

Lesson 14: Solving Inequalities

Student Outcomes

- Students solve word problems leading to inequalities that compare px + q and r, where p, q, and r are specific rational numbers.
- Students interpret the solutions in the context of the problem.

Classwork

Opening (1 minute)

Start the lesson by discussing some summertime events that students may attend. One event may be a carnival or a fair. The problems that the students complete today are all about a local carnival in their town that lasts $5\frac{1}{2}$ days.

Exercise (12 minutes)	 Scaffolding: Use integers to lead to the inequality. What would be the profit if 10 people rode the rollercoaster? 		
Opening Exercise The annual County Carnival is being held this summer and will last $5\frac{1}{2}$ days. Use this information and the other given information to answer each problem.			
You are the owner of the biggest and newest rollercoaster called the Gentle Giant. The rollercoaster costs \$6 to ride. The operator of the ride must pay \$200 per day for the ride rental and \$65 per day for a safety inspection. If you want to make a profit of at least \$1,000 each day, what is the minimum number of people that must ride the rollercoaster?	 6(10) - 200 - 65 = -205 What does the answer mean within the context of the problem? <i>There was not a profit; the owner lost</i> \$205. What would be the profit if 50 people rode the rollercoaster? 6(50) - 200 - 65 = \$35 		
Write an inequality that can be used to find the minimum number of people, p, that must ride the rollercoaster each day to make the daily profit. 6p - 200 - 65 > 1000			
Solve the inequality. $6p - 200 - 65 \ge 1000$ $6p - 265 \ge 1000$ $6p - 265 + 265 \ge 1000 + 265$ $6p + 0 \ge 1265$ $\left(\frac{1}{6}\right)(6p) \ge \left(\frac{1}{6}\right)(1265)$ $p \ge 210\frac{5}{6}$	 What would be the profit if 200 people rode the rollercoaster? 6(200) - 200 - 65 = \$935 Recall that profit is the revenue (money received) less the expense (money spent). 		
Interpret the solution. There needs to be a minimum of 211 people to ride the rollercoaster every day to make a	a daily profit of at least \$1,000.		



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Discussion

- Recall the formula for profit as revenue expenses. In this example, what expression represents the revenue, and what expression represents the expenses?
 - *The revenue is the money coming in. This would be* \$6 *per person.*
 - The expenses are the money spent or going out. This would be the daily cost of renting the ride, \$200, and the daily cost of safety inspections, \$65.
- Why was the inequality \geq used?
 - The owner would be satisfied if the profit was at least \$1,000 or more. The phrase at least means greater than or equal to.
- Was it necessary to *flip* or *reverse* the inequality sign? Explain why or why not.
 - No, it was not necessary to reverse the inequality sign. This is because we did not multiply or divide by a negative number.
- Describe the if-then moves used in solving the inequality.
 - After combining like terms, 265 was added to both sides. Adding a number to both sides of the inequality does not change the solution of the inequality. Lastly $\frac{1}{6}$ was multiplied to both sides to isolate the variable.
- Why is the answer 211 people versus 210 people?
 - The answer has to be greater than or equal to $210\frac{5}{6}$ people. You cannot have $\frac{5}{6}$ of a person. If only 210 people purchased tickets, the profit would be \$995, which is less than \$1,000. Therefore, we round up to assure the profit of at least \$1,000.
- The variable p represents the number of people who ride the rollercoaster each day. Explain the importance of clearly defining p as people riding the rollercoaster per day versus people who ride it the entire carnival time. How would the inequality change if p were the number of people who rode the rollercoaster the entire time?
 - Since the expenses and profit were given as daily figures, then p would represent the number of people who rode the ride daily. The units have to be the same. If p were for the entire time the carnival was in town, then the desired profit would be \$1,000 for the entire $5\frac{1}{2}$ days instead of daily. However, the expenses were given as daily costs. Therefore, to determine the number of tickets that need to be sold to achieve a profit of at least \$1,000 for the entire time the carnival is in town, we will need to calculate the total expense by multiplying the daily expenses by $5\frac{1}{2}$. The new inequality would be $6p - 5.5(265) \ge 1,000$, which would change the answer to 410 people overall.
- What if the expenses were charged for a whole day versus a half day? How would that change the inequality and answer?
 - The expenses would be multiplied by 6, which would change the answer to 432 people.
- What if the intended profit was still 1,000 per day, but p was the number of people who rode the rollercoaster the entire time the carnival was in town?
 - The expenses and desired profit would be multiplied by 5.5. The answer would change to a total of 1,160 people.





