Lesson 11: Angle Problems and Solving Equations

Student Outcomes

- Students use facts about supplementary, complementary, vertical, and adjacent angles in a multistep problem to write and solve simple equations for an unknown angle in a figure.

Lesson Notes

Lesson 11 continues where Lesson 10 ended and incorporates slightly more difficult problems. At the heart of each problem is the need to model the angle relationships in an equation, and then solve for the unknown angle. The diagrams are all drawn to scale; students should verify their answers by using a protractor to measure relevant angles.

Classwork

Opening Exercise (8 minutes)

Students describe the angle relationship in the diagram and set up and solve an equation that models it. Have students verify their answers by measuring the unknown angle with a protractor.

Note to Teacher:

You may choose or offer a choice of difficulty to students in the Opening Exercise.
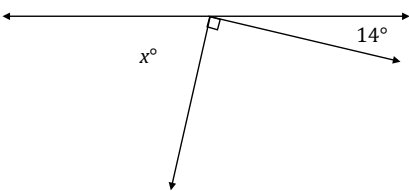
Opening Exercise

a. In a complete sentence, describe the angle relationship in the diagram. Write an equation for the angle relationship shown in the figure and solve for x. Confirm your answers by measuring the angle with a protractor.

The angles marked by $x°$, $90°$, and $14°$ are angles on a line and have a sum of $180°$.

$$x + 90 + 14 = 180$$
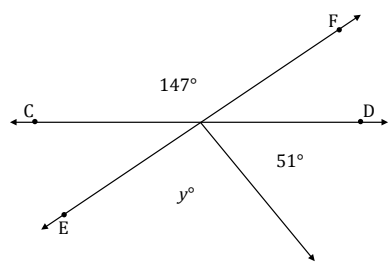$$x + 104 = 180$$
$$x + 104 - 104 = 180 - 104$$
$$x = 76$$

b. CD and EF are intersecting lines. In a complete sentence, describe the angle relationship in the diagram. Write an equation for the angle relationship shown in the figure and solve for y. Confirm your answers by measuring the angle with a protractor.

The adjacent angles marked by $y°$ and $51°$ together form the angle that is vertically opposite and equal to the angle measuring $147°$.

$$y + 51 = 147$$
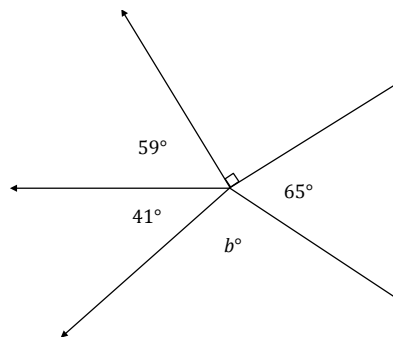$$y + 51 - 51 = 147 - 51$$
$$y = 96$$

c. In a complete sentence, describe the angle relationship in the diagram. Write an equation for the angle relationship shown in the figure and solve for b. Confirm your answers by measuring the angle with a protractor.

The adjacent angles marked by 59°, 41°, b°, 65°, and 90° are angles at a point and together have a sum of 360°.

$$59 + 41 + b + 65 + 90 = 360$$
$$b + 255 = 360 - 255$$
$$b = 105$$

d. The following figure shows three lines intersecting at a point. In a complete sentence, describe the angle relationship in the diagram. Write an equation for the angle relationship shown in the figure and solve for z. Confirm your answers by measuring the angle with a protractor.

The angles marked by z°, 158°, and z° are angles on a line and have a sum of 180°.

$$z + 158 + z = 180$$
$$2z + 158 = 180$$
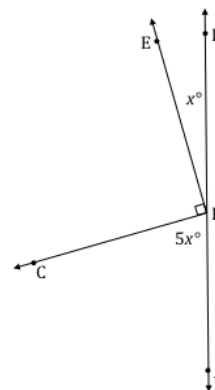$$2z + 158 - 158 = 180 - 158$$
$$2z = 22$$
$$z = 11$$
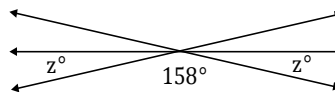
e. Write an equation for the angle relationship shown in the figure and solve for x. In a complete sentence, describe the angle relationship in the diagram. Find the measurements of $\angle EPB$ and $\angle CPA$. Confirm your answers by measuring the angle with a protractor.

