



Lesson 12: Properties of Inequalities

Student Outcomes

- Students justify the properties of inequalities that are denoted by $<$ (less than), $\leq$ (less than or equal), $>$ (greater than), and $\geq$ (greater than or equal).

Classwork

Sprint (10 minutes): Equations

Students complete a two round Sprint exercise where they practice their knowledge of solving linear equations in the form $px + q = r$ and $p(x + q) = r$. Provide one minute for each round of the Sprint. Refer to the Sprints and Sprint Delivery Script sections in the Module Overview for directions to administer a Sprint. Be sure to provide any answers not completed by the students.

Example 1 (2 minutes)

Review the descriptions of preserves the inequality symbol and reverses the inequality symbol with students.

Example 1

Preserves the inequality symbol: *means the inequality symbol stays the same.*

Reverses the inequality symbol: *means the inequality symbol switches less than with greater than and less than or equal to with greater than or equal to.*

Exploratory Challenge (20 minutes)

Split students into four groups. Discuss the directions.

There are four stations. Provide each station with two cubes containing integers. (Cube templates provided at the end of the document.) At each station, students record their results in their student materials. (An example is provided for each station.)

MP.2
&
MP.4

- Roll each die, recording the numbers under the first and third columns. Students are to write an inequality symbol that makes the statement true. Repeat this four times to complete the four rows in the table.
- Perform the operation indicated at the station (adding or subtracting a number, writing opposites, multiplying or dividing by a number), and write a new inequality statement.
- Determine if the inequality symbol is preserved or reversed when the operation is performed.
- Rotate to a new station after five minutes.

Station 1: Add or Subtract a Number to Both Sides of the Inequality

Station 1

Die 1	Inequality	Die 2	Operation	New Inequality	Inequality Symbol Preserved or Reversed?
−3	<	5	Add 2	$-3 + 2 < 5 + 2$ $-1 < 7$	Preserved
			Add −3		
			Subtract 2		
			Subtract −1		
			Add 1		

Scaffolding:

Guide students in writing a statement using the following:

- When a number is added or subtracted to both numbers being compared, the symbol _____; therefore, the inequality symbol is _____.

Examine the results. Make a statement about what you notice, and justify it with evidence.

When a number is added or subtracted to both numbers being compared, the symbol stays the same, and the inequality symbol is preserved.

Station 2: Multiply each term by −1

Station 2

Die 1	Inequality	Die 2	Operation	New Inequality	Inequality Symbol Preserved or Reversed?
−3	<	4	Multiply by −1	$(-1)(-3) < (-1)(4)$ $3 < -4$ $3 > -4$	Reversed
			Multiply by −1		
			Multiply by −1		
			Multiply by −1		
			Multiply by −1		

Scaffolding:

Guide students in writing a statement using the following:

- When −1 is multiplied to both numbers, the symbol _____; therefore, the inequality symbol is _____.

Examine the results. Make a statement about what you notice and justify it with evidence.

When both numbers are multiplied by −1, the symbol changes, and the inequality symbol is reversed.

Station 3: Multiply or Divide Both Sides of the Inequality by a Positive Number

Station 3

Die 1	Inequality	Die 2	Operation	New Inequality	Inequality Symbol Preserved or Reversed?
-2	>	-4	Multiply by $\frac{1}{2}$	$(-2)\left(\frac{1}{2}\right) > (-4)\left(\frac{1}{2}\right)$ $-1 > -2$	Preserved
			Multiply by 2		
			Divide by 2		
			Divide by $\frac{1}{2}$		
			Multiply by 3		

Scaffolding:

Guide students in writing a statement using the following:

- When a positive number is multiplied or divided to both numbers being compared, the symbol _____; therefore, the inequality symbol is _____.

Examine the results. Make a statement about what you notice, and justify it with evidence.

When a positive number is multiplied or divided to both numbers being compared, the symbol stays the same, and the inequality symbol is preserved.

