Lesson 3: Comparing Quantities with Percent

Student Outcomes

- Students use the context of a word problem to determine which of two quantities represents the whole.
- Students understand that the whole is 100% and think of one quantity as a percent of another using the formula $\text{Quantity} = \text{Percent} \times \text{Whole}$ to problem-solve when given two terms out of three from a quantity, whole, and percent.
- When comparing two quantities, students compute percent more or percent less using algebraic, numeric, and visual models.

Lesson Notes

In this lesson, students compare two quantities using a percent. They will build on their understanding of the relationship between the part, whole, and percent. It is important for students to understand that the part in a percent problem may be greater than the whole, especially in problems that compare two disjoint (or separate) quantities (for example, a quantity of dogs versus a quantity of cats). For this reason, the formula $\text{Part} = \text{Percent} \times \text{Whole}$ will be changed to $\text{Quantity} = \text{Percent} \times \text{Whole}$ from this point forward. This wording will work for problems that compare a part to the whole and in problems comparing one quantity to another. Students continue to relate the algebraic model to visual and arithmetic models and come to understand that an algebraic model will always work for any numbers and is often more efficient than constructing a visual model. Students are prompted to consider when a percent is greater than a quantity as well as times that a percent is less than a quantity as a bridge to concepts related to percent increase and decrease in Lesson 4.

Classwork

Opening Exercises (3 minutes)

Since many of the problems in this lesson represent percents greater than 100, these exercises will review different models that represent percents greater than 100.

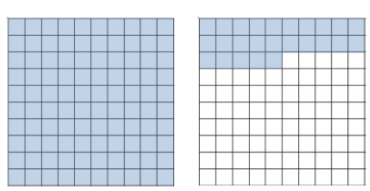
Scaffolding:

Some students may recognize that 125% contains exactly 5 regions of 25%. In this case, they would simply multiply $10 \cdot 5 = 50$ to show that the shaded region represents 50 students. This recognition is okay, but allow the students to make this observation for themselves.

Opening Exercise

If each 10×10 unit square represents one whole, then what percent is represented by the shaded region?

125%



In the model above, 25% represents a quantity of 10 students. How many students does the shaded region represent?

If 25% represents 10 students, then 1% represents $\frac{10}{25}$, or $\frac{2}{5}$, of a student. The shaded region covers 125 square units, or 125%, so since $\frac{2}{5} \cdot 125 = 50$, the shaded region represents 50 students.

Example 1 (20 minutes)

Model Example 1, part (a) with students using a visual model; then, shift to numeric and algebraic approaches in parts (b) and (c). To highlight MP.1, give students an opportunity to engage with the parts of Example 1 before modeling with them. Students are equipped to understand the problems based on knowledge of percents. Use scaffolding questions as needed to assist students in their reasoning.

Example 1

a. **The members of a club are making friendship bracelets to sell to raise money. Anna and Emily made 54 bracelets over the weekend. They need to produce 300 bracelets by the end of the week. What percent of the bracelets were they able to produce over the weekend?**

- What quantity represents the whole, and how do you know?
  - *The total number of bracelets is the whole because the number of bracelets that Anna and Emily produced is being compared to it.*

It will often be helpful to include a percent number line in visual models to show that 100% corresponds with the whole quantity. This will be used to a greater extent in future lessons.

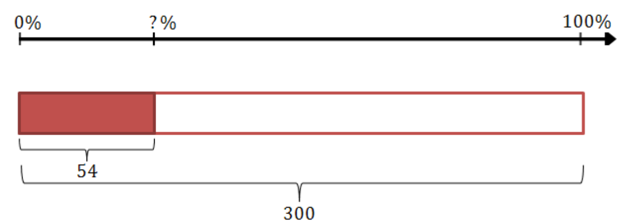
300 represents 100% of the bracelets.

$$\frac{54}{300} = \frac{18}{100}$$

$$\frac{18}{100} = 0.18 = 18\%$$

54 bracelets represents 18% of the whole.

Anna and Emily were able to produce 18% of the total number of bracelets over the weekend.



Next, solve the problem using the percent formula. Compare the steps used to solve the equation to the arithmetic steps previously used with the tape diagram.

$\text{Quantity} = \text{Percent} \times \text{Whole}$

Let p represent the unknown percent.

$$54 = p(300)$$

$$\frac{1}{300}(54) = \frac{1}{300}(300)p$$

$$\frac{54}{300} = 1p$$

$$\frac{18}{100} = p$$

$$\frac{18}{100} = 0.18 = 18\%$$

Anna and Emily were able to produce 18% of the total bracelets over the weekend.

- What similarities do you observe between the arithmetic method and the algebraic method?
  - *In both cases, we divided the part* (54) *by the whole quantity* (300) *to get the quotient* 0.18.

b. Anna produced 32 of the 54 bracelets produced by Emily and Anna over the weekend. Write the number of bracelets that Emily produced as a percent of those that Anna produced.

