Lesson 25: Volume and Surface Area

Student Outcomes

- Students solve real-world and mathematical problems involving volume and surface areas of three-dimensional objects composed of cubes and right prisms.

Lesson Notes

In this lesson, students apply what they learned in Lessons 22–24 to solve real-world problems. As students work on the problems, encourage them to present their approaches for determining the volume and surface area. The initial questions specifically ask for volume, but later in the lesson, students must interpret the context of the problem to know which measurement to choose. Several problems involve finding the height of a prism if the volume and two other dimensions are given. Students work with cubic units and units of liquid measure on the volume problems.

Classwork

Opening (2 minutes)

In the Opening Exercise, students are asked to find the volume and surface area of a right rectangular prism. This exercise provides information about students who may need some additional support during the lesson if they have difficulty solving this problem. Tell the class that today they will be applying what they learned about finding the surface area and volume of prisms to real-world problems.

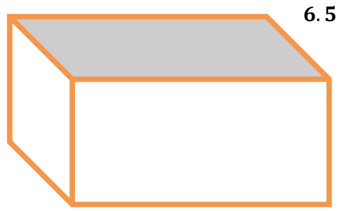
Opening Exercise (3 minutes)

Scaffolding:

This lesson builds gradually to more and more complicated problems. Provide additional practice at each stage if you find students are struggling.

Opening Exercise

What is the surface area and volume of the right rectangular prism?



Surface Area $= 2(11\text{ in.})(6.5\text{ in.}) + 2(10\text{ in.})(6.5\text{ in.}) + 2(11\text{ in.})(10\text{ in.}) = 493\text{ in}^2$

Volume $= 11\text{ in.} \cdot 6.5\text{ in.} \cdot 10\text{ in.} = 715\text{ in}^3$

Example 1 (10 minutes): Volume of a Fish Tank

This example uses the prism from the Opening Exercise and applies it to a real-world situation. Either guide students through this example, allow them to work with a partner, or allow them to work in small groups, depending on their level. If you have students work with a partner or a group, be sure to present different solutions and monitor the groups' progress.

MP.2

For part (a) below.

- How did you identify this as a *volume* problem?
  - *The term gallon refers to capacity or volume*.

Be sure that students recognize the varying criteria for calculating surface area and volume.

For part (c) below.

- What helped you to understand that this is a surface area problem?"
  - *Square inches are measures of area, not volume.*
  - *"Covering the sides" requires using an area calculation, not a volume calculation.*

Example 1: Volume of a Fish Tank

Jay has a small fish tank. It is the same shape and size as the right rectangular prism shown in the Opening Exercise.

a. The box it came in says that it is a 3 gallon tank. Is this claim true? Explain your reasoning. Recall that $1 \text{ gal} = 231 \text{ in}^3$.

The volume of the tank is 715 in^3. To convert cubic inches to gallons, divide by 231.

$$715 \text{ in}^3 \cdot \frac{1 \text{ gallon}}{231 \text{ in}^3} = 3.09 \text{ gallons}$$

The claim is true if you round to the nearest whole gallon.

b. The pet store recommends filling the tank to within 1.5 in. of the top. How many gallons of water will the tank hold if it is filled to the recommended level?

Use 8.5 in. instead of 10 in. to calculate the volume. $V = 11 \text{ in} \cdot 6.5 \text{ in} \cdot 8.5 \text{ in} = 607.75 \text{ in}^3$.

$$607.75 \text{ in}^3 \cdot \frac{1 \text{ gallon}}{231 \text{ in}^3} = 2.63 \text{ gallons}$$

c. Jay wants to cover the back, left, and right sides of the tank with a background picture. How many square inches will be covered by the picture?

Back side area $= 10 \text{ in.} \cdot 11 \text{ in.} = 110 \text{ in}^2$.

Left and right side area $= 2(6.5 \text{ in.})(10 \text{ in.}) = 130 \text{ in}^2$.

The total area to be covered with the background picture is 240 in^2.

d. Water in the tank evaporates each day, causing the water level to drop. How many gallons of water have evaporated by the time the water in the tank is four inches deep? Assume the tank was filled to within 1.5 in. of the top to start.

When the water is filled to within 1.5 in. of the top, the volume is 607.75 in^3. When the water is 4 in. deep, the volume is $11 \text{ in.} \cdot 6.5 \text{ in.} \cdot 4 \text{ in.} = 286 \text{ in}^3$. The difference in the two volumes is $607.75 \text{ in}^3 - 286 \text{ in}^3 = 321.75 \text{ in}^3$. Converting cubic inches to gallons by dividing by 231 gives a difference of 1.39 gal, which means 1.39 gal of water have evaporated.