Station 4: Multiply or Divide Both Sides of the Inequality by a Negative Number

Station 4

Die 1	Inequality	Die 2	Operation	New Inequality	Inequality Symbol Preserved or Reversed?
3	>	-2	Multiply by -2	$3(-2) > (-2)(-2)$ $-6 < 4$	Reversed
			Multiply by -3		
			Divide by -2		
			Divide by $-\frac{1}{2}$		
			Multiply by $-\frac{1}{2}$		

Scaffolding:

Guide students in writing a statement using the following:

- When a negative number is multiplied by or divided by a negative number, the symbol _____; therefore, the inequality symbol is _____.

Examine the results. Make a statement about what you notice and justify it with evidence.

When a negative number is multiplied or divided to both numbers being compared, the symbol changes, and the inequality symbol is reversed.

Discussion

Summarize the findings and complete the lesson summary in the student materials.

- To summarize, when does an inequality change (reverse), and when does it stay the same (preserve)?
 - *The inequality reverses when we multiply or divide the expressions on both sides of the inequality by a negative number. The inequality stays the same for all other cases.*

Exercise (5 minutes)

Exercise			
Complete the following chart using the given inequality, and determine an operation in which the inequality symbol is preserved and an operation in which the inequality symbol is reversed. Explain why this occurs.			
<i>Solutions may vary. A sample student response is below.</i>			
Inequality	Operation and New Inequality Which Preserves the Inequality Symbol	Operation and New Inequality which Reverses the Inequality Symbol	Explanation
$2 < 5$	Add 4 to both sides. $2 < 5$ $2 + 4 < 5 + 4$ $6 < 9$	Multiply both sides by -4 . $-8 > -20$	Adding a number to both sides of an inequality preserves the inequality symbol. Multiplying a negative number to both sides of an inequality reverses the inequality symbol.
$-4 > -6$	Subtract 3 from both sides. $-4 > -6$ $-4 - 3 > -6 - 3$ $-7 > -9$	Divide both sides by -2 . $2 < 3$	Subtracting a number to both sides of an inequality preserves the inequality symbol. Dividing a negative number to both sides of an inequality reverses the inequality symbol.
$-1 \leq 2$	Multiply both sides by 3. $-1 \leq 2$ $-1(3) \leq 2(3)$ $-3 \leq 6$	Multiply both sides by -1 . $1 \geq -2$	Multiplying a positive number to both sides of an inequality preserves the inequality symbol. Multiplying a negative number to both sides of an inequality reverses the inequality symbol.
$-2 + (-3)$ $< -3 - 1$	Add 5 to both sides. $-2 + (-3) < -3 - 1$ $-2 + (-3) + 5 < -3 - 1 + 5$ $0 < 1$	Multiply each side by $-\frac{1}{2}$. $-2 + (-3) < -3 - 1$ $-5 < -4$ $\left(-\frac{1}{2}\right)(-5) > \left(-\frac{1}{2}\right)(-4)$ $\frac{5}{2} > 2$	Adding a number to both sides of an inequality preserves the inequality symbol. Multiplying a negative number to both sides of an inequality reverses the inequality symbol.

Closing (3 minutes)

- What does it mean for an inequality to be preserved? What does it mean for the inequality to be reversed?
 - *When an operation is done to both sides and the inequality does not change, it is preserved. If the inequality does change, it is reversed. For example, less than would become greater than.*

- When does a greater than become a less than?
 - *When both sides are multiplied or divided by a negative, the inequality is reversed.*

Lesson Summary

When both sides of an inequality are added or subtracted by a number, the inequality symbol stays the same, and the inequality symbol is said to be preserved.

When both sides of an inequality are multiplied or divided by a positive number, the inequality symbol stays the same, and the inequality symbol is said to be preserved.

When both sides of an inequality are multiplied or divided by a negative number, the inequality symbol switches from $<$ to $>$ or from $>$ to $<$. The inequality symbol is reversed.

