Lesson 14: Solving Inequalities

Student Outcomes

- Students solve word problems leading to inequalities that compare $px + q$ and r, where p, q, and r are specific rational numbers.
- Students interpret the solutions in the context of the problem.

Classwork

Opening (1 minute)

Start the lesson by discussing some summertime events that students may attend. One event may be a carnival or a fair. The problems that the students complete today are all about a local carnival in their town that lasts $5\frac{1}{2}$ days.

Opening Exercise (12 minutes)

Opening Exercise

The annual County Carnival is being held this summer and will last $5\frac{1}{2}$ days. Use this information and the other given information to answer each problem.

You are the owner of the biggest and newest rollercoaster called the Gentle Giant. The rollercoaster costs $\$6$ to ride. The operator of the ride must pay $\$200$ per day for the ride rental and $\$65$ per day for a safety inspection. If you want to make a profit of at least $\$1,000$ each day, what is the minimum number of people that must ride the rollercoaster?

Write an inequality that can be used to find the minimum number of people, p, that must ride the rollercoaster each day to make the daily profit.

$$6p - 200 - 65 \geq 1000$$

Solve the inequality.

$$6p - 200 - 65 \geq 1000$$
$$6p - 265 \geq 1000$$
$$6p - 265 + 265 \geq 1000 + 265$$
$$6p + 0 \geq 1265$$
$$\left(\frac{1}{6}\right)(6p) \geq \left(\frac{1}{6}\right)(1265)$$
$$p \geq 210\frac{5}{6}$$

Interpret the solution.

There needs to be a minimum of 211 people to ride the rollercoaster every day to make a daily profit of at least $\$1,000$.

Scaffolding:

- Use integers to lead to the inequality.
- What would be the profit if 10 people rode the rollercoaster?
  - $6(10) - 200 - 65 = -205$
- What does the answer mean within the context of the problem?
  - *There was not a profit; the owner lost* $\$205$.
- What would be the profit if 50 people rode the rollercoaster?
  - $6(50) - 200 - 65 = \$35$
- What would be the profit if 200 people rode the rollercoaster?
  - $6(200) - 200 - 65 = \$935$
- Recall that profit is the revenue (money received) less the expenses (money spent).

Discussion

- Recall the formula for profit as $\text{revenue} - \text{expenses}$. In this example, what expression represents the revenue, and what expression represents the expenses?
    - *The revenue is the money coming in. This would be* $\$6$ *per person.*
    - *The expenses are the money spent or going out. This would be the daily cost of renting the ride,* $\$200$*, and the daily cost of safety inspections,* $\$65$*.*
- Why was the inequality $\geq$ used?
    - *The owner would be satisfied if the profit was at least* $\$1{,}000$ *or more. The phrase at least means greater than or equal to.*
- Was it necessary to *flip* or *reverse* the inequality sign? Explain why or why not.
    - *No, it was not necessary to reverse the inequality sign. This is because we did not multiply or divide by a negative number.*
- Describe the if-then moves used in solving the inequality.
    - *After combining like terms,* 265 *was added to both sides. Adding a number to both sides of the inequality does not change the solution of the inequality. Lastly* $\frac{1}{6}$ *was multiplied to both sides to isolate the variable.*

MP.2 & MP.6

- Why is the answer 211 people versus 210 people?
    - *The answer has to be greater than or equal to* $210\frac{5}{6}$ *people. You cannot have* $\frac{5}{6}$ *of a person. If only* 210 *people purchased tickets, the profit would be* $\$995$*, which is less than* $\$1{,}000$*. Therefore, we round up to assure the profit of at least* $\$1{,}000$*.*
- The variable p represents the number of people who ride the rollercoaster each day. Explain the importance of clearly defining p as people riding the rollercoaster per day versus people who ride it the entire carnival time. How would the inequality change if p were the number of people who rode the rollercoaster the entire time?
    - *Since the expenses and profit were given as daily figures, then* p *would represent the number of people who rode the ride daily. The units have to be the same. If* p *were for the entire time the carnival was in town, then the desired profit would be* $\$1{,}000$ *for the entire* $5\frac{1}{2}$ *days instead of daily. However, the expenses were given as daily costs. Therefore, to determine the number of tickets that need to be sold to achieve a profit of at least* $\$1{,}000$ *for the entire time the carnival is in town, we will need to calculate the total expense by multiplying the daily expenses by* $5\frac{1}{2}$*. The new inequality would be* $6p - 5.5(265) \geq 1{,}000$*, which would change the answer to* 410 *people overall.*
- What if the expenses were charged for a whole day versus a half day? How would that change the inequality and answer?
    - *The expenses would be multiplied by* 6*, which would change the answer to* 432 *people.*
- What if the intended profit was still $\$1{,}000$ per day, but p was the number of people who rode the rollercoaster the entire time the carnival was in town?
    - *The expenses and desired profit would be multiplied by* 5.5*. The answer would change to a total of* $1{,}160$ *people.*

