Lesson 14: Solving Inequalities

Classwork

Opening Exercise

The annual County Carnival is being held this summer and will last $5\frac{1}{2}$ days. Use this information and the other given information to answer each problem.

You are the owner of the biggest and newest rollercoaster called the Gentle Giant. The rollercoaster costs $\$6$ to ride. The operator of the ride must pay $\$200$ per day for the ride rental and $\$65$ per day for a safety inspection. If you want to make a profit of at least $\$1,000$ each day, what is the minimum number of people that must ride the rollercoaster?

Write an inequality that can be used to find the minimum number of people, $p$, that must ride the rollercoaster each day to make the daily profit.

Solve the inequality.

Interpret the solution.

**Example 1**

A youth summer camp has budgeted $\$2,000$ for the campers to attend the carnival. The cost for each camper is $\$17.95$, which includes general admission to the carnival and two meals. The youth summer camp must also pay $\$250$ for the chaperones to attend the carnival and $\$350$ for transportation to and from the carnival. What is the greatest number of campers who can attend the carnival if the camp must stay within its budgeted amount?

**Example 2**

The carnival owner pays the owner of an exotic animal exhibit $\$650$ for the entire time the exhibit is displayed. The owner of the exhibit has no other expenses except for a daily insurance cost. If the owner of the animal exhibit wants to make more than $\$500$ in profits for the $5\frac{1}{2}$ days, what is the greatest daily insurance rate he can afford to pay?

**Example 3**

Several vendors at the carnival sell products and advertise their businesses. Shane works for a recreational company that sells ATVs, dirt bikes, snowmobiles, and motorcycles. His boss paid him $\$500$ for working all of the days at the carnival plus $5\%$ commission on all of the sales made at the carnival. What was the minimum amount of sales Shane needed to make if he earned more than $\$1,500$?

Lesson Summary

The key to solving inequalities is to use If-then moves to make $0$s and $1$s to get the inequality into the form $x>$ a number or $x<$ a number. Adding or subtracting opposites will make $0$s. According to the if-then move, a number that is added or subtracted to each side of an inequality does not change the solution to the inequality. Multiplying and dividing numbers makes $1$s. A positive number that is multiplied or divided to each side of an inequality does not change the solution of the inequality. However, multiplying or dividing each side of an inequality by a negative number does reverse the inequality sign.

Given inequalities containing decimals, equivalent inequalities can be created which have only integer coefficients and constant terms by repeatedly multiplying every term by ten until all coefficients and constant terms are integers.

Given inequalities containing fractions, equivalent inequalities can be created which have only integer coefficients and constant terms by multiplying every term by the least common multiple of the values in the denominators.

Problem Set

1. As a salesperson, Jonathan is paid $\$50$ per week plus $3\%$ of the total amount he sells. This week, he wants to earn at least $\$100$. Write an inequality with integer coefficients for the total sales needed to earn at least $\$100$, and describe what the solution represents.
2. Systolic blood pressure is the higher number in a blood pressure reading. It is measured as the heart muscle contracts. Heather was with her grandfather when he had his blood pressure checked. The nurse told him that the upper limit of his systolic blood pressure is equal to half his age increased by $110$.
	1. $a$ is the age in years, and $p$ is the systolic blood pressure in mmHg (milliliters of Mercury). Write an inequality to represent this situation.
	2. Heather’s grandfather is $76$ years old. What is “normal” for his systolic blood pressure?
3. Traci collects donations for a dance marathon. One group of sponsors will donate a total of $\$6$ for each hour she dances. Another group of sponsors will donate $\$75$ no matter how long she dances. What number of hours, to the nearest minute, should Traci dance if she wants to raise at least $\$1,000$?
4. Jack’s age is three years more than twice his younger brother’s, Jimmy’s, age. If the sum of their ages is at most $18$, find the greatest age that Jimmy could be.
5. Brenda has $\$500$ in her bank account. Every week she withdraws $\$40$ for miscellaneous expenses. How many weeks can she withdraw the money if she wants to maintain a balance of a least $\$200$?
6. A scooter travels $10$ miles per hour faster than an electric bicycle. The scooter traveled for $3$ hours, and the bicycle traveled for $5\frac{1}{2}$ hours. All together, the scooter and bicycle traveled no more than $285$ miles. Find the maximum speed of each.