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Example 1 (8 minutes)

Example 1

A youth summer camp has budgeted \$2,000 for the campers to attend the carnival. The cost for each camper is \$17.95, which includes general admission to the carnival and two meals. The youth summer camp must also pay \$250 for the chaperones to attend the carnival and \$350 for transportation to and from the carnival. What is the greatest number of campers who can attend the carnival if the camp must stay within its budgeted amount?

Let c represent the number of campers to attend the carnival.

 $\begin{aligned} 17.95c + 250 + 350 &\leq 2000 \\ 17.95c + 600 &\leq 2000 \\ 17.95c + 600 - 600 &\leq 2000 - 600 \\ 17.95c &\leq 1400 \\ \left(\frac{1}{17.95}\right)(17.95c) &\leq \left(\frac{1}{17.95}\right)(1400) \\ c &\leq 77.99 \end{aligned}$

In order for the camp to stay in budget, the greatest number of campers who can attend the carnival is 77 campers.

- Why is the inequality \leq used?
 - ^a The camp can spend less than the budgeted amount or the entire amount, but cannot spend more.
- Describe the if-then moves used to solve the inequality.
 - ^D Once like terms were collected, then the goal was to isolate the variable to get 0s and 1s. If a number, such as 600 is subtracted from each side of an inequality, then the solution of the inequality does not change. If a positive number, $\frac{1}{17.95}$, is multiplied to each side of the inequality, then the solution of the inequality does not change.
- Why did we round down instead of rounding up?
 - In the context of the problem, the number of campers has to be less than 77.99 campers. Rounding up to 78 would be greater than 77.99; thus, we rounded down.
- How can the equation be written to clear the decimals, resulting in an inequality with integer coefficients?
 Write the equivalent inequality.
 - Since the decimal terminates in the 100^{th} place, to clear the decimals we can multiply every term by 100. The equivalent inequality would be $1,795 + 60,000 \le 200,000$.









Example 2 (8 minutes)

Example 2

The carnival owner pays the owner of an exotic animal exhibit \$650 for the entire time the exhibit is displayed. The owner of the exhibit has no other expenses except for a daily insurance cost. If the owner of the animal exhibit wants to make more than \$500 in profits for the $5\frac{1}{2}$ days, what is the greatest daily insurance rate he can afford to pay?

Let i represent the daily insurance cost.

650 - 5.5i > 500-5.5i + 650 - 650 > 500 - 650-5.5i + 0 > -150 $\left(\frac{1}{-5.5}\right)(-5.5i) > \left(\frac{1}{-5.5}\right)(-150)$ i < 27.27

The maximum daily cost the owner can pay for insurance is \$27.27.

Encourage students to verbalize the If-then moves used to obtain a solution.

