Lesson 2: Part of a Whole as a Percent

Student Outcomes

- Students understand that the whole is 100% and use the formula $\text{Part} = \text{Percent} \times \text{Whole}$ to problem-solve when given two terms out of three from the part, whole, and percent.
- Students solve word problems involving percent using expressions, equations, and numeric and visual models.

Lesson Notes

This lesson serves as an introduction to general percent problems by considering problems of which a part of a whole is represented as a percent of the whole. Students solve percent problems using visual models and proportional reasoning and then make connections to solving percent problems using numeric and algebraic methods. This lesson focuses on the relationship: $\text{Part} = \text{Percent} \times \text{Whole}$.

Classwork

Opening (2 minutes)

One of the challenges students face when solving word problems involving percents is deciding which of the given quantities represents the whole unit and which represents the part of that whole unit. Discuss with students how the value of a nickel coin ($\$0.05$) compares to the value of a dollar ($\$1.00$) using percents.

- As a percent, how does the value of a nickel coin compare to the value of a dollar?
  - *A dollar is 100 cents; therefore, the quantity 100 cents is 100% of a dollar. A nickel coin has a value of 5 cents, which is 5 of 100 cents, or $\frac{5}{100} = 5\%$ of a dollar.*
- Part-of-a-whole percent problems involve the following:
  - A comparison of generic numbers (e.g., 25% of 12 is 3), or
  - A comparison of a quantity that is a part of another quantity (e.g., the number of boys in a classroom is part of the total number of students in the classroom).
- The number or quantity that another number or quantity is being compared to is called the *whole*. The number or quantity that is compared to the whole is called the *part* because it is part (or a piece) of the whole quantity.
- In our comparison of the value of a nickel coin to the value of a dollar, which quantity is considered the part and which is considered the whole? Explain your answer.
  - *The value of the nickel coin is the part because it is being compared to the value of the whole dollar. The dollar represents the whole because the value of the nickel coin is being compared to the value of the dollar.*

Opening Exercise (4 minutes)

Part (a) of the Opening Exercise asks students to practice identifying the whole in given percent scenarios. In part (b), students are presented with three different approaches to a given scenario but need to make sense of each approach to identify the part, the whole, and the percent.

Opening Exercise

a. **What is the whole unit in each scenario?**

Scenario	Whole Unit
15 is what percent of 90?	*The number* 90
What number is 10% of 56?	*The number* 56
90% of a number is 180.	*The unknown number*
A bag of candy contains 300 pieces, and 25% of the pieces in the bag are red.	*The* 300 *pieces of candy*
Seventy percent (70%) of the students earned a B on the test.	*All the students in the class*
The 20 girls in the class represented 55% of the students in the class.	*All the students in the class*

MP.1 After students complete part (a) with a partner, ask the following question:

- How did you decide on the whole unit in each of the given scenarios?
  - *In each case, we looked for the number or quantity that another number or quantity was being compared to.*

b. **Read each problem and complete the table to record what you know.**

Problem	Part	Percent	Whole
40% of the students on the field trip love the museum. If there are 20 students on the field trip, how many love the museum?	*?*	40%	20 *students*
40% of the students on the field trip love the museum. If 20 students love the museum, how many are on the field trip?	20 *students*	40%	*?*
20 students on the field trip love the museum. If there are 40 students on the field trip, what percent love the museum?	20 *students*	*?*	40 *students*

When students complete part (b), encourage them to share how they decided which number in the problem represents the whole and which represents the part.

Example 1 (5 minutes): Visual Approaches to Finding a Part, Given a Percent of the Whole

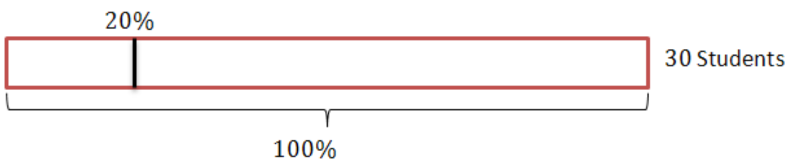
Present the following problem to students. Show how to solve the problem using visual models; then, generalize a numeric method through discussion. Have students record each method in their student materials.

Example 1: Visual Approaches to Finding a Part, Given a Percent of the Whole

In Ty's math class, 20% of students earned an A on a test. If there were 30 students in the class, how many got an A?