- What is the whole quantity, and how do you know?
  - *The whole quantity is the number of bracelets that Anna produced because the problem asks us to compare the number of bracelets that Emily produced to the number that Anna produced.*
- How does the context of part (b) differ from the context of part (a)?
  - *The whole quantity is not the same. In part (a) the whole quantity was the total number of bracelets to be produced, and in part (b) the whole quantity was the number of bracelets that Anna produced over the weekend.*
  - *In part (a) the number of bracelets that Anna and Emily produced was a part of the whole quantity of bracelets. In part (b) the number of bracelets that Emily produced was not part of the whole quantity. The quantities being compared are separate quantities.*
- Why are we able to compare one of these quantities to the other?
  - *Because the quantities are measured using the same unit, the number of bracelets.*

MP.2

Solve part (b) using both the arithmetic method and the algebraic method.

Arithmetic Method:

32 ***is the whole or*** 100% ***of the bracelets.***

$$\frac{22}{32} = 0.6875$$

$$0.6875 \times 100\% = 68.75\%$$

Algebraic Method:

$\textbf{Quantity} = \textbf{Percent} \times \textbf{Whole}$

Let p ***represent the unknown percent.***

$$22 = p(32)$$

$$\frac{1}{32}(22) = \frac{1}{32}(32)p$$

$$\frac{22}{32} = 1p$$

$$0.6875 = p$$

$$0.6875 = 68.75\%$$

22 ***bracelets are*** 68.75% ***of the number of bracelets that Anna produced. Emily produced*** 22 ***bracelets; therefore, she produced*** 68.75% ***of the number of bracelets that Anna produced.***

- How does each method compare?
  - *In each case we divided the part by the whole quantity and then converted the quotient to a percent.*
- Do you prefer one method over another? Why?
  - *Answers will vary.*

Ask students to solve part (c) using either the arithmetic or the algebraic method.

> **c. Write the number of bracelets that Anna produced as a percent of those that Emily produced.**

MP.2

- What is the whole quantity, and how do you know?
  - *The whole quantity is the number of bracelets that Emily produced over the weekend because the problem asks us to compare the number of bracelets that Anna produced to the number that Emily produced.*
- How do you think this will affect the percent and why?
  - *The percent should be greater than 100% because the part (Anna's 32 bracelets) is greater than the whole (Emily's 22 bracelets).*

> ***Arithmetic Method:***
>
> ***22 is the whole or 100% of the bracelets.***
>
> $$\frac{32}{22}=\frac{16}{11}$$
> $$\frac{16}{11}\cdot 100\% = \frac{1600}{11}\%$$
> $$\frac{1600}{11}\% = 145\frac{5}{11}\%$$
>
> ***Algebraic Method:***
>
> $$\text{Quantity} = \text{Percent} \times \text{Whole}$$
>
> ***Let p represent the unknown percent.***
>
> $$32 = p(22)$$
> $$\frac{1}{22}(32) = \frac{1}{22}(22)p$$
> $$\frac{32}{22} = 1p$$
> $$\frac{16}{11} = p$$
> $$1\frac{5}{11} = p$$
> $$1\frac{5}{11} = 1\frac{5}{11}\times 100\%$$
> $$= 145\frac{5}{11}\%$$
>
> ***32 bracelets are $145\frac{5}{11}\%$ of the number of bracelets that Emily produced. Anna produced 32 bracelets over the weekend, so Anna produced $145\frac{5}{11}\%$ of the number of bracelets that Emily produced.***

Scaffolding:

- The following progression can help students understand why $1\frac{5}{11} = 145\frac{5}{11}\%$:

$1\frac{5}{11}\cdot 1$	Multiplicative identity
$1\frac{5}{11}\cdot 100\%$	Since $100\% = 1$
$\left(1+\frac{5}{11}\right)\cdot 100\%$	Since $1+\frac{5}{11} = 1\frac{5}{11}$
$1(100\%)+\frac{5}{11}(100\%)$	Distributive property
$100\% + \left(\frac{500}{11}\right)\%$	
$100\% + 45\frac{5}{11}\% = 145\frac{5}{11}\%$	

- What percent more did Anna produce in bracelets than Emily? What percent fewer did Emily produce than Anna? Are these numbers the same? Why?
  - *Anna produced $45\frac{5}{11}\%$ more bracelets than Emily. This is because Anna produced more than Emily did, so her quantity is 100% of Emily's quantity plus an additional $45\frac{5}{11}\%$ more.*
  - *Emily produced 31.25% fewer bracelets than Anna. This is because the difference of what Anna produced and what Emily produced is $100\% - 68.75\% = 31.25\%$.*
  - *The numbers are not the same because in each case the percent is calculated using a different whole quantity.*

Fluency Exercise (12 minutes): Part, Whole, or Percent

Students complete two rounds of the Sprint provided at the end of this lesson (Part, Whole, or Percent). Provide one minute for each round of the Sprint. Refer to the Sprints and Sprint Delivery Script sections in the Module Overview for directions to administer a Sprint.