Use these questions with the whole class or small groups as discussion points.

- Which problems involve measuring the surface area? Which problems involve measuring the volume?
  - *Covering the sides of the tank involved surface area. The other problems asked about the amount of water the tank would hold, which required us to measure the volume of the tank.*
- How do you convert cubic inches to gallons?
  - *You need to divide the total cubic inches by the number of cubic inches in one gallon.*
- What are some different ways to answer part (c)?
  - *Answers will vary. You could do each side separately, or you could do the left side and multiply it by* 2*, then add the area of the back side.*

Exercise 1 (10 minutes): Fish Tank Designs

MP.3

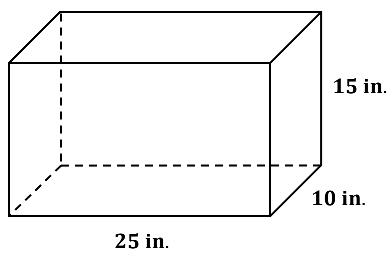
In this exercise, students compare the volume of two different right prisms. They consider the differences in the surface areas and volumes of differently shaped tanks. This example presents two solid figures where a figure with larger volume has a smaller surface area. In part (c), students explore whether or not this is always true. After completing the exercise, have the class consider the following questions as you discuss this exercise. If time permits, encourage students to consider how a company that manufactures fish tanks might decide on its designs. Encourage students to make claims and respond to the claims of others. Below are some possible discussion questions to pose to students after the exercises are completed.

- When comparing the volumes and the surface areas, the larger-volume tank has the smaller surface area. Why? Will it always be like that?
  - *Changing the dimensions of the base affects the surface area. Shapes that are more like a cube will have a smaller surface area. For a rectangular base tank, where the area of the base is a long and skinny rectangle, the surface area is much greater. For example, a tank with a base that is* 50 in. *by* 5 in. *has a surface area of* $2(5\text{ in.})(50\text{ in.}) + 2(5\text{ in.})(15\text{ in.}) + 2(50\text{ in.})(15\text{ in.}) = 2150\text{ in}^2$*. The surface area is more than the trapezoid base tank, but the volume is the same.*
- Why might a company be interested in building a fish tank that has a smaller surface area for a larger volume? What other parts of the design might make a difference when building a fish tank?
  - *The company that makes tanks might set its prices based on the amount of material used. If the volumes are the same, then the tank with fewer materials would be cheaper to make. The company might make designs that are more interesting to buyers, such as the trapezoidal prism.*

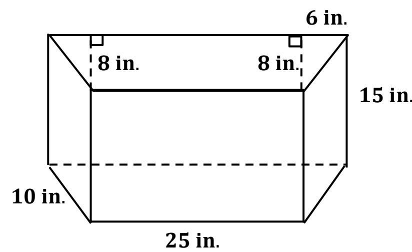
Exercise 1: Fish Tank Designs

Two fish tanks are shown below, one in the shape of a right rectangular prism (R) and one in the shape of a right trapezoidal prism (T).

Tank R



Tank T



a. **Which tank holds the most water? Let $Vol(R)$ represent the volume of the right rectangular prism and $Vol(T)$ represent the volume of the right trapezoidal prism. Use your answer to fill in the blanks with $Vol(R)$ and $Vol(T)$.**

Volume of right rectangular prism: $(25\text{ in.} \times 10\text{ in.}) \times 15\text{ in.} = 3{,}750\text{ in}^3$

Volume of right trapezoidal prism: $(31\text{ in.} \times 8\text{ in.}) \times 15\text{ in.} = 3{,}720\text{ in}^3$

The right rectangular prism holds the most water.

$\underline{\quad Vol(T) \quad} < \underline{\quad Vol(R) \quad}$

b. **Which tank has the most surface area? Let $SA(R)$ represent the surface area of the right rectangular prism and $SA(T)$ represent the surface area of the right trapezoidal prism. Use your answer to fill in the blanks with $SA(R)$ and $SA(T)$.**

The surface area of the right rectangular prism:
$2(25\text{ in.} \times 10\text{ in.}) + 2(25\text{ in.} \times 15\text{ in.}) + 2(10\text{ in.} \times 15\text{ in.}) = 500\text{ in}^2 + 750\text{ in}^2 + 300\text{ in}^2 = 1{,}550\text{ in}^2$

The surface area of the right trapezoidal prism:

$$\begin{aligned} &2(31\text{ in.} \times 8\text{ in.}) + 2(10\text{ in.} \times 15\text{ in.}) + (25\text{ in.} \times 15\text{ in.}) + (31\text{ in.} \times 15\text{ in.}) \\ &= 496\text{ in}^2 + 300\text{ in}^2 + 375\text{ in}^2 + 555\text{ in}^2 = 1726\text{ in}^2 \end{aligned}$$

The right trapezoidal prism has the most surface area.