- Is 30 the whole unit or part of the whole?
  - *It is the whole unit; the number of students that earned an A on the test is the part and is compared to the total number of students in the class.*
- What percentage of Ty's class does the quantity "30 students" represent?
  - 100% *of Ty's class*

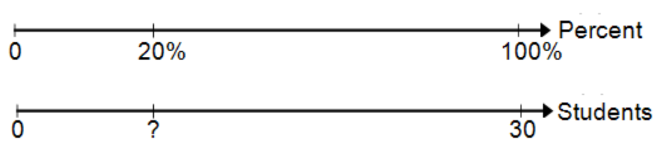
Solve the problem first using a tape diagram.



> *Scaffolding:*
> Some students may recognize that there are 5 intervals of 20% in the tape diagram and want to divide 30 students into 5 groups. That is okay. However, if students do not immediately recognize this, do not force it upon them. Further practice scaffolds this shortcut while also supporting primary understanding of how the percent problems work.

- 30 students make up 100% of the class. Let's divide the 100% into 100 slices of 1% and also divide the quantity of 30 students into 100 slices. What number of students does each 1% correspond to?
  - $\frac{30}{100} = 0.3$; 0.3 *of a student represents* 1% *of Ty's class.*
- If this is 1% of Ty's class, then how do we find 20% of Ty's class?
  - $(1\%) \times 20 = 20\%$, *so we can multiply* $(0.3) \times 20 = 6$; 6 *students are* 20% *of Ty's class; therefore,* 6 *students got an A on the test.*

Revisit the problem using a double number line.



- 30 students represents the whole class, so 30 aligns with 100%. There are 100 intervals of 1% on the percent number line. What number of students does each 1% correspond to?
  - $\frac{30}{100} = 0.3$; 0.3 *of a student represents* 1% *of Ty's class.*
- To help us keep track of quantities and their corresponding percents, we can use arrows to show the correspondences in our sequences of reasoning:

$$\div \mathbf{100} \quad \begin{array}{c} 30 \to 100\% \\ 0.3 \to 1\% \end{array} \quad \div \mathbf{100}$$

- If this is 1% of Ty's class, how do we find 20% of Ty's class?
  - *Multiply by* 20; $0.3 \cdot 20 = 6$; 6 *students are* 20% *of Ty's class, so* 6 *students got an A on the test.*

$$\times \mathbf{20} \quad \begin{array}{c} 0.3 \to 1\% \\ 6 \to 20\% \end{array} \quad \times \mathbf{20}$$

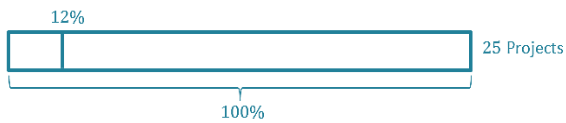
- What similarities do you notice in each of these visual models?
  - *In both models,* 30 *corresponds with the* 100%*; so, we divided* 30 *by* 100 *to get the number of students that corresponds with* 1% *and then multiplied that by* 20 *to get the number of students that corresponds with* 20%.

Exercise 1 (3 minutes)

Students use visual methods to solve a problem similar to Example 1. After completing the exercise, initiate a discussion about the similarities of the problems, and generalize a numeric approach to the problems. This numeric approach will be used to generalize an algebraic equation that can be used in solving percent problems.

Exercise 1

In Ty's art class, 12% of the Flag Day art projects received a perfect score. There were 25 art projects turned in by Ty's class. How many of the art projects earned a perfect score? (Identify the whole.)



The whole is the number of art projects turned in by Ty's class, 25.

$\frac{25}{100} = 0.25$; $0.25 \cdot 12 = 3$; *12% of 25 is 3, so 3 art projects in Ty's class received a perfect score.*

Discussion (2 minutes)

MP.8

- What similarities do you recognize in Example 1 and Exercise 1?
  - *In each case, the whole corresponded with 100%, and dividing the whole by 100 resulted in 1% of the whole. Multiplying this number by the percent resulted in the part.*
- Describe and show how the process seen in the visual models can be generalized into a numeric approach.
  - *Divide the whole by 100 to get 1%, and then multiply by the percent needed.*
  - Whole $\rightarrow 100\%$.