Exit Ticket (5 minutes)



Name _____

Date _____

Lesson 12: Properties of Inequalities

Exit Ticket

1. Given the initial inequality $-4 < 7$, state possible values for c that would satisfy the following inequalities.
 - a. $c(-4) < c(7)$
 - b. $c(-4) > c(7)$
 - c. $c(-4) = c(7)$
2. Given the initial inequality $2 > -4$, identify which operation preserves the inequality symbol and which operation reverses the inequality symbol. Write the new inequality after the operation is performed.
 - a. Multiply both sides by -2 .
 - b. Add -2 to both sides.
 - c. Divide both sides by 2 .
 - d. Multiply both sides by $-\frac{1}{2}$.
 - e. Subtract -3 from both sides.

Exit Ticket Sample Solutions

1. Given the initial inequality $-4 < 7$, state possible values for c that would satisfy the following inequalities.

a. $c(-4) < c(7)$

$$c > 0$$

b. $c(-4) > c(7)$

$$c < 0$$

c. $c(-4) = c(7)$

$$c = 0$$

2. Given the initial inequality $2 > -4$, identify which operation preserves the inequality symbol and which operation reverses the inequality symbol. Write the new inequality after the operation is performed.

- a. Multiply both sides by -2 .

Inequality symbol is reversed.

$$\begin{aligned} 2 &> -4 \\ 2(-2) &< -4(-2) \\ -4 &< 8 \end{aligned}$$

- b. Add -2 to both sides.

Inequality symbol is preserved.

$$\begin{aligned} 2 &> -4 \\ 2 + (-2) &> -4 + (-2) \\ 0 &> -6 \end{aligned}$$

- c. Divide both sides by 2.

Inequality symbol is preserved.

$$\begin{aligned} 2 &> -4 \\ 2 \div 2 &> -4 \div 2 \\ 1 &> -2 \end{aligned}$$

- d. Multiply both sides by $-\frac{1}{2}$.

Inequality symbol is reversed.

$$\begin{aligned} 2 &> -4 \\ 2\left(-\frac{1}{2}\right) &< -4\left(-\frac{1}{2}\right) \\ -1 &< 2 \end{aligned}$$

- e. Subtract -3 from both sides.

Inequality symbol is preserved.

$$\begin{aligned}2 &> -4 \\2 - (-3) &> -4 - (-3) \\5 &> -1\end{aligned}$$

Problem Set Sample Solutions

1. For each problem, use the properties of inequalities to write a true inequality statement. The two integers are -2 and -5 .

- a. Write a true inequality statement.

$$-5 < -2$$

- b. Subtract -2 from each side of the inequality. Write a true inequality statement.

$$-7 < -4$$

- c. Multiply each number by -3 . Write a true inequality statement.

$$15 > 6$$

2. On a recent vacation to the Caribbean, Kay and Tony wanted to explore the ocean elements. One day they went in a submarine 150 feet below sea level. The second day they went scuba diving 75 feet below sea level.

- a. Write an inequality comparing the submarine's elevation and the scuba diving elevation.

$$-150 < -75$$

- b. If they only were able to go one-fifth of the capable elevations, write a new inequality to show the elevations they actually achieved.

$$-30 < -15$$

- c. Was the inequality symbol preserved or reversed? Explain.

The inequality symbol was preserved because the number that was multiplied to both sides was NOT negative.

3. If a is a negative integer, then which of the number sentences below is true? If the number sentence is not true, give a reason.

a. $5 + a < 5$

True.

b. $5 + a > 5$

False because adding a negative number to 5 will decrease 5, which will not be greater than 5.

c. $5 - a > 5$

True.

d. $5 - a < 5$

False because subtracting a negative number is adding a number to 5, which will be larger than 5.

e. $5a < 5$

True.

f. $5a > 5$

False because a negative number multiplied by a positive number is negative, which will be less than 5.

g. $5 + a > a$

True.

h. $5 + a < a$

False because adding 5 to a negative number is greater than the negative number itself.

i. $5 - a > a$

True.

j. $5 - a < a$

False because subtracting a negative number is the same as adding the number, which is greater than the negative number itself.

k. $5a > a$

False because a negative number multiplied by a 5 is negative and will be 5 times smaller than a.

l. $5a < a$

True.