$\underline{\quad SA(R) \quad} < \underline{\quad SA(T) \quad}$

c. **Water evaporates from each aquarium. After the water level has dropped $\frac{1}{2}$ inch in each aquarium, how many cubic inches of water are required to fill up each aquarium? Show work to support your answers.**

The right rectangular prism will need 125 in^3 *of water. The right trapezoidal prism will need* 124 in^3 *of water. I decreased the height of each prism by a half inch and recalculated the volumes. Then, I subtracted each answer from the original volume of each prism.*

$NewVol(R) = (25)(10)(14.5)\text{ in}^3 = 3{,}625\text{ in}^3$

$3{,}750\text{ in}^3 - 3{,}625\text{ in}^3 = 125\text{ in}^3$

$NewVol(T) = (31)(8)(14.5)\text{ in}^3 = 3{,}596\text{ in}^3$

$3{,}720\text{ in}^3 - 3{,}596\text{ in}^3 = 124\text{ in}^3$

Exercise 2 (15 minutes): Design Your Own Fish Tank

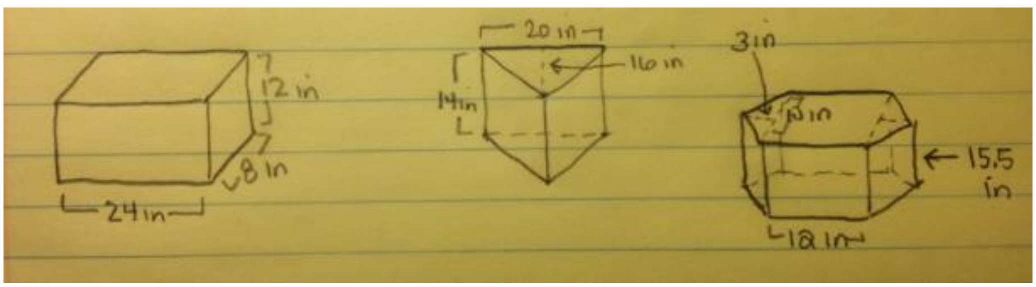
MP.4

This is a very open-ended task. If students have struggled with the first example and exercise, you may wish to move them directly to some of the Problem Set exercises. Three possible solutions are presented below, but there are others. None of these solutions has a volume of exactly 10 gallons. Encourage students to find reasonable dimensions that are close to 10 gallons. The volume in cubic inches of a 10 gallon tank is 2310 in^3. Students may try various approaches to this problem. Encourage them to select values for the dimensions of the tank that are realistic. For example, a rectangular prism tank that is 23 in. by 20 in. by 5 in. is probably not a reasonable choice, even though the volume is exactly 2310 in^3.

Exercise 2: Design Your Own Fish Tank

Design at least three fish tanks that will hold approximately 10 gallons of water. All of the tanks should be shaped like right prisms. Make at least one tank have a base that is not a rectangle. For each tank, make a sketch, and calculate the volume in gallons to the nearest hundredth.

Three possible designs are shown below.



10 gal *is* $2,310\ \text{in}^3$

Rectangular Base: Volume $= 2,304\ \text{in}^3$ *or* 9.97 gal

Triangular Base: Volume $= 2,240\ \text{in}^3$ *or* 9.70 gal

Hexagonal Base: Volume $= 2,325\ \text{in}^3$ *or* 10.06 gal

Challenge: Each tank is to be constructed from glass that is $\frac{1}{4}$ in. thick. Select one tank that you designed and determine the difference between the volume of the total tank (including the glass) and the volume inside the tank. Do not include a glass top on your tank.

Height $= 12\ \text{in.} - \frac{1}{4}\ \text{in.} = 11.75\ \text{in.}$

Length $= 24\ \text{in.} - \frac{1}{2}\ \text{in.} = 23.5\ \text{in.}$

Width $= 8\ \text{in.} - \frac{1}{2}\ \text{in.} = 7.5\ \text{in.}$

Inside Volume $= 2,070.9\ \text{in}^3$

The difference between the two volumes is $233.1\ \text{in}^3$, *which is approximately* 1 gal.