Note: The end of this lesson is designed for teacher flexibility. The Sprint enriches students' fluencies with percents and helps them to be more efficient in future work with percents. However, an alternate set of exercises (Exercises 1–4) is included below if the teacher assesses that students need further practice before attempting problems independently.

Alternate Exercises 1–4 (12 minutes)

Have students use an equation for each problem and justify their solution with a visual or numeric model. After 10 minutes, ask students to present their solutions to the class. Compare and contrast different methods, and emphasize how the algebraic, numeric, and visual models are related. This also provides an opportunity for differentiation.

Exercises

1. **There are 750 students in the seventh-grade class and 625 students in the eighth-grade class at Kent Middle School.**

   a. **What percent is the seventh-grade class of the eighth-grade class at Kent Middle School?**

   The number of eighth graders is the whole amount. Let p represent the percent of seventh graders compared to eighth graders.

   $\text{Quantity} = \text{Percent} \times \text{Whole}$

   Let p represent the unknown percent.

$$750 = p(625)$$
$$750\left(\frac{1}{625}\right) = p(625)\left(\frac{1}{625}\right)$$
$$1.2 = p$$
$$1.2 = 120\%$$

   The number of seventh graders is 120% of the number of eighth graders.

Teacher may choose to ask what percent more are seventh graders than eighth graders.

   There are 20% more seventh graders than eighth graders.

   Alternate solution: There are 125 more seventh graders. $125 = p(625)$, $p = 0.20$. There are 20% more seventh graders than eighth graders.

   b. **The principal will have to increase the number of eighth-grade teachers next year if the seventh-grade enrollment exceeds 110% of the current eighth-grade enrollment. Will she need to increase the number of teachers? Explain your reasoning.**

   The principal will have to increase the number of teachers next year. In part (a), we found out that the seventh grade enrollment was 120% of the number of eighth graders, which is greater than 110%.

2. **At Kent Middle School, there are 104 students in the band and 80 students in the choir. What percent of the number of students in the choir is the number of students in the band?**

   The number of students in the choir is the whole. Let b represent the number of students in the band.

   $\text{Quantity} = \text{Percent} \times \text{Whole}$

   Let p represent the unknown percent.

$$104 = p(80)$$

$$p = 1.3$$
$$1.3 = 130\%$$

The number of students in the band is 130% of the number of students in the choir.

3. **At Kent Middle School, breakfast costs $\$1.25$ and lunch costs $\$3.75$. What percent of the cost of lunch is the cost of breakfast?**

$\text{Quantity} = \text{Percent} \times \text{Whole}$

Let p represent the unknown percent.

$$1.25 = p(3.75)$$
$$1.25\left(\frac{1}{3.75}\right) = p(3.75)\left(\frac{1}{3.75}\right)$$
$$p = \frac{1.25}{3.75}$$
$$p = \frac{1}{3}$$
$$\frac{1}{3} = \frac{1}{3}(100\%) = 33\frac{1}{3}\%$$

$\div 3$ $\$3.75$ *lunch* → $\$1.25$ *breakfast*; 100% → $33\frac{1}{3}\%$ $\div 3$

The cost of breakfast is $33\frac{1}{3}\%$ of the cost of lunch.

Teacher may ask students what percent less than the cost of lunch is the cost of breakfast.

The cost of breakfast is $66\frac{2}{3}\%$ less than the cost of lunch.

Teacher may ask what percent more is the cost of lunch than the cost of breakfast.

Let p represent the percent of lunch to breakfast.

$$3.75 = p(1.25)$$
$$3.75\left(\frac{1}{1.25}\right) = p(1.25)\left(\frac{1}{1.25}\right)$$
$$p = \frac{3.75}{1.25} = 3 = 300\%$$

$\times 3$ $\$1.25$ *breakfast* → $\$3.75$ *lunch*; 100% → 300% $\times 3$

The cost of lunch is 300% of the cost of breakfast.

4. Describe a real-world situation that could be modeled using the equation $398.4 = 0.83(x)$. Describe how the elements of the equation correspond with the real-world quantities in your problem. Then, solve your problem.

Word problems will vary. Sample problem: A new tablet is on sale for 83% of its original sale price. The tablet is currently priced at $\$398.40$. What was the original price of the tablet?