- Since the desired profit was greater than (>) \$500, the inequality used was >. Why, then, is the answer i < 27.27?</p>
 - When solving the inequality, we multiplied both sides by a negative number. When you multiply or divide by a negative number, the inequality is NOT preserved, and it is reversed.
- Why was the answer rounded to 2 decimal places?
 - ^a Since *i* represents the daily cost, in cents, then when we are working with money, the decimal is rounded to the hundredth place, or two decimal places.
- The answer is i < 27.27. Notice that the inequality is not less than or equal to. The largest number less than 27.27 is 27.26. However, the daily cost is still \$27.27. Why is the maximum daily cost \$27.27 and not \$27.26?
 - The profit had to be more than \$500, not equal to \$500. The precise answer is 27.27272727. Since the answer is rounded to \$27.27, the actual profit, when 27.27 is substituted into the expression, would be 500.01, which is greater than \$500.
- Write an equivalent inequality clearing the decimals.
 - □ 6,500 − 55*i* > 5,000
- Why do we multiply by 10 and not 100 to clear the decimals?
 - D The smallest decimal terminates in the tenths place.



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Example 3 (8 minutes)

Example 3

Several vendors at the carnival sell products and advertise their businesses. Shane works for a recreational company that sells ATVs, dirt bikes, snowmobiles, and motorcycles. His boss paid him \$500 for working all of the days at the carnival plus 5% commission on all of the sales made at the carnival. What was the minimum amount of sales Shane needed to make if he earned more than \$1,500?

Let s represent the sales, in dollars, made during the carnival.

 $500 + \frac{5}{100}s > 1,500$ $\frac{5}{100}s + 500 > 1,500$ $\frac{5}{100}s + 500 - 500 > 1,500 - 500$ $\frac{5}{100}s + 0 > 10,00$ $\left(\frac{100}{5}\right) \left(\frac{5}{100}s\right) > \left(\frac{100}{5}\right) (1,000)$ s > 20,000 The sales had to be more than \$20,000 for Shane to earn more than \$1,500.

Encourage students to verbalize the if-then moves used in obtaining the solution.

Recall from Module 2 how to work with a percent. Percents are out of 100, so what fraction or decimal represents 5%?

$$\frac{5}{100}$$
 or 0.05

- How can we write an equivalent inequality containing only integer coefficients and constant terms? Write the equivalent inequality.
 - Every term can be multiplied by the common denominator of the fraction. In this case, the only denominator is 100. After clearing the fraction the equivalent inequality is 50,000 + 5x > 150,000.
- Solve the new inequality.

$$50,000 + 5x > 150,000$$

$$5x + 50,000 - 50,000 > 150,000 - 50,000$$

$$5x + 0 > 100,000$$

$$\left(\frac{1}{5}\right)(5x) > \left(\frac{1}{5}\right)(100,000)$$





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Closing (3 minutes)

- What did all of the situations that required an inequality to solve have in common?
- How is a solution of an inequality interpreted?

Lesson Summary

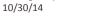
The key to solving inequalities is to use If-then moves to make 0s and 1s to get the inequality into the form x > anumber or x < a number. Adding or subtracting opposites will make 0s. According to the if-then move, a number that is added or subtracted to each side of an inequality does not change the solution to the inequality. Multiplying and dividing numbers makes 1s. A positive number that is multiplied or divided to each side of an inequality does not change the solution of the inequality. However, multiplying or dividing each side of an inequality by a negative number does reverse the inequality sign.

Given inequalities containing decimals, equivalent inequalities can be created which have only integer coefficients and constant terms by repeatedly multiplying every term by ten until all coefficients and constant terms are integers.

Given inequalities containing fractions, equivalent inequalities can be created which have only integer coefficients and constant terms by multiplying every term by the least common multiple of the values in the denominators.

Exit Ticket (5 minutes)







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Lesson 14: Solving Inequalities

Exit Ticket

Games at the carnival cost \$3 each. The prizes awarded to winners cost the owner \$145.65. How many games must be played for the owner of the game to make at least \$50?