Closing (2 minutes)

When discussing the third bulleted item below with students, emphasize the point by using two containers with the same volume (perhaps a rectangular cake pan and a more cube-like container). Pour water (or rice) into both. Ask students which container has a greater surface area (the rectangular cake pan). Then, pour the contents of the two containers into separate one-gallon milk jugs to see that, while the surface areas are different, the volume held by each is the same.

- When the water is removed from a right prism-shaped tank, and the volume of water is reduced, which other measurement(s) also change? Which measurement(s) stay the same?
  - *The height is also reduced, but the area of the base stays the same.*
- How do you decide whether a problem asks you to find the surface area or the volume of a solid figure?
  - *The decision is based on whether you are measuring the area of the sides of the solid or whether you are measuring the space inside. If you are filling a tank with water or a liquid, then the question is about volume. If you are talking about the materials required to build the solid, then the question is about surface area.*
- Does a bigger volume always mean a bigger surface area?
  - *No. Two right prisms can have the same volume but different surface areas. If you increase the volume by making the shape more like a cube, the surface area might be less than if it was as a solid with a smaller volume.*

Exit Ticket (3 minutes)

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Name ___________________________________ Date ____________________

Lesson 25: Volume and Surface Area

Exit Ticket

Melody is planning a raised bed for her vegetable garden.

a. How many square feet of wood does she need to create the bed?

b. She needs to add soil. Each bag contains 1.5 ft^3. How many bags will she need to fill the vegetable garden?

Exit Ticket Sample Solutions

Melody is planning a raised bed for her vegetable garden.

a. How many square feet of wood does she need to create the bed?

The dimensions in feet are 4 ft. *by* 1.25 ft. *by* 2.5 ft. *The lateral area is* $2(4\text{ ft.})(1.25\text{ ft.}) + 2(2.5\text{ ft.})(1.25\text{ ft.}) = 16.25\text{ ft}^2$.

b. She needs to add soil. Each bag contains 1.5 cubic feet. How many bags will she need to fill the vegetable garden?

The volume is $4\text{ ft.} \cdot 1.25\text{ ft.} \cdot 2.5\text{ ft.} = 12.5\text{ ft}^3$. *Divide the total cubic feet by* 1.5 ft^3 *to determine the number of bags.* $12.5 \div 1.5 = 8\frac{1}{3}$ *bags. Melody will need to purchase* $nine$ *bags of soil to fill the garden bed.*

Note that if students fail to recognize the need to round up to nine bags, this should be addressed. Also, if the thickness of the wood were given, then there would be soil left over and possibly only 8 bags would be needed, depending on the thickness.

Problem Set Sample Solutions

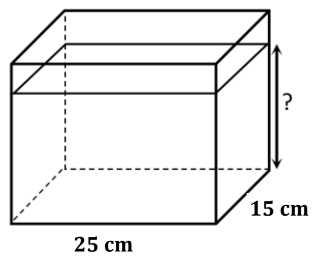
1. The dimensions of several right rectangular fish tanks are listed below. Find the volume in cubic centimeters, the capacity in liters ($1\text{ L} = 1000\text{ cm}^3$), and the surface area in square centimeters for each tank. What do you observe about the change in volume compared with the change in surface area between the small tank and the extra-large tank?

Tank Size	Length (cm)	Width (cm)	Height (cm)
Small	24	18	15
Medium	30	21	20
Large	36	24	25
Extra-Large	40	27	30

Tank Size	Volume (cm^3)	Capacity (L)	Surface Area (cm^2)
Small	$6,480$	6.48	$2,124$
Medium	$12,600$	12.6	$3,300$
Large	$21,600$	21.6	$4,728$
Extra-Large	$32,400$	32.4	$6,180$

While the volume of the extra-large tank is about five times the volume of the small tank, its surface area is less than three times that of the small tank.

2. A rectangular container 15 cm long by 25 cm wide contains 2.5 L of water.



a. Find the height of the water level in the container. $(1\text{ L} = 1000\text{ cm}^3)$

$2.5\text{ L } = 2,500\text{ cm}^3$

To find the height of the water level, divide the volume in cubic centimeters by the area of the base.

$$\frac{2,500\text{ cm}^3}{25\text{ cm} \cdot 15\text{ cm}} = 6\frac{2}{3}\text{ cm}$$

b. If the height of the container is 18 cm, how many more liters of water would it take to completely fill the container?