$0.83 = \frac{83}{100} = 83\%$*, so* 0.83 *represents the percent that corresponds with the current price. The current price (*$\$398.40$*) is part of the original price; therefore, it is represented by* 398.4*. The original price is represented by* x *and is the whole quantity in this problem.*

$$398.4 = 0.83x$$
$$\frac{1}{0.83}(398.4) = \frac{1}{0.83}(0.83)x$$
$$\frac{398.4}{0.83} = 1x$$
$$480 = x$$

The original price of the tablet was $\$480.00$*.*

Closing (5 minutes)

- What formula can we use to relate the part, whole, and percent?
  - $\text{Quantity} = \text{Percent} \times \text{Whole}$
- Why did the word *part* change to *quantity* in the percent formula?
  - *When we compare two separate quantities, one quantity is not a part of the other.*
- What are the advantages of using an algebraic representation to solve percent problems?
  - *It can be a quicker way to solve the problem. Sometimes the numbers do not divide evenly, which makes the visual model more complex.*
- Explain how to decide which quantity in a problem should represent the whole.
  - *You need to focus on identifying the quantity that we are finding a percent "of." That quantity will be the whole in the equation or equal to* 100% *when you use a visual or arithmetic model.*

Lesson Summary

- **Visual models or arithmetic methods can be used to solve problems that compare quantities with percents.**
- **Equations can be used to solve percent problems using the basic equation**

$$\textbf{Quantity} = \textbf{Percent} \times \textbf{Whole}.$$

- ***Quantity* in the new percent formula is the equivalent of *part* in the original percent formula.**

Exit Ticket (5 minutes)

Name ____ Date ____________________

Lesson 3: Comparing Quantities with Percent

Exit Ticket

Solve each problem below using at least two different approaches.

1. Jenny's great-grandmother is 90 years old. Jenny is 12 years old. What percent of Jenny's great-grandmother's age is Jenny's age?

2. Jenny's mom is 36 years old. What percent of Jenny's mother's age is Jenny's great-grandmother's age?

Exit Ticket Sample Solutions

Solve each problem below using at least two different approaches.

1. **Jenny's great-grandmother is 90 years old. Jenny is 12 years old. What percent of Jenny's great-grandmother's age is Jenny's age?**

   Algebraic Solution:

   $\text{Quantity} = \text{Percent} \times \text{Whole}$. *Let p represent the unknown percent. Jenny's age is the whole.*

$$12 = p(90)$$
$$12 \cdot \frac{1}{90} = p(90) \cdot \frac{1}{90}$$
$$2 \cdot \frac{1}{15} = p(1)$$
$$\frac{2}{15} = p$$
$$\frac{2}{15} = \frac{2}{15}(100\%) = 13\frac{1}{3}\%$$

   Jenny's age is $13\frac{1}{3}\%$ of her great-grandmother's age.

   Numeric Solution:

   90 is the whole or 100%.

$$\frac{12}{90} = \frac{2}{15}$$
$$\frac{2}{15}(100\%) = \frac{200}{15}\%$$
$$\frac{200}{15}\% = 13\frac{1}{3}\%$$

   Alternative Numeric Solution:

   90 represents 100% of the whole.
   Therefore, 9 represents 10% of the whole.

   3 represents $\frac{10}{3}\%$.

   By scaling up, I can determine that 12 represents $\frac{40}{3}\%$.

   So, 12 represents $13\frac{1}{3}\%$ of the grandmother's age.

2. **Jenny's mom is 36 years old. What percent of Jenny's mother's age is Jenny's great-grandmother's age?**

   $\text{Quantity} = \text{Percent} \times \text{Whole}$. *Let p represent the unknown percent. Jenny's mother's age is the whole.*

$$90 = p(36)$$
$$90 \cdot \frac{1}{36} = p(36) \cdot \frac{1}{36}$$
$$5 \cdot \frac{1}{2} = p(1)$$
$$2.5 = p$$
$$2.5 = 250\%$$

   Jenny's great grandmother's age is 250% of Jenny's mother's age.

Problem Set Sample Solutions

Encourage students to solve these problems using an equation. They can check their work with a visual or arithmetic model if needed. Problem 2, part (e) is a very challenging problem, and most students will likely solve it using arithmetic reasoning rather than an equation.

1. **Solve each problem using an equation.**

   a. **49.5 is what percent of 33?**

   $$49.5 = p(33)$$
   $$p = 1.5$$
   $$1.5 = 150\%$$

   b. **72 is what percent of 180?**

   $$72 = p(180)$$
   $$p = 0.4$$
   $$0.4 = 40\%$$

   c. **What percent of 80 is 90?**

   $$90 = p(80)$$
   $$p = 1.125$$
   $$1.125 = 112.5\%$$

2. **This year, Benny is 12 years old, and his mom is 48 years old.**

   a. **What percent of his mom's age is Benny's age?**

   Let p represent the percent of Benny's age to his mom's age.

   $$12 = p(48)$$
   $$p = 0.25 = 25\%$$

   Benny's age is 25% of his mom's age.

   b. **What percent of Benny's age is his mom's age?**

   Let p represent the percent of his mom's age to Benny's age.

   $$48 = p(12)$$
   $$p = 4 = 400\%$$

   Benny's mom's age is 400% of Benny's age.

   c. **In two years, what percent of his age will Benny's mom's age be at that time?**

   In two years, Benny will be 14, and his mom will be 50.