$\angle CPA$, $\angle CPE$, and $\angle EPB$ are angles on a line and have a sum of 180°.

$$5x + 90 + x = 180$$
$$6x + 90 = 180$$
$$6x + 90 - 90 = 180 - 90$$
$$6x = 90$$
$$\left(\frac{1}{6}\right) 6x = \left(\frac{1}{6}\right) 90$$
$$x = 15$$
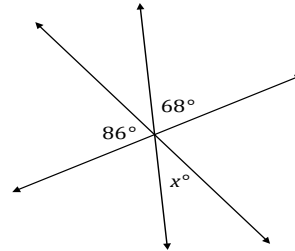
$$\angle EPB = 15°$$
$$\angle CPA = 5(15°) = 75°$$

Example 1 (4 minutes)

Example 1

The following figure shows three lines intersecting at a point. In a complete sentence, describe the angle relationship in the diagram. Write an equation for the angle relationship shown in the figure and solve for x. Confirm your answers by measuring the angle with a protractor.

The angles $86°$, $68°$, and the angle between them, which is vertically opposite and equal in measure to x, are angles on a line and have a sum of $180°$.

$$86 + x + 68 = 180$$
$$x + 154 = 180$$
$$x + 154 - 154 = 180 - 154$$
$$x = 26$$

Exercise 1 (5 minutes)

Exercise 1

The following figure shows four lines intersecting at a point. In a complete sentence, describe the angle relationships in the diagram. Write an equation for the angle relationship shown in the figure and solve for x and y. Confirm your answers by measuring the angle with a protractor.

The angles $x°$, $25°$, $y°$, and $40°$ are angles on a line and have a sum of $180°$; the angle marked $y°$ is vertically opposite and equal to $96°$.

$y = 96°$, vert. $\angle s$

$$x + 25 + (96) + 40 = 180$$
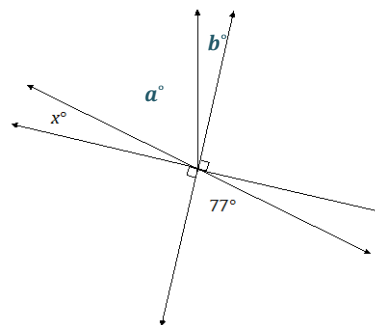$$x + 161 = 180$$
$$x + 161 - 161 = 180 - 161$$
$$x = 19$$
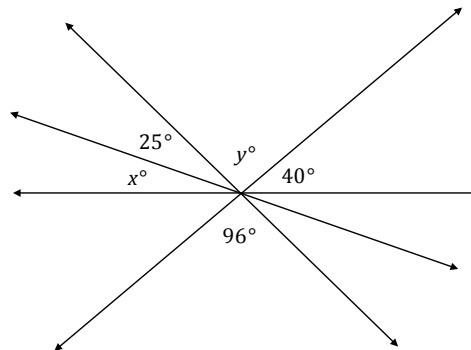
Example 2 (4 minutes)

Example 2

In a complete sentence, describe the angle relationships in the diagram. You may label the diagram to help describe the angle relationships. Write an equation for the angle relationship shown in the figure and solve for x. Confirm your answers by measuring the angle with a protractor.

The angle formed by adjacent angles $a°$ and $b°$ is vertically opposite to the $77°$ angle. The angles $x°$, $a°$, and $b°$ are adjacent angles that have a sum of $90°$ (since the adjacent angle is a right angle and together the angles are on a line).

$$x + 77 = 90$$
$$x + 77 - 77 = 90 - 77$$
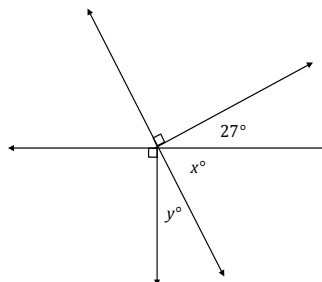$$x = 13$$

Exercise 2 (4 minutes)

Exercise 2

In a complete sentence, describe the angle relationships in the diagram. Write an equation for the angle relationship shown in the figure and solve for x and y. Confirm your answers by measuring the angle with a protractor.