Volume of tank: $(25\text{ cm} \times 15\text{ cm}) \times 18\text{ cm} = 6,750\text{ cm}^3$

Capacity of tank: 6.75 L

Difference: $6.75\text{ L} - 2.5\text{ L} = 4.25\text{ L}$

c. What percentage of the tank is filled when it contains 2.5 L of water?

$$\frac{2.5\text{ L}}{6.75\text{ L}} = 0.37 = 37\%$$

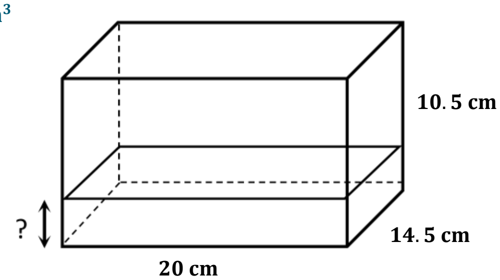
3. A rectangular container measuring 20 cm by 14.5 cm by 10.5 cm is filled with water to its brim. If 300 cm^3 are drained out of the container, what will be the height of the water level? If necessary, round to the nearest tenth.

Volume: $(20\text{ cm} \times 14.5\text{ cm}) \times 10.5\text{ cm} = 3,045\text{ cm}^3$

Volume after draining: $2,745\text{ cm}^3$

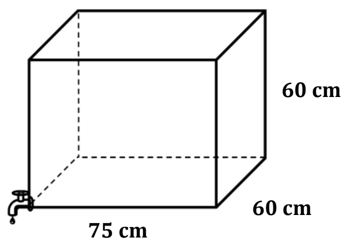
Height (divide the volume by the area of the base):

$$\frac{2745\text{ cm}^3}{20\text{ cm} \times 14.5\text{ cm}} \approx 9.5\text{ cm}$$

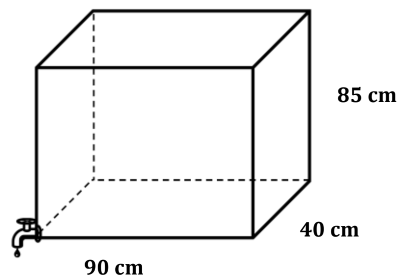


4. **Two tanks are shown below. Both are filled to capacity, but the owner decides to drain them. Tank 1 is draining at a rate of 8 liters per minute. Tank 2 is draining at a rate of 10 liters per minute. Which tank empties first?**

Tank 1



Tank 2



Tank 1 Volume: $75 \text{ cm} \times 60 \text{ cm} \times 60 \text{ cm} = 270,000 \text{ cm}^3$

Tank 2 Volume: $90 \text{ cm} \times 40 \text{ cm} \times 85 \text{ cm} = 306,000 \text{ cm}^3$

Tank 1 Capacity: 270 L

Tank 2 Capacity: 306 L

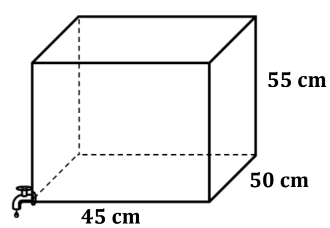
To find the time to drain each tank, divide the capacity by the rate (liters per minute).

Time to drain tank 1: $\dfrac{270 \text{ L}}{8 \frac{\text{L}}{\text{min}}} = 33.75 \text{ min}.$

Time to drain tank 2: $\dfrac{306 \text{ L}}{10 \frac{\text{L}}{\text{min}}} = 30.6 \text{ min}.$

Tank 2 empties first.

5. **Two tanks are shown below. One tank is draining at a rate of 8 liters per minute into the other one, which is empty. After 10 minutes, what will be the height of the water level in the second tank? If necessary, round to the nearest minute.**



Volume of the top tank: $45 \text{ cm} \times 50 \text{ cm} \times 55 \text{ cm} = 123,750 \text{ cm}^3$

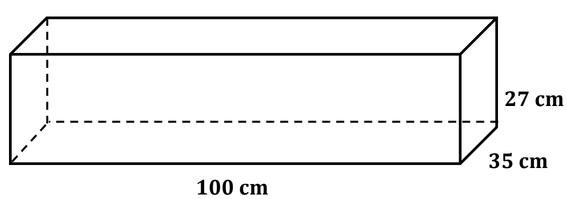
Capacity of the top tank: 123.75 L

At $8 \frac{\text{L}}{\text{min}}$ *for* 10 *minutes,* 80 L *will have drained into the bottom tank after* 10 *minutes.*