   Let p represent the percent that Benny's mom's age is of his age.

   14 is the whole or 100% of Benny's age, and 50 is the quantity.

   $$\frac{50}{14} = \frac{25}{7}$$
   $$\frac{25}{7}(100\%) = \frac{2500}{7}\%$$
   $$\frac{2500}{7}\% = 357\frac{1}{7}\%$$

   His mom's age will be $357\frac{1}{7}\%$ of Benny's age at that time.

d. In 10 years, what percent will Benny's mom's age be of his age?

In 10 years, Benny will be 22 years old, and his mom will be 58 years old.

Let p represent the percent that Benny's mom's age is of his age.

Now, 22 represents the whole or 100% of Benny's age.

$$\frac{58}{22} = \frac{29}{11}$$
$$\frac{29}{11}(100\%) = \frac{2900}{11}\%$$
$$\frac{2900}{11}\% = 263\frac{7}{11}\%$$

In 10 years, Benny's mom's age will be $263\frac{7}{11}\%$ of Benny's age at that time.

e. In how many years will Benny be 50% of his mom's age?

Benny will be 50% of his mom's age when she is 200% of his age (or twice his age). Benny and his mom are always 36 years apart. When Benny is 36, his mom will be 72, and he will be 50% of her age. So, in 24 years, Benny will be 50% of his mom's age.

f. As Benny and his mom get older, Benny thinks that the percent of difference between their ages will decrease as well. Do you agree or disagree? Explain your reasoning.

Student responses will vary. Some students might argue that they are not getting closer since they are always 36 years apart. However, if you compare the percents, you can see that Benny's age is getting closer to 100% of his mom's age, even though their ages are not getting any closer.

3. This year, Benny is 12 years old. His brother Lenny's age is 175% of Benny's age. How old is Lenny?

Let L represent Lenny's age. Benny's age is the whole.

$L = 1.75(12)$

$L = 21$

Lenny is 21 years old.

4. When Benny's sister Penny is 24, Benny's age will be 125% of her age.

a. How old will Benny be then?

Let b represent Benny's age when Penny is 24.

$$b = 1.25(24)$$
$$b = 30$$

b. If Benny is 12 years old now, how old is Penny now? Explain your reasoning.

Penny is 6 years younger than Benny. If Benny is 12 now, then Penny is 6.

5. Benny's age is currently 200% of his sister Jenny's age. What percent of Benny's age will Jenny's age be in 4 years?

If Benny is 200% of Jenny's age, then he is twice her age, and she is half of his age. Half of 12 is 6. Jenny is currently 6 years old. In 4 years, Jenny will be 10 years old, and Benny will be 16 years old.

$\text{Quantity} = \text{Percent} \times \text{Whole}$. *Let p represent the unknown percent. Benny's age is the whole.*

$$10 = p(16)$$
$$p = \frac{10}{16} = \frac{5}{8}$$
$$p = 0.625 = 62.5\%$$

In 4 years, Jenny will be 62.5% of Benny's age.

6. At the animal shelter, there are 15 dogs, 12 cats, 3 snakes, and 5 parakeets.

a. What percent of the number of cats is the number of dogs?

$\frac{15}{12} = 1.25$. *That is 125%. The number of dogs is 125% the number of cats.*

b. What percent of the number of cats is the number of snakes?

$\frac{3}{12} = \frac{1}{4} = 0.25$. *There are 25% as many snakes as cats.*

c. What percent less parakeets are there than dogs?

$\frac{5}{15} = \frac{1}{3}$. *That is $33\frac{1}{3}\%$. There are $66\frac{2}{3}\%$ less parakeets than dogs.*

d. Which animal has 80% of the number of another animal?

$\frac{12}{15} = \frac{4}{5} = \frac{8}{10} = 0.$ *The number of cats is 80% the number of dogs.*

e. Which animal makes up approximately 14% of the animals in the shelter?

$\text{Quantity} = \text{Percent} \times \text{Whole}$. *The total number of animals is the whole.*

$$q = 0.14(35)$$
$$q = 4.9$$

The quantity closest to 4.9 is 5, the number of parakeets.

7. Is 2 hours and 30 minutes more or less than 10% of a day? Explain your answer.

$2\text{ hr. }30\text{ min.} \rightarrow 2.5\text{ hr.}$; *$24$ hours is a whole day and represents the whole quantity in this problem.*

10% of 24 hours is 2.4 hours.

$2.5 > 2.4$, so 2 hours and 30 minutes is more than 10% of a day.

8. A club's membership increased from 25 to 30 members.

a. Express the new membership as a percent of the old membership.

The old membership is the whole.