Example 1

$$30 \rightarrow 100\% \quad (\div 100)$$
$$\frac{30}{100} \rightarrow 1\% \quad (\times 20)$$
$$20\left(\frac{30}{100}\right) \rightarrow 20\%$$
$$6 \rightarrow 20\%$$

Exercise 1

$$25 \rightarrow 100\% \quad (\div 100)$$
$$\frac{25}{100} \rightarrow 1\% \quad (\times 12)$$
$$12\left(\frac{25}{100}\right) \rightarrow 12\%$$
$$3 \rightarrow 12\%$$

Example 2 (3 minutes): A Numeric Approach to Finding a Part, Given a Percent of the Whole

Present the following problem to students. Have them guide you through solving the problem using the arithmetic method from the previous discussion. When complete, generalize an arithmetic method through further discussion.

Example 2: A Numeric Approach to Finding a Part, Given a Percent of the Whole

In Ty's English class, 70% of the students completed an essay by the due date. There are 30 students in Ty's English class. How many completed the essay by the due date?

- First, identify the whole quantity in the problem.
  - *The number of students that completed the essay by the due date is being compared to the total number of students in Ty's class, so the total number of students in the class* (30) *is the whole.*

Whole $\rightarrow$ $\mathbf{100\%}$

$\mathbf{30 \rightarrow 100\%}$

$\mathbf{\frac{30}{100} \rightarrow 1\%}$

$\mathbf{70 \cdot \frac{30}{100} \rightarrow 21\%}$

$\mathbf{21 \rightarrow 70\%}$

70% of 30 is 21, so 21 of the students in Ty's English class completed their essays on time.

Discussion (2 minutes)

MP.7

This discussion is an extension of Example 2 and serves as a bridge to Example 3.

- Is the expression $\frac{70}{100} \cdot 30$ equivalent to $70 \cdot \frac{30}{100}$ from the steps above? Why or why not?
  - *The expressions are equivalent by the any order, any grouping property of multiplication.*
- What does $\frac{70}{100}$ represent? What does 30 represent? What does their product represent?
  - $\frac{70}{100} = 70\%$*,* 30 *represents the whole, and their product* (21) *represents the part, or* 70% *of the students in Ty's English class.*
- Write a true multiplication sentence relating the part (21), the whole (30), and the percent $\left(\frac{70}{100}\right)$ in this problem.
  - $21 = \frac{70}{100} \cdot (30)$
- Translate your sentence into words. Is the sentence valid?
  - *Twenty-one is seventy percent of thirty. Yes, the sentence is valid because* 21 *students represents* 70% *of the* 30 *students in Ty's English class.*
- Generalize the terms in your multiplication sentence by writing what each term represents.
  - $\text{Part} = \text{Percent} \times \text{Whole}$

Example 3 (4 minutes): An Algebraic Approach to Finding a Part, Given a Percent of the Whole

- In percent problems, the percent equation $(\text{Part} = \text{Percent} \times \text{Whole})$ can be used to solve the problem when given two of its three terms. To solve a percent word problem, first identify the whole quantity in the problem, and then identify the part and percent. Use a letter (variable) to represent the term whose value is unknown.

Example 3: An Algebraic Approach to Finding a Part, Given a Percent of the Whole

A bag of candy contains 300 pieces of which 28% are red. How many pieces are red?

Which quantity represents the whole?

The total number of candies in the bag, 300, is the whole because the number of red candies is being compared to it.

Which of the terms in the percent equation is unknown? Define a letter (variable) to represent the unknown quantity.

We do not know the part, which is the number of red candies in the bag. Let r represent the number of red candies in the bag.

Write an expression using the percent and the whole to represent the number of pieces of red candy.

$\frac{28}{100}\cdot(300)$*, or* $0.28\cdot(300)$*, is the amount of red candy since the number of red candies is* 28% *of the* 300 *pieces of candy in the bag.*

Write and solve an equation to find the unknown quantity.

$$\text{Part} = \text{Percent} \times \text{Whole}$$
$$r = \frac{28}{100}\cdot(300)$$
$$r = 28\cdot 3$$
$$r = 84$$

There are 84 red pieces of candy in the bag.

Exercise 2 (4 minutes)

This exercise is a continuation of Example 3.