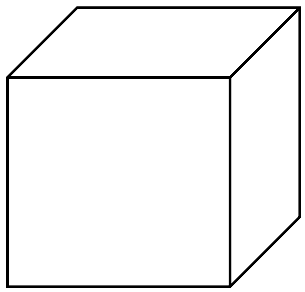
That is $80,000 \text{ cm}^3$*. To find the height, divide the volume by the area of the base.*

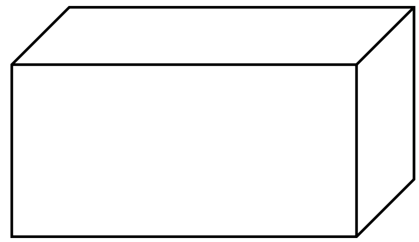
$$\frac{80000 \text{ cm}^3}{100 \text{ cm} \cdot 35 \text{ cm}} \approx 22.9 \text{ cm}$$

After 10 *minutes, the height of the water in the bottom tank will be about* 23 *cm.*



6. **Two tanks with equal volumes are shown below. The tops are open. The owner wants to cover one tank with a glass top. The cost of glass is $\$0.05$ per square inch. Which tank would be less expensive to cover? How much less?**





Dimensions: 12 in. long by 8 in. wide by 10 in. high

Surface area: 96 in^2

Cost: $0.05\frac{\$}{\text{in}^2}\cdot 96\text{ in}^2 = \$4.80$

The first tank is less expensive. It is $\$1.20$ cheaper.

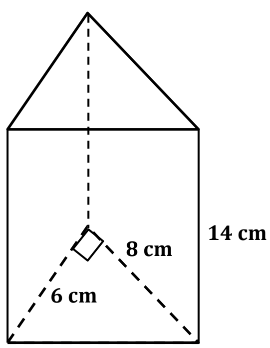
Dimensions: 15 in. long by 8 in. wide by 8 in. high

Surface area: 120 in^2

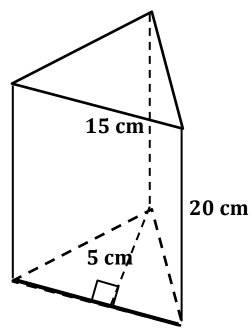
Cost: $0.05\frac{\$}{\text{in}^2}\cdot 120\text{ in}^2 = \$6.00$

7. **Each prism below is a gift box sold at the craft store.**

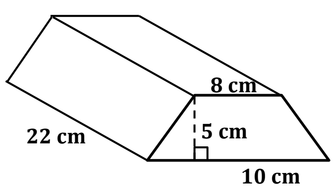
(a)



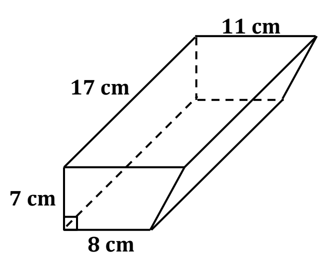
(b)



(c)



(d)



a. **What is the volume of each prism?**

(a) Volume $= 336\text{ cm}^3$, *(b) Volume* $= 750\text{ cm}^3$, *(c) Volume* $= 990\text{ cm}^3$, *(d) Volume* $= 1130.5\text{ cm}^3$

b. **Jenny wants to fill each box with jelly beans. If one ounce of jelly beans is approximately 30 cm^3, estimate how many ounces of jelly beans Jenny will need to fill all four boxes? Explain your estimates.**

Divide each volume in cubic centimeters by 30.

(a) 11.2 *ounces* *(b)* 25 *ounces* *(c)* 33 *ounces* *(d)* 37.7 *ounces*

Jenny would need a total of 106.9 *ounces.*

8. Two rectangular tanks are filled at a rate of 0.5 cubic inches per minute. How long will it take each tank to be half-full?

   a. Tank 1 Dimensions: 15 in. by 10 in. by 12.5 in.

      Volume: $1,875\ \text{in}^3$

      Half of the volume is $937.5\ \text{in}^3$*.*

      To find the number of minutes, divide the volume by the rate in cubic inches per minute.

      Time: $1,875$ *minutes.*

   b. Tank 2 Dimensions: $2\frac{1}{2}$ in. by $3\frac{3}{4}$ in. by $4\frac{3}{8}$ in.

      Volume: $\frac{2625}{64}\ \text{in}^3$

      Half of the volume is $\frac{2625}{128}\ \text{in}^3$*.*

      To find the number of minutes, divide the volume by the rate in cubic inches per minute.

      Time: 41 *minutes*