$\text{Quantity} = \text{Percent} \times \text{Whole}$. *Let p represent the unknown percent.*

$$30 = p(25)$$
$$p = 1.2 = 120\%$$

The new membership is 120% of the old membership.

b. Express the old membership as a percent of the new membership.

The new membership is the whole.

$$30 \rightarrow 100\%$$
$$1 \rightarrow \frac{100}{30}\%$$
$$25 \rightarrow 25 \cdot \frac{100}{30}\%$$
$$25 \rightarrow 5 \cdot \frac{100}{6}\%$$
$$25 \rightarrow \frac{500}{6}\% = 83\frac{1}{3}\%$$

The old membership is $83\frac{1}{3}\%$ of the new membership.

9. The number of boys in a school is 120% the number of girls at the school.

a. Find the number of boys if there are 320 girls.

The number of girls is the whole.

$\text{Quantity} = \text{Percent} \times \text{Whole}$. *Let b represent the unknown number of boys at the school.*

$$b = 1.2(320)$$
$$b = 384$$

If there are 320 girls, then there are 384 boys at the school.

b. Find the number of girls if there are 360 boys.

The number of girls is still the whole.

$\text{Quantity} = \text{Percent} \times \text{Whole}$. *Let g represent the unknown number of girls at the school.*

$$360 = 1.2(g)$$
$$g = 300$$

If there are 360 boys at the school, then there are 300 girls.

10. The price of a bicycle was increased from $\$300$ to $\$450$.

a. What percent of the original price is the increased price?

The original price is the whole.

$\text{Quantity} = \text{Percent} \times \text{Whole}$. *Let p represent the unknown percent.*

$$450 = p(300)$$
$$p = 1.5$$
$$1.5 = \frac{150}{100} = 150\%$$

The increased price is 150% of the original price.

b. **What percent of the increased price is the original price?**

The increased price, $\$450$, is the whole.

$$\frac{300}{450}=\frac{30}{45}$$
$$1\frac{30}{45}=\frac{2}{3}$$
$$\frac{2}{3}(100\%)=\frac{200}{3}\%$$
$$\frac{200}{3}\%=66\frac{2}{3}\%$$

The original price is $66\frac{2}{3}\%$ of the increased price.

11. **The population of Appleton is 175% of the population of Cherryton.**

a. **Find the population in Appleton if the population in Cherryton is $4,000$ people.**

The population of Cherryton is the whole.

$\text{Quantity} = \text{Percent} \times \text{Whole}$. *Let a represent the unknown population of Appleton.*

$a = 1.75(4,000)$
$a = 7,000$

If the population of Cherryton is $4,000$ people, then the population of Appleton is $7,000$ people.

b. **Find the population in Cherryton if the population in Appleton is $10,500$ people.**

The population of Cherryton is still the whole.

$\text{Quantity} = \text{Percent} \times \text{Whole}$. *Let c represent the unknown population of Cherryton.*

$$10,500 = 1.75c$$
$$c = 10,500 \div 1.75$$
$$c = 6000$$

If the population of Appleton is $10,500$ people, then the population of Cherryton is $6,000$ people.

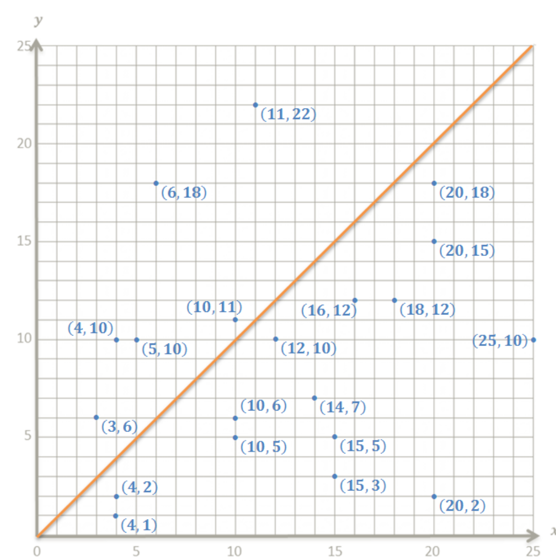
12. A statistics class collected data regarding the number of boys and the number of girls in each classroom at their school during homeroom. Some of their results are shown in the table below.

a. Complete the blank cells of the table using your knowledge about percent.

Number of Boys (x)	Number of Girls (y)	Number of Girls as a Percent of the Number of Boys
10	5	50%
4	1	25%
18	12	$66\frac{2}{3}\%$
5	10	200%
4	2	50%
20	18	90%
4	10	250%
10	6	60%
11	22	200%
15	5	$33\frac{1}{3}\%$
15	3	20%
20	15	75%
6	18	300%
25	10	40%
10	11	110%
20	2	10%
16	12	75%
14	7	50%
3	6	200%
12	10	$83\frac{1}{3}\%$

b. Using a coordinate plane and grid paper, locate and label the points representing the ordered pairs (x, y).