Number Correct: _____

Equations—Round 1

Directions: Write the solution for each equation as quickly and accurately as possible within the allotted time.

1.	$x + 1 = 5$	
2.	$x + 2 = 5$	
3.	$x + 3 = 5$	
4.	$x + 4 = 5$	
5.	$x + 5 = 5$	
6.	$x + 6 = 5$	
7.	$x + 7 = 5$	
8.	$x - 5 = 2$	
9.	$x - 5 = 4$	
10.	$x - 5 = 6$	
11.	$x - 5 = 8$	
12.	$x - 5 = 10$	
13.	$3x = 15$	
14.	$3x = 12$	
15.	$3x = 6$	
16.	$3x = 0$	
17.	$3x = -3$	
18.	$-9x = 18$	
19.	$-6x = 18$	
20.	$-3x = 18$	
21.	$-1x = 18$	
22.	$3x = -18$	

23.	$\frac{1}{7}x = 5$	
24.	$\frac{2}{7}x = 10$	
25.	$\frac{3}{7}x = 15$	
26.	$\frac{4}{7}x = 20$	
27.	$-\frac{5}{7}x = -25$	
28.	$2x + 4 = 12$	
29.	$2x + 5 = 13$	
30.	$2x + 6 = 14$	
31.	$3x + 6 = 18$	
32.	$4x + 6 = 22$	
33.	$-x - 3 = -10$	
34.	$-x - 3 = -8$	
35.	$-x - 3 = -6$	
36.	$-x - 3 = -4$	
37.	$-x - 3 = -2$	
38.	$-x - 3 = 0$	
39.	$2(x + 3) = 4$	
40.	$3(x + 3) = 6$	
41.	$5(x + 3) = 10$	
42.	$5(x - 3) = 10$	
43.	$-2(x - 3) = 8$	
44.	$-3(x + 4) = 3$	

Equations—Round 1 [KEY]

Directions: Write the solution for each equation as quickly and accurately as possible within the allotted time.

1.	$x + 1 = 5$	4
2.	$x + 2 = 5$	3
3.	$x + 3 = 5$	2
4.	$x + 4 = 5$	1
5.	$x + 5 = 5$	0
6.	$x + 6 = 5$	-1
7.	$x + 7 = 5$	-2
8.	$x - 5 = 2$	7
9.	$x - 5 = 4$	9
10.	$x - 5 = 6$	11
11.	$x - 5 = 8$	13
12.	$x - 5 = 10$	15
13.	$3x = 15$	5
14.	$3x = 12$	4
15.	$3x = 6$	2
16.	$3x = 0$	0
17.	$3x = -3$	-1
18.	$-9x = 18$	-2
19.	$-6x = 18$	-3
20.	$-3x = 18$	-6
21.	$-1x = 18$	-18
22.	$3x = -18$	-6

23.	$\frac{1}{7}x = 5$	35
24.	$\frac{2}{7}x = 10$	35
25.	$\frac{3}{7}x = 15$	35
26.	$\frac{4}{7}x = 20$	35
27.	$-\frac{5}{7}x = -25$	35
28.	$2x + 4 = 12$	4
29.	$2x + 5 = 13$	4
30.	$2x + 6 = 14$	4
31.	$3x + 6 = 18$	4
32.	$4x + 6 = 22$	4
33.	$-x - 3 = -10$	7
34.	$-x - 3 = -8$	5
35.	$-x - 3 = -6$	3
36.	$-x - 3 = -4$	1
37.	$-x - 3 = -2$	-1
38.	$-x - 3 = 0$	-3
39.	$2(x + 3) = 4$	-1
40.	$3(x + 3) = 6$	-1
41.	$5(x + 3) = 10$	-1
42.	$5(x - 3) = 10$	5
43.	$-2(x - 3) = 8$	-1
44.	$-3(x + 4) = 3$	-5



Number Correct: _____

Improvement: _____

Equations—Round 2

Directions: Write the solution for each equation as quickly and accurately as possible within the allotted time.