Exercise 2

A bag of candy contains 300 pieces of which 28% are red. How many pieces are not red?

a. **Write an equation to represent the number of pieces that are not red, n.**

$$\text{Part} = \text{Percent} \times \text{Whole}$$
$$n = (100\% - 28\%)(300)$$

b. **Use your equation to find the number of pieces of candy that are not red.**

If 28% of the candies are red, then the difference of 100% and 28% must be candies that are not red.

$$n = (100\% - 28\%)(300)$$
$$n = (72\%)(300)$$
$$n = \frac{72}{100}(300)$$
$$n = 72\cdot 3$$
$$n = 216$$

There are 216 pieces of candy in the bag that are not red.

c. **Jah-Lil told his math teacher that he could use the answer from part (b) and mental math to find the number of pieces of candy that are not red. Explain what Jah-Lil meant by that.**

He meant that once you know there are 84 red pieces of candy in a bag that contains 300 pieces of candy total, you just subtract 84 from 300 to know that 216 pieces of candy are not red.

Students saw in Module 3 that it is possible to find a solution to a formula, or algebraic equation, by using the properties of operations and if-then moves to rewrite the expressions in an equation in a form in which a solution can be easily seen. Examples 4 and 5 use the algebraic formula $\text{Part} = \text{Percent} \times \text{Whole}$ to solve percent word problems where they are given two of the following three terms: part, percent, and whole.

Example 4 (5 minutes): Comparing Part of a Whole to the Whole with the Percent Formula

Students use the percent formula and algebraic reasoning to solve a percent problem in which they are given the part and the percent.

> **Example 4: Comparing Part of a Whole to the Whole with the Percent Formula**
>
> **Zoey inflated 24 balloons for decorations at the middle school dance. If Zoey inflated 15% of the balloons that are inflated for the dance, how many balloons are there total? Solve the problem using the percent formula, and verify your answer using a visual model.**

- What is the whole quantity? How do you know?
  - *The total number of balloons at the dance is the whole quantity because the number of balloons that Zoey inflated is compared to the total number of balloons for the dance.*
- What do the 24 balloons represent?
  - *24 balloons are part of the total number of balloons for the dance.*
- Write the percent formula, and determine which term is unknown.

$\mathbf{Part = Percent \times Whole}$

The part is 24 balloons, and the percent is 15%, so let t represent the unknown total number of balloons.

$24 = \frac{15}{100}t$	***If $a = b$, then $ac = bc$.***
$\frac{100}{15}(24) = \frac{100}{15}\left(\frac{15}{100}\right)t$	***Multiplicative inverse***
$\frac{2400}{15} = 1t$	***Multiplicative identity property of 1 and equivalent fractions***
$160 = t$	***The total number of balloons to be inflated for the dance was 160 balloons.***

$$15\% \rightarrow 24$$

$$1\% \rightarrow \frac{24}{15}$$

$$100\% \rightarrow \frac{24}{15} \cdot 100$$

$$100\% \rightarrow \frac{24}{3} \cdot 20 = 160$$

We want the quantity that corresponds with 100%, so first we find 1%.*

****Student may also find 5% as is shown in the tape diagram above.***

- Is the solution from the equation consistent with the visual and numeric solution?
  - *Yes.*

Example 5 (5 minutes): Finding the Percent Given a Part of the Whole and the Whole

Students use the percent formula and algebraic reasoning to solve a percent problem in which they are given the part and the whole.

Example 5: Finding the Percent Given a Part of the Whole and the Whole

Haley is making admission tickets to the middle school dance. So far she has made 112 tickets, and her plan is to make 320 tickets. What percent of the admission tickets has Haley produced so far? Solve the problem using the percent formula, and verify your answer using a visual model.

- What is the whole quantity? How do you know?
  - *The total number of admission tickets, 320, is the whole quantity because the number of tickets that Haley has already made is compared to the total number of tickets that she needs to make.*
- What does the quantity 112 tickets represent?
  - 112 *tickets is part of the total number of tickets for the dance.*
- Write the percent formula, and determine which term is unknown.

$\textbf{Part} = \textbf{Percent} \times \textbf{Whole}$

The part is 112 tickets, and the whole is 320 tickets, so let p represent the unknown percent.

$$112 = p(320) \qquad \textit{If } a = b\textit{, then } ac = bc.$$

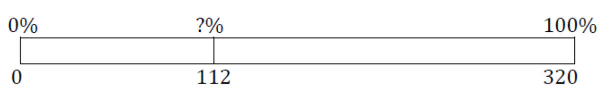
$$112 \cdot \frac{1}{320} = p(320) \cdot \frac{1}{320} \qquad \textit{Multiplicative inverse}$$

$$\frac{112}{320} = p(1) \qquad \textit{Multiplicative identity property of } 1$$

$$\frac{7}{20} = p$$

$$0.35 = p$$

$0.35 = \frac{35}{100} = 35\%$***, so Haley has made 35% of the tickets for the dance.***



We need to know the percent that corresponds with 112, so first we find the percent that corresponds with 1 ticket.