See graph to the right.



c. Locate all points on the graph that would represent classrooms in which the number of girls y is 100% of the number of boys x. Describe the pattern that these points make.

The points lie on a line that includes the origin; therefore, it is a proportional relationship.

d. Which points represent the classrooms in which the number of girls is greater than 100% of the number of boys? Which points represent the classrooms in which the number of girls is less than 100% of the number of boys? Describe the locations of the points in relation to the points in part (c).

All points where $y > x$ are above the line and represent classrooms where the number of girls is greater than 100% of the number of boys. All points where $y < x$ are below the line and represent classrooms where the number of girls is less than 100% of the boys.

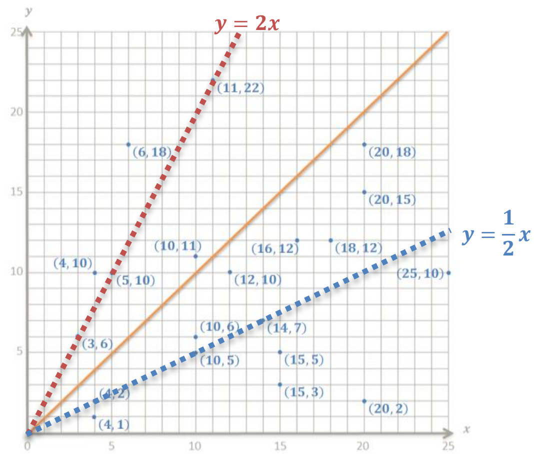
e. Find three ordered pairs from your table representing classrooms where the number of girls is the same percent of the number of boys. Do these points represent a proportional relationship? Explain your reasoning.

There are two sets of points that satisfy this question:

$\{(3,6), (5,10), \text{and } (11,22)\}$*: The points do represent a proportional relationship because there is a constant of proportionality* $k = \frac{y}{x} = 2$.

$\{(4,2), (10,5), \text{and } (14,7)\}$*: The points do represent a proportional relationship because there is a constant of proportionality* $k = \frac{y}{x} = \frac{1}{2}$.

f. Show the relationship(s) from part (e) on the graph, and label them with the corresponding equation(s).



g. What is the constant of proportionality in your equation(s), and what does it tell us about the number of girls and the number of boys at each point on the graph that represents it? What does the constant of proportionality represent in the table in part (a)?

In the equation $y = 2x$, the constant of proportionality is 2, and it tells us that the number of girls will be twice the number of boys, or 200% of the number of boys, as shown in the table in part (a).

In the equation $y = \frac{1}{2}x$, the constant of proportionality is $\frac{1}{2}$, and it tells us that the number of girls will be half the number of boys, or 50% of the number of boys, as shown in the table in part (a).

Part, Whole, or Percent—Round 1

Number Correct: ______

Directions: Find each missing value.

1.	1% of 100 is?		23.	10% of 22 is?	
2.	2% of 100 is?		24.	20% of 22 is?	
3.	3% of 100 is?		25.	30% of 22 is?	
4.	4% of 100 is?		26.	50% of 22 is?	
5.	5% of 100 is?		27.	25% of 22 is?	
6.	9% of 100 is?		28.	75% of 22 is?	
7.	10% of 100 is?		29.	80% of 22 is?	
8.	10% of 200 is?		30.	85% of 22 is?	
9.	10% of 300 is?		31.	90% of 22 is?	
10.	10% of 500 is?		32.	95% of 22 is?	
11.	10% of 550 is?		33.	5% of 22 is?	
12.	10% of 570 is?		34.	15% of 80 is?	
13.	10% of 470 is?		35.	15% of 60 is?	
14.	10% of 170 is?		36.	15% of 40 is?	
15.	10% of 70 is?		37.	30% of 40 is?	
16.	10% of 40 is?		38.	30% of 70 is?	
17.	10% of 20 is?		39.	30% of 60 is?	
18.	10% of 25 is?		40.	45% of 80 is?	
19.	10% of 35 is?		41.	45% of 120 is?	
20.	10% of 36 is?		42.	120% of 40 is?	
21.	10% of 37 is?		43.	120% of 50 is?	
22.	10% of 37.5 is?		44.	120% of 55 is?	

Part, Whole, or Percent—Round 1 [KEY]

Directions: Find each missing value.