Example 1 (8 minutes)

Example 1

A youth summer camp has budgeted $\$2,000$ for the campers to attend the carnival. The cost for each camper is $\$17.95$, which includes general admission to the carnival and two meals. The youth summer camp must also pay $\$250$ for the chaperones to attend the carnival and $\$350$ for transportation to and from the carnival. What is the greatest number of campers who can attend the carnival if the camp must stay within its budgeted amount?

Let c represent the number of campers to attend the carnival.

$$17.95c + 250 + 350 \leq 2000$$
$$17.95c + 600 \leq 2000$$
$$17.95c + 600 - 600 \leq 2000 - 600$$
$$17.95c \leq 1400$$
$$\left(\frac{1}{17.95}\right)(17.95c) \leq \left(\frac{1}{17.95}\right)(1400)$$
$$c \leq 77.99$$

In order for the camp to stay in budget, the greatest number of campers who can attend the carnival is 77 campers.

- Why is the inequality $\leq$ used?
  - *The camp can spend less than the budgeted amount or the entire amount, but cannot spend more.*
- Describe the if-then moves used to solve the inequality.
  - *Once like terms were collected, then the goal was to isolate the variable to get 0s and 1s. If a number, such as 600 is subtracted from each side of an inequality, then the solution of the inequality does not change. If a positive number, $\frac{1}{17.95}$, is multiplied to each side of the inequality, then the solution of the inequality does not change.*
- Why did we round down instead of rounding up?
  - *In the context of the problem, the number of campers has to be less than 77.99 campers. Rounding up to 78 would be greater than 77.99; thus, we rounded down.*
- How can the equation be written to clear the decimals, resulting in an inequality with integer coefficients? Write the equivalent inequality.
  - *Since the decimal terminates in the 100th place, to clear the decimals we can multiply every term by 100. The equivalent inequality would be $1{,}795 + 60{,}000 \leq 200{,}000$.*

Example 2 (8 minutes)

Example 2

The carnival owner pays the owner of an exotic animal exhibit $\$650$ for the entire time the exhibit is displayed. The owner of the exhibit has no other expenses except for a daily insurance cost. If the owner of the animal exhibit wants to make more than $\$500$ in profits for the $5\frac{1}{2}$ days, what is the greatest daily insurance rate he can afford to pay?

Let i represent the daily insurance cost.

$$650 - 5.5i > 500$$
$$-5.5i + 650 - 650 > 500 - 650$$
$$-5.5i + 0 > -150$$
$$\left(\frac{1}{-5.5}\right)(-5.5i) > \left(\frac{1}{-5.5}\right)(-150)$$
$$i < 27.27$$

The maximum daily cost the owner can pay for insurance is $\$27.27$.

Encourage students to verbalize the If-then moves used to obtain a solution.

- Since the desired profit was greater than ($>$) $\$500$, the inequality used was $>$. Why, then, is the answer $i < 27.27$?
  - *When solving the inequality, we multiplied both sides by a negative number. When you multiply or divide by a negative number, the inequality is NOT preserved, and it is reversed.*
- Why was the answer rounded to 2 decimal places?
  - *Since i represents the daily cost, in cents, then when we are working with money, the decimal is rounded to the hundredth place, or two decimal places.*

MP.6

- The answer is $i < 27.27$. Notice that the inequality is not less than or equal to. The largest number less than 27.27 is 27.26. However, the daily cost is still \$27.27. Why is the maximum daily cost \$27.27 and not \$27.26?
  - *The profit had to be more than $\$500$, not equal to $\$500$. The precise answer is 27.27272727. Since the answer is rounded to $\$27.27$, the actual profit, when 27.27 is substituted into the expression, would be 500.01, which is greater than $\$500$.*
- Write an equivalent inequality clearing the decimals.
  - $6{,}500 - 55i > 5{,}000$
- Why do we multiply by 10 and not 100 to clear the decimals?
  - *The smallest decimal terminates in the tenths place.*

Example 3 (8 minutes)

Example 3

Several vendors at the carnival sell products and advertise their businesses. Shane works for a recreational company that sells ATVs, dirt bikes, snowmobiles, and motorcycles. His boss paid him $\$500$ for working all of the days at the carnival plus 5% commission on all of the sales made at the carnival. What was the minimum amount of sales Shane needed to make if he earned more than $\$1,500$?