$320 \rightarrow 100\%$

$1 \rightarrow \left(\frac{100}{320}\right)\%$

$112 \rightarrow 112 \cdot \left(\frac{100}{320}\right)\%$

$112 \rightarrow 112 \cdot \left(\frac{5}{16}\right)\%$

$112 \rightarrow 7 \cdot (5)\% = 35\%$

- Is the solution from the equation consistent with the visual and numeric solution?
  - *Yes.*

Closing (2 minutes)

- What formula can we use to relate the part, the whole, and the percent of the whole? Translate the formula into words.
  - $\text{Part} = \text{Percent} \times \text{Whole}$. *The part is some percent of the whole.*
- What are the advantages of using an algebraic representation to solve percent problems?
  - *If you can identify the whole, part, and percent, the algebraic approach is very fast and efficient.*
- Explain how to use a visual model and an equation to determine how many calories are in a candy bar if 75% of its 200 calories is from sugar.
  - *Use a double number line or tape diagram. The whole (total calories) corresponds with* 100%*.* 200 *calories divided into* 100 *intervals shows that every* 1% *will be* 2 *calories. That means there are* 150 *calories from sugar in the candy bar.*

Lesson Summary

- **Visual models or numeric methods can be used to solve percent problems.**
- **An equation can be used to solve percent problems:**

$$\textbf{Part} = \textbf{Percent} \times \textbf{Whole}.$$

Exit Ticket (4 minutes)

Note to the teacher: Students using the visual or numeric approaches for problems in the Exit Ticket do not necessarily need to find 1% first. Alternatively, if they recognize that they can instead find 4%, 5%, 10%, 20%, or other factors of 100%, then they can multiply by the appropriate factor to obtain 100%.

Name ____________________________________ Date ____________________

Lesson 2: Part of a Whole as a Percent

Exit Ticket

1. On a recent survey, 60% of those surveyed indicated that they preferred walking to running.
   a. If 540 people preferred walking, how many people were surveyed?

   b. How many people preferred running?

2. Which is greater: 25% of 15 or 15% of 25? Explain your reasoning using algebraic representations or visual models.

Exit Ticket Sample Solutions

1. On a recent survey, 60% of those surveyed indicated that they preferred walking to running.

   a. If 540 people preferred walking, how many people were surveyed?

   Let n represent the number of people surveyed.

   $0.60n$ is the number of people who preferred walking.

   Since 540 people preferred walking,

   $0.60n = 540$

   $$n = \frac{540}{0.6} = \frac{5400}{6} = 900$$

   Therefore, 900 people were surveyed.

   b. How many people preferred running?

   Subtract 540 from 900.

   $900 - 540 = 360$

   Therefore, 360 people preferred running.

2. Which is greater: 25% of 15 or 15% of 25? Explain your reasoning using algebraic representations or visual models.

   They are the same.

   $$0.25 \times 15 = \frac{25}{100} \times 15 = 3.75$$

   $$0.15 \times 25 = \frac{15}{100} \times 25 = 3.75$$

   Also, you can see they are the same without actually computing the product because of any order, any grouping of multiplication.

   $$\frac{25}{100} \times 15 = 25 \times \frac{1}{100} \times 15 = 25 \times \frac{15}{100}$$

Problem Set Sample Solutions

Students should be encouraged to solve these problems using an algebraic approach.

1. Represent each situation using an equation. Check your answer with a visual model or numeric method.

   a. What number is 40% of 90?

   $n = 0.40(90)$
   $n = 36$

   b. What number is 45% of 90?

   $n = 0.45(90)$
   $n = 40.5$

c. 27 is 30% of what number?

$$27 = 0.3n$$
$$\frac{27}{0.3} = 1n$$
$$90 = n$$

d. 18 is 30% of what number?

$$0.30n = 18$$
$$1n = \frac{18}{0.3}$$
$$n = 60$$

e. 25.5 is what percent of 85?

$$25.5 = p(85)$$
$$\frac{25.5}{85} = 1p$$
$$0.3 = p$$
$$0.3 = \frac{30}{100} = 30\%$$

f. 21 is what percent of 60?

$$21 = p(60)$$
$$0.35 = p$$
$$0.35 = \frac{35}{100} = 35\%$$

2. 40% of the students on a field trip love the museum. If there are 20 students on the field trip, how many love the museum?