Angles $x°$ and $y°$ have a sum of $90°$; angles $x°$ and $27°$ have a sum of $90°$.

$$x + 27 = 90$$
$$x + 27 - 27 = 90 - 27$$
$$x = 63$$

$$(63) + y = 90$$
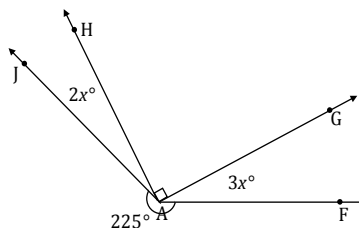$$63 - 63 + y = 90 - 63$$
$$y = 27$$

Example 3 (5 minutes)

Example 3

In a complete sentence, describe the angle relationships in the diagram. Write an equation for the angle relationship shown in the figure and solve for x. Find the measures of $\angle JAH$ and $\angle GAF$. Confirm your answers by measuring the angle with a protractor.

The sum of the degree measurements of $\angle JAH$, $\angle GAH$, $\angle GAF$, and the arc that subtends $\angle JAF$ is $360°$.

$$225 + 2x + 90 + 3x = 360$$
$$315 + 5x = 360$$
$$315 - 315 + 5x = 360 - 315$$
$$5x = 45$$
$$\left(\frac{1}{5}\right) 5x = \left(\frac{1}{5}\right) 45$$
$$x = 9$$

$$m\angle JAH = 2(9°) = 18° \qquad m\angle GAF = 3(9°) = 27°$$

Exercise 3 (4 minutes)

Exercise 3

In a complete sentence, describe the angle relationships in the diagram. Write an equation for the angle relationship shown in the figure and solve for x. Find the measure of $\angle JKG$. Confirm your answers by measuring the angle with a protractor.

The sum of the degree measurements of $\angle LKJ$, $\angle JKG$, $\angle GKM$, and the arc that subtends $\angle LKM$ is $360°$.

$$5x + 24 + x + 90 = 360$$
$$6x + 114 = 360$$
$$6x + 114 - 114 = 360 - 114$$
$$6x = 246$$
$$\left(\frac{1}{6}\right) 6x = \left(\frac{1}{6}\right) 246$$
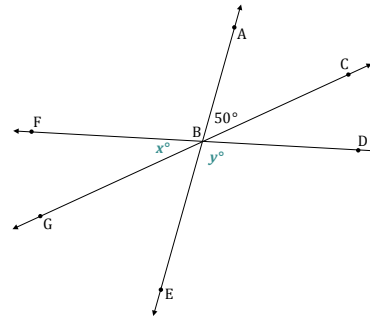$$x = 41$$

$$m\angle JKG = (41) = 41°$$
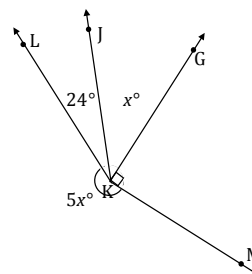
Example 4 (5 minutes)

Example 4

In the accompanying diagram, $\angle DBE$ is four times the measure of $\angle FBG$.

a.   Label $\angle DBE$ as $y°$ and $\angle FBG$ as $x°$. Write an equation that describes the relationship between $\angle DBE$ and $\angle FBG$.

$$y = 4x$$

b.   Find the value of x.

$$50 + x + 4x = 180$$
$$50 + 5x = 180$$
$$5x + 50 - 50 = 180 - 50$$
$$5x = 130$$
$$\left(\frac{1}{5}\right)(5x) = \left(\frac{1}{5}\right)(130)$$
$$x = 26$$

c.   Find the measures of $\angle FBG$, $\angle CBD$, $\angle ABF$, $\angle GBE$, and $\angle DBE$.

$m\angle FBG = 26°$

$m\angle CBD = 26°$

$m\angle ABF = 4(26) = 104°$

$m\angle GBE = 50°$

$m\angle DBE = 104°$

d.   What is the measure of $\angle ABG$? Identify the angle relationship used to get your answer.

$$\angle ABG = \angle ABF + \angle FBG$$
$$\angle ABG = 104 + 26$$
$$\angle ABG = 130$$