Let s represent the sales, in dollars, made during the carnival.

$$500 + \frac{5}{100}s > 1,500$$
$$\frac{5}{100}s + 500 > 1,500$$
$$\frac{5}{100}s + 500 - 500 > 1,500 - 500$$
$$\frac{5}{100}s + 0 > 10,00$$
$$\left(\frac{100}{5}\right)\left(\frac{5}{100}s\right) > \left(\frac{100}{5}\right)(1,000)$$
$$s > 20,000$$

The sales had to be more than $\$20,000$ for Shane to earn more than $\$1,500$.

Encourage students to verbalize the if-then moves used in obtaining the solution.

- Recall from Module 2 how to work with a percent. Percents are out of 100, so what fraction or decimal represents 5%?
  - $\frac{5}{100}$ *or* 0.05
- How can we write an equivalent inequality containing only integer coefficients and constant terms? Write the equivalent inequality.
  - *Every term can be multiplied by the common denominator of the fraction. In this case, the only denominator is* 100. *After clearing the fraction the equivalent inequality is* $50{,}000 + 5x > 150{,}000$.
- Solve the new inequality.
  - $$50{,}000 + 5x > 150{,}000$$
    $$5x + 50{,}000 - 50{,}000 > 150{,}000 - 50{,}000$$
    $$5x + 0 > 100{,}000$$
    $$\left(\frac{1}{5}\right)(5x) > \left(\frac{1}{5}\right)(100{,}000)$$
    $$x > 20{,}000$$

Closing (3 minutes)

- What did all of the situations that required an inequality to solve have in common?
- How is a solution of an inequality interpreted?

Lesson Summary

The key to solving inequalities is to use If-then moves to make 0s and 1s to get the inequality into the form $x >$ a number or $x <$ a number. Adding or subtracting opposites will make 0s. According to the if-then move, a number that is added or subtracted to each side of an inequality does not change the solution to the inequality. Multiplying and dividing numbers makes 1s. A positive number that is multiplied or divided to each side of an inequality does not change the solution of the inequality. However, multiplying or dividing each side of an inequality by a negative number does reverse the inequality sign.

Given inequalities containing decimals, equivalent inequalities can be created which have only integer coefficients and constant terms by repeatedly multiplying every term by ten until all coefficients and constant terms are integers.

Given inequalities containing fractions, equivalent inequalities can be created which have only integer coefficients and constant terms by multiplying every term by the least common multiple of the values in the denominators.

Exit Ticket (5 minutes)

Name ____ Date____________________

Lesson 14: Solving Inequalities

Exit Ticket

Games at the carnival cost $\$3$ each. The prizes awarded to winners cost the owner $\$145.65$. How many games must be played for the owner of the game to make at least $\$50$?

Exit Ticket Sample Solutions

Games at the carnival cost $\$3$ each. The prizes awarded to winners cost $\$145.65$. How many games must be played to make at least $\$50$?

Let g represent the number of games played.

$$3g - 145.65 \geq 50$$
$$3g - 145.65 + 145.65 \geq 50 + 145.65$$
$$3g + 0 \geq 195.65$$
$$\left(\frac{1}{3}\right)(3g) \geq \left(\frac{1}{3}\right)(195.65)$$
$$g \geq 65.217$$

There must be at least 66 games played to make at least $\$50$.

Problem Set Sample Solutions

1. As a salesperson, Jonathan is paid $\$50$ per week plus 3% of the total amount he sells. This week, he wants to earn at least $\$100$. Write an inequality with integer coefficients for the total sales needed to earn at least $\$100$, and describe what the solution represents.