1.	$x + 7 = 9$	
2.	$x + 6 = 9$	
3.	$x + 5 = 9$	
4.	$x + 4 = 9$	
5.	$x + 3 = 9$	
6.	$x + 2 = 9$	
7.	$x + 1 = 9$	
8.	$x - 8 = 2$	
9.	$x - 8 = 4$	
10.	$x - 8 = 6$	
11.	$x - 8 = 8$	
12.	$x - 10 = 10$	
13.	$4x = 12$	
14.	$4x = 8$	
15.	$4x = 4$	
16.	$4x = 0$	
17.	$4x = -4$	
18.	$-8x = 24$	
19.	$-6x = 24$	
20.	$-3x = 24$	
21.	$-2x = 24$	
22.	$6x = -24$	

23.	$\frac{1}{5}x = 10$	
24.	$\frac{2}{5}x = 20$	
25.	$\frac{3}{5}x = 30$	
26.	$\frac{4}{5}x = 40$	
27.	$\frac{5}{5}x = 50$	
28.	$3x + 2 = 14$	
29.	$3x + 3 = 15$	
30.	$3x + 4 = 16$	
31.	$2x + 4 = 12$	
32.	$x + 4 = 8$	
33.	$-2x - 1 = 0$	
34.	$-2x - 1 = 2$	
35.	$-2x - 1 = 4$	
36.	$-2x - 1 = 6$	
37.	$-2x - 1 = 7$	
38.	$-2x - 1 = 8$	
39.	$3(x + 2) = 9$	
40.	$4(x + 2) = 12$	
41.	$5(x + 2) = 15$	
42.	$5(x - 2) = -5$	
43.	$-3(2x - 1) = -9$	
44.	$-5(4x + 1) = 15$	



Equations—Round 2 [KEY]

Directions: Write the solution for each equation as quickly and accurately as possible within the allotted time.

1.	$x + 7 = 9$	2
2.	$x + 6 = 9$	3
3.	$x + 5 = 9$	4
4.	$x + 4 = 9$	5
5.	$x + 3 = 9$	6
6.	$x + 2 = 9$	7
7.	$x + 1 = 9$	8
8.	$x - 8 = 2$	10
9.	$x - 8 = 4$	12
10.	$x - 8 = 6$	14
11.	$x - 8 = 8$	16
12.	$x - 10 = 10$	20
13.	$4x = 12$	3
14.	$4x = 8$	2
15.	$4x = 4$	1
16.	$4x = 0$	0
17.	$4x = -4$	-1
18.	$-8x = 24$	-3
19.	$-6x = 24$	-4
20.	$-3x = 24$	-8
21.	$-2x = 24$	-12
22.	$6x = -24$	-4

23.	$\frac{1}{5}x = 10$	50
24.	$\frac{2}{5}x = 20$	50
25.	$\frac{3}{5}x = 30$	50
26.	$\frac{4}{5}x = 40$	50
27.	$\frac{5}{5}x = 50$	50
28.	$3x + 2 = 14$	4
29.	$3x + 3 = 15$	4
30.	$3x + 4 = 16$	4
31.	$2x + 4 = 12$	4
32.	$x + 4 = 8$	4
33.	$-2x - 1 = 0$	$-\frac{1}{2}$
34.	$-2x - 1 = 2$	$-\frac{3}{2}$
35.	$-2x - 1 = 4$	$-\frac{5}{2}$
36.	$-2x - 1 = 6$	$-\frac{7}{2}$
37.	$-2x - 1 = 7$	-4
38.	$-2x - 1 = 8$	$-\frac{9}{2}$
39.	$3(x + 2) = 9$	1
40.	$4(x + 2) = 12$	1
41.	$5(x + 2) = 15$	1
42.	$5(x - 2) = -5$	1
43.	$-3(2x - 1) = -9$	2
44.	$-5(4x + 1) = 15$	-1

Die Templates