Exit Ticket Sample Solutions

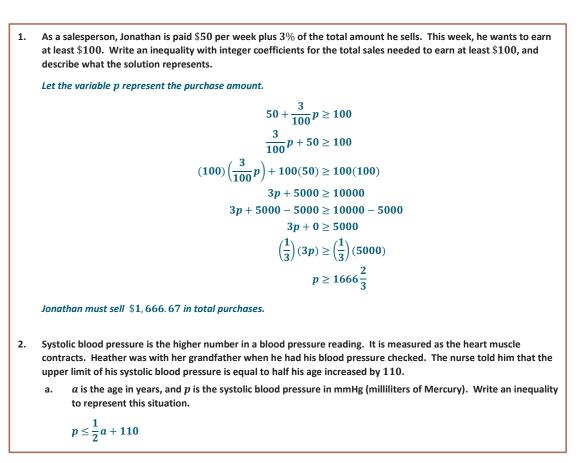
Games at the carnival cost \$3 each. The prizes awarded to winners cost \$145.65. How many games must be played to make at least \$50?

Let g represent the number of games played.

$$\begin{array}{c} 3g-145.65 \geq 50\\ 3g-145.65+145.65 \geq 50+145.65\\ 3g+0 \geq 195.65\\ \left(\frac{1}{3}\right)(3g) \geq \left(\frac{1}{3}\right)(195.65)\\ g \geq 65.217 \end{array}$$

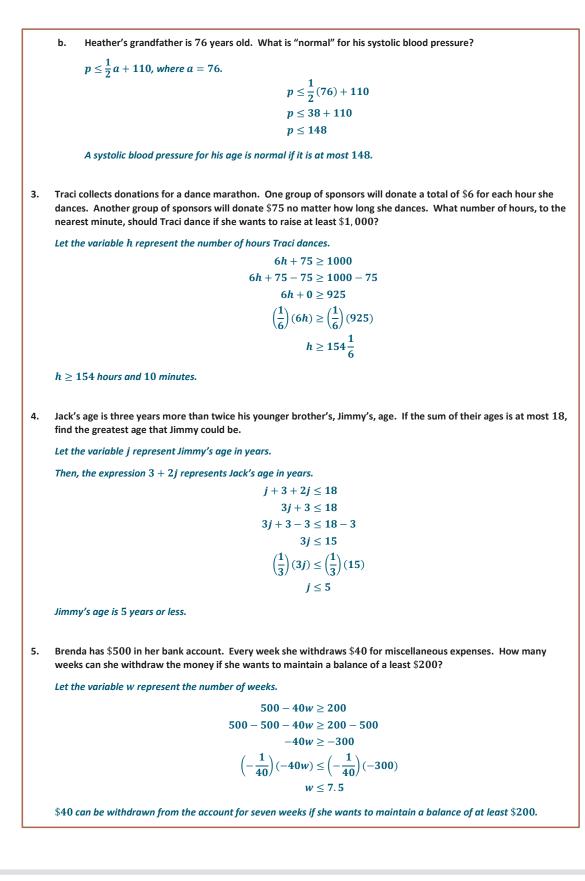
There must be at least 66 games played to make at least \$50.

Problem Set Sample Solutions











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6. A scooter travels 10 miles per hour faster than an electric bicycle. The scooter traveled for 3 hours, and the bicycle traveled for $5\frac{1}{2}$ hours. All together, the scooter and bicycle traveled no more than 285 miles. Find the maximum speed of each.

		Speed	Time	Distance			
	Scooter	<i>x</i> + 10	3	3 (<i>x</i> + 10)			
	Bicycle	x	$5\frac{1}{2}$	$5\frac{1}{2}x$			
$3(x+10) + 5\frac{1}{2}x \le 285$							
$3x + 30 + 5\frac{1}{2}x \le 285$							
$8\frac{1}{2}x+30\leq 285$							
$8\frac{1}{2}x + 30 - 30 \le 285 - 30$							
$8\frac{1}{2}x \le 255$							
$\frac{17}{2}x \le 255$							
$\left(\frac{2}{17}\right)\left(\frac{17}{2}x\right) \le (255)\left(\frac{2}{17}\right)$							
			$x \leq 30$				
The maximum speed the bicycle traveled was 30 miles per hour, and the maximum speed the scooter traveled was 40 miles per hour.							







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