   Let the variable p represent the purchase amount.

$$50 + \frac{3}{100}p \geq 100$$
$$\frac{3}{100}p + 50 \geq 100$$
$$(100)\left(\frac{3}{100}p\right) + 100(50) \geq 100(100)$$
$$3p + 5000 \geq 10000$$
$$3p + 5000 - 5000 \geq 10000 - 5000$$
$$3p + 0 \geq 5000$$
$$\left(\frac{1}{3}\right)(3p) \geq \left(\frac{1}{3}\right)(5000)$$
$$p \geq 1666\frac{2}{3}$$

   Jonathan must sell $\$1,666.67$ in total purchases.

2. Systolic blood pressure is the higher number in a blood pressure reading. It is measured as the heart muscle contracts. Heather was with her grandfather when he had his blood pressure checked. The nurse told him that the upper limit of his systolic blood pressure is equal to half his age increased by 110.

   a. a is the age in years, and p is the systolic blood pressure in mmHg (milliliters of Mercury). Write an inequality to represent this situation.

   $$p \leq \frac{1}{2}a + 110$$

b. **Heather's grandfather is 76 years old. What is "normal" for his systolic blood pressure?**

$p \leq \frac{1}{2}a + 110$, where $a = 76$.

$$p \leq \frac{1}{2}(76) + 110$$
$$p \leq 38 + 110$$
$$p \leq 148$$

A systolic blood pressure for his age is normal if it is at most 148.

3. **Traci collects donations for a dance marathon. One group of sponsors will donate a total of $\$6$ for each hour she dances. Another group of sponsors will donate $\$75$ no matter how long she dances. What number of hours, to the nearest minute, should Traci dance if she wants to raise at least $\$1,000$?**

Let the variable h represent the number of hours Traci dances.

$$6h + 75 \geq 1000$$
$$6h + 75 - 75 \geq 1000 - 75$$
$$6h + 0 \geq 925$$
$$\left(\frac{1}{6}\right)(6h) \geq \left(\frac{1}{6}\right)(925)$$
$$h \geq 154\frac{1}{6}$$

$h \geq 154$ hours and 10 minutes.

4. **Jack's age is three years more than twice his younger brother's, Jimmy's, age. If the sum of their ages is at most 18, find the greatest age that Jimmy could be.**

Let the variable j represent Jimmy's age in years.

Then, the expression $3 + 2j$ represents Jack's age in years.

$$j + 3 + 2j \leq 18$$
$$3j + 3 \leq 18$$
$$3j + 3 - 3 \leq 18 - 3$$
$$3j \leq 15$$
$$\left(\frac{1}{3}\right)(3j) \leq \left(\frac{1}{3}\right)(15)$$
$$j \leq 5$$

Jimmy's age is 5 years or less.

5. **Brenda has $\$500$ in her bank account. Every week she withdraws $\$40$ for miscellaneous expenses. How many weeks can she withdraw the money if she wants to maintain a balance of a least $\$200$?**

Let the variable w represent the number of weeks.

$$500 - 40w \geq 200$$
$$500 - 500 - 40w \geq 200 - 500$$
$$-40w \geq -300$$
$$\left(-\frac{1}{40}\right)(-40w) \leq \left(-\frac{1}{40}\right)(-300)$$
$$w \leq 7.5$$

$\$40$ can be withdrawn from the account for seven weeks if she wants to maintain a balance of at least $\$200$.

6. A scooter travels 10 miles per hour faster than an electric bicycle. The scooter traveled for 3 hours, and the bicycle traveled for $5\frac{1}{2}$ hours. All together, the scooter and bicycle traveled no more than 285 miles. Find the maximum speed of each.

	Speed	*Time*	*Distance*
Scooter	$x+10$	3	$3(x+10)$
Bicycle	x	$5\frac{1}{2}$	$5\frac{1}{2}x$

$$3(x+10)+5\frac{1}{2}x \le 285$$
$$3x+30+5\frac{1}{2}x \le 285$$
$$8\frac{1}{2}x+30 \le 285$$
$$8\frac{1}{2}x+30-30 \le 285-30$$
$$8\frac{1}{2}x \le 255$$
$$\frac{17}{2}x \le 255$$
$$\left(\frac{2}{17}\right)\left(\frac{17}{2}x\right) \le (255)\left(\frac{2}{17}\right)$$
$$x \le 30$$

The maximum speed the bicycle traveled was 30 miles per hour, and the maximum speed the scooter traveled was 40 miles per hour.