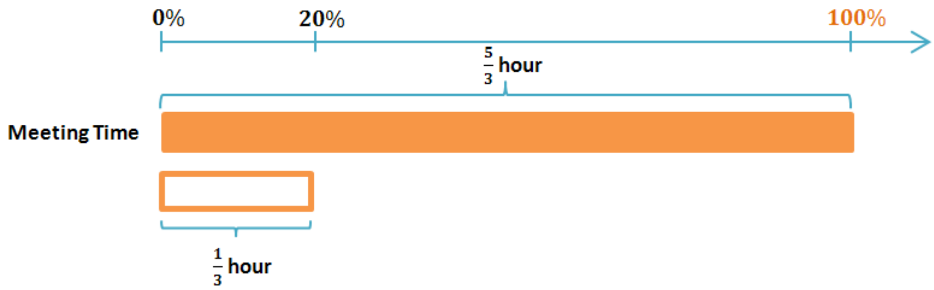
So, using the scale factor:

$\frac{1}{2}(\text{length}_B) = 5 \text{ cm}$; $\text{length}_B = 10 \text{ cm}$

$\frac{1}{2}(\text{width}_B) = 4 \text{ cm}$; $\text{width}_B = 8 \text{ cm}$

The dimensions of Rectangle B are 8 cm and 10 cm.

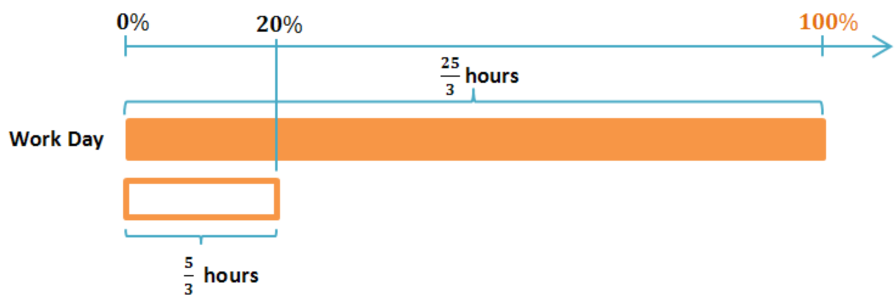
17. Ted is a supervisor and spends 20% of his typical work day in meetings and 20% of that meeting time in his daily team meeting. If he starts each day at 7:30 a.m., and his daily team meeting is from 8:00 a.m. to 8:20 a.m., when does Ted's typical work day end?



20 minutes is $\frac{1}{3}$ of an hour since $\frac{20}{60} = \frac{1}{3}$.

Ted spends $\frac{1}{3}$ hour in his daily team meeting, so $\frac{1}{3}$ corresponds to 20% of his meeting time. There are 5 intervals of 20% in 100%, and $5\left(\frac{1}{3}\right) = \frac{5}{3}$, so Ted spends $\frac{5}{3}$ hours in meetings.

$\frac{5}{3}$ of an hour corresponds to 20% of Ted's work day.



There are 5 intervals of 20% in 100%, and $5\left(\frac{5}{3}\right) = \frac{25}{3}$, so Ted spends $\frac{25}{3}$ hours working. $\frac{25}{3}$ hours $= 8\frac{1}{3}$ hours. Since $\frac{1}{3}$ hour $= 20$ minutes, Ted works a total of 8 hours 20 minutes. If he starts at 7:30 a.m., he works 4 hours 30 minutes until 12:00 p.m., and since $8\frac{1}{3} - 4\frac{1}{2} = 3\frac{5}{6}$, Ted works another $3\frac{5}{6}$ hours after 12:00 p.m.

$\frac{1}{6}$ hour $= 10$ minutes, and $\frac{5}{6}$ hour $= 50$ minutes, so Ted works 3 hours 50 minutes after 12:00 p.m., which is 3:50 p.m. Therefore, Ted's typical work day ends at 3:50 p.m.