Let s represent the number of students who love the museum.

$$s = 0.40(20)$$
$$s = 8$$

Therefore, 8 students love the museum.

3. Maya spent 40% of her savings to pay for a bicycle that cost her $\$85$.

a. How much money was in her savings to begin with?

Let s represent the unknown amount of money in Maya's savings.

$$85 = 0.4s$$
$$212.5 = s$$

Maya originally had $\$212.50$ in her savings.

b. How much money does she have left in her savings after buying the bicycle?

$$\$212.50 - \$85.00 = \$127.50$$

She has $\$127.50$ left in her savings after buying the bicycle.

4. Curtis threw 15 darts at a dartboard. 40% of his darts hit the bull's-eye. How many darts did not hit the bull's-eye?

Let d represent the number of darts that hit the bull's-eye.

$d = 0.4(15)$

$d = 6$

6 darts hit the bull's-eye. $15 - 6 = 9$, so 9 darts did not hit the bull's-eye.

5. A tool set is on sale for $\$424.15$. The original price of the tool set was $\$499.00$. What percent of the original price is the sale price?

Let p represent the unknown percent.

$424.15 = p(499)$

The sale price is 85% of the original price.

6. Matthew scored a total of 168 points in basketball this season. He scored 147 of those points in the regular season and the rest were scored in his only playoff game. What percent of his total points did he score in the playoff game?

Matthew scored 21 points during the playoff game because $169 - 147 = 21$.

Let p represent the unknown percent.

$21 = p(168)$

The points that Matthew scored in the playoff game were 12.5% of his total points scored in basketball this year.

7. Brad put 10 crickets in his pet lizard's cage. After one day, Brad's lizard had eaten 20% of the crickets he had put in the cage. By the end of the next day, the lizard had eaten 25% of the remaining crickets. How many crickets were left in the cage at the end of the second day?

Day 1:

$n = 0.2(10)$

$n = 2$

At the end of the first day, Brad's lizard had eaten 2 of the crickets.

Day 2:

$n = 0.75(10 - 2)$

$n = 0.75(8)$

$n = 6$

At the end of the second day, Brad's lizard had eaten a total of 4 crickets, leaving 6 crickets in the cage.

8. A furnace used 40% of the fuel in its tank in the month of March and then used 25% of the remaining fuel in the month of April. At the beginning of March, there were 240 gallons of fuel in the tank. How much fuel (in gallons) was left at the end of April?

March:

$n = 0.4(240)$

$n = 96$

Therefore, 96 gallons were used during the month of March, which means 144 gallons remain.

April:

$n = 0.25(144)$

$n = 36$

Therefore, 36 gallons were used during the month of April, which means 108 gallons remain.

There were 144 gallons of fuel remaining in the tank at the end of March and 108 gallons of fuel remaining at the end of April.

9. In Lewis County, there were $2,277$ student athletes competing in spring sports in 2014. That was 110% of the number from 2013, which was 90% of the number from the year before. How many student athletes signed up for a spring sport in 2012?

2013:

$2,277 = 1.10a$

$2,070 = a$

Therefore, $2,070$ student athletes competed in spring sports in 2013.

2012:

$2,070 = 0.9a$

$2,300 = a$

Therefore, $2,300$ student athletes competed in spring sports in 2012.

There were $2,070$ students competing in spring sports in 2013 and $2,300$ students in 2012.

10. Write a real-world word problem that could be modeled by the equation below. Identify the elements of the percent equation and where they appear in your word problem, and then solve the problem.

$$57.5 = p(250)$$

Answers will vary. Greig is buying sliced almonds for a baking project. According to the scale, his bag contains 57.5 grams of almonds. Greig needs 250 grams of sliced almonds for his project. What percent of his total weight of almonds does Greig currently have?

The quantity 57.5 represents the part of the almonds that Greig currently has on the scale, the quantity 250 represents the 250 grams of almonds that he plans to purchase, and the variable p represents the unknown percent of the whole quantity that corresponds to the quantity 57.5.

$$57.5 = p(250)$$
$$\frac{1}{250}(57.5) = p\left(\frac{1}{250}\right)(250)$$
$$\frac{57.5}{250} = p(1)$$
$$0.23 = p$$
$$0.23 = \frac{23}{100} = 23\%$$

Greig currently has 23% of the total weight of almonds that he plans to buy.