1.	1% of 100 is?	**1**	23.	10% of 22 is?	**2.2**
2.	2% of 100 is?	**2**	24.	20% of 22 is?	**4.4**
3.	3% of 100 is?	**3**	25.	30% of 22 is?	**6.6**
4.	4% of 100 is?	**4**	26.	50% of 22 is?	**11**
5.	5% of 100 is?	**5**	27.	25% of 22 is?	**5.5**
6.	9% of 100 is?	**9**	28.	75% of 22 is?	**16.5**
7.	10% of 100 is?	**10**	29.	80% of 22 is?	**17.6**
8.	10% of 200 is?	**20**	30.	85% of 22 is?	**18.7**
9.	10% of 300 is?	**30**	31.	90% of 22 is?	**19.8**
10.	10% of 500 is?	**50**	32.	95% of 22 is?	**20.9**
11.	10% of 550 is?	**55**	33.	5% of 22 is?	**1.1**
12.	10% of 570 is?	**57**	34.	15% of 80 is?	**12**
13.	10% of 470 is?	**47**	35.	15% of 60 is?	**9**
14.	10% of 170 is?	**17**	36.	15% of 40 is?	**6**
15.	10% of 70 is?	**7**	37.	30% of 40 is?	**12**
16.	10% of 40 is?	**4**	38.	30% of 70 is?	**21**
17.	10% of 20 is?	**2**	39.	30% of 60 is?	**18**
18.	10% of 25 is?	**2.5**	40.	45% of 80 is?	**36**
19.	10% of 35 is?	**3.5**	41.	45% of 120 is?	**54**
20.	10% of 36 is?	**3.6**	42.	120% of 40 is?	**48**
21.	10% of 37 is?	**3.7**	43.	120% of 50 is?	**60**
22.	10% of 37.5 is?	**3.75**	44.	120% of 55 is?	**66**

Part, Whole, or Percent—Round 2

Number Correct: ______

Improvement: ______

Directions: Find each missing value.

1.	20% of 100 is?	
2.	21% of 100 is?	
3.	22% of 100 is?	
4.	23% of 100 is?	
5.	25% of 100 is?	
6.	25% of 200 is?	
7.	25% of 300 is?	
8.	25% of 400 is?	
9.	25% of 4000 is?	
10.	50% of 4000 is?	
11.	10% of 4000 is?	
12.	10% of 4700 is?	
13.	10% of 4600 is?	
14.	10% of 4630 is?	
15.	10% of 463 is?	
16.	10% of 46.3 is?	
17.	10% of 18 is?	
18.	10% of 24 is?	
19.	10% of 3.63 is?	
20.	10% of 0.336 is?	
21.	10% of 37 is?	
22.	10% of 37.5 is?	

23.	10% of 4 is?	
24.	20% of 4 is?	
25.	30% of 4 is?	
26.	50% of 4 is?	
27.	25% of 4 is?	
28.	75% of 4 is?	
29.	80% of 4 is?	
30.	85% of 4 is?	
31.	90% of 4 is?	
32.	95% of 4 is?	
33.	5% of 4 is?	
34.	15% of 40 is?	
35.	15% of 30 is?	
36.	15% of 20 is?	
37.	30% of 20 is?	
38.	30% of 50 is?	
39.	30% of 90 is?	
40.	45% of 90 is?	
41.	90% of 120 is?	
42.	125% of 40 is?	
43.	125% of 50 is?	
44.	120% of 60 is?	

Part, Whole, or Percent—Round 2 [KEY]

Directions: Find each missing value.

1.	20% of 100 is?	**20**
2.	21% of 100 is?	**21**
3.	22% of 100 is?	**22**
4.	23% of 100 is?	**23**
5.	25% of 100 is?	**25**
6.	25% of 200 is?	**50**
7.	25% of 300 is?	**75**
8.	25% of 400 is?	**100**
9.	25% of 4000 is?	**1000**
10.	50% of 4000 is?	**2000**
11.	10% of 4000 is?	**400**
12.	10% of 4700 is?	**470**
13.	10% of 4600 is?	**460**
14.	10% of 4630 is?	**463**
15.	10% of 463 is?	**46.3**
16.	10% of 46.3 is?	**4.63**
17.	10% of 18 is?	**1.8**
18.	10% of 24 is?	**2.4**
19.	10% of 3.63 is?	**0.363**
20.	10% of 0.336 is?	**0.0363**
21.	10% of 37 is?	**3.7**
22.	10% of 37.5 is?	**3.75**

23.	10% of 4 is?	**0.4**
24.	20% of 4 is?	**0.8**
25.	30% of 4 is?	**1.2**
26.	50% of 4 is?	**2**
27.	25% of 4 is?	**1**
28.	75% of 4 is?	**3**
29.	80% of 4 is?	**3.2**
30.	85% of 4 is?	**3.4**
31.	90% of 4 is?	**3.6**
32.	95% of 4 is?	**3.8**
33.	5% of 4 is?	**0.2**
34.	15% of 40 is?	**6**
35.	15% of 30 is?	**4.5**
36.	15% of 20 is?	**3**
37.	30% of 20 is?	**6**
38.	30% of 50 is?	**15**
39.	30% of 90 is?	**27**
40.	45% of 90 is?	**40.5**
41.	90% of 120 is?	**108**
42.	125% of 40 is?	**50**
43.	125% of 50 is?	**62.5**
44.	120% of 60 is?	**72**