$m\angle ABG = 130°$

To determine the measure of $\angle ABG$, you need to add the measures of adjacent angles $\angle ABF$ and $\angle FBG$.
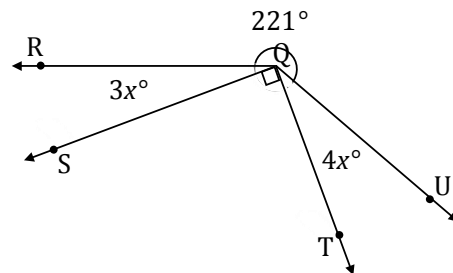
Exit Ticket (6 minutes)

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Lesson 11: Angle Problems and Solving Equations

Exit Ticket

Write an equation for the angle relationship shown in the figure and solve for x. Find the measures of $\angle RQS$ and $\angle TQU$.
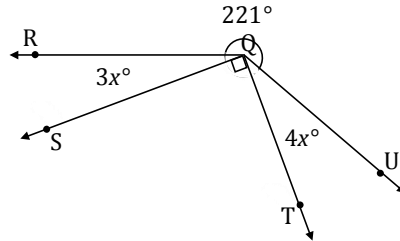
Exit Ticket Sample Solutions

Write an equation for the angle relationship shown in the figure and solve for x. Find the measures of $\angle RQS$ and $\angle TQU$.

$$3x + 90 + 4x + 221 = 360$$
$$7x + 311 = 360$$
$$7x + 311 - 311 = 360 - 311$$
$$7x = 49$$
$$\left(\frac{1}{7}\right)7x = \left(\frac{1}{7}\right)49$$
$$x = 7$$

$m\angle RQS = 3(7°) = 21°$

$m\angle TQU = 4(7°) = 28°$

Problem Set Sample Solutions

In a complete sentence, describe the angle relationships in each diagram. Write an equation for the angle relationship(s) shown in the figure, and solve for the indicated unknown angle. You can check your answers by measuring each angle with a protractor.

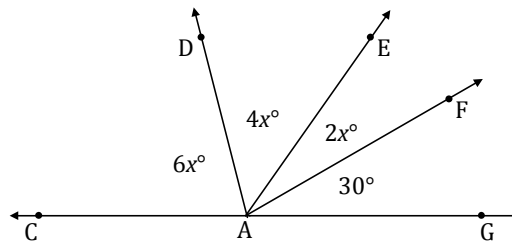1. Find the measures of $\angle EAF$, $\angle DAE$, and $\angle CAD$.

    $\angle GAF$, $\angle EAF$, $\angle DAE$, and $\angle CAD$ are angles on a line and have a sum of $180°$.

    $$6x + 4x + 2x + 30 = 180$$
    $$12x + 30 = 180$$
    $$12x + 30 - 30 = 180 - 30$$
    $$12x = 150$$
    $$x = 12.5$$

    $m\angle EAF = 2(12.5°) = 25°$

    $m\angle DAE = 4(12.5°) = 50°$

    $m\angle CAD = 6(12.5°) = 75°$

2. Find the measure of a.

    Angles $a°$, $26°$, $a°$, and $126°$ are angles at a point and have a sum of $360°$.

    $$a + 126 + a + 26 = 360$$
    $$2a + 152 = 360$$
    $$2a + 152 - 152 = 360 - 152$$
    $$2a = 208$$
    $$\left(\frac{1}{2}\right)2a = \left(\frac{1}{2}\right)208$$
    $$a = 104$$
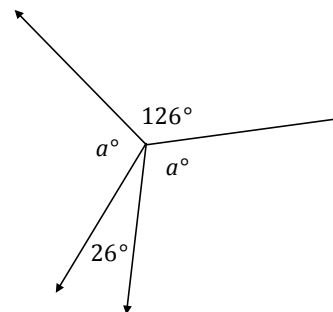
3. Find the measures of x and y.

Angles $y°$ and $65°$ and angles $25°$ and $x°$ have a sum of $90°$.

$$x + 25 = 90$$
$$x + 25 - 25 = 90 - 25$$
$$x = 65$$

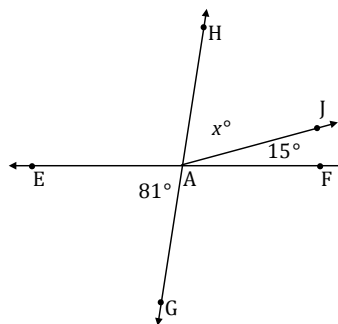$$65 + y = 90$$
$$65 + y = 90$$
$$65 - 65 + y = 90 - 65$$
$$y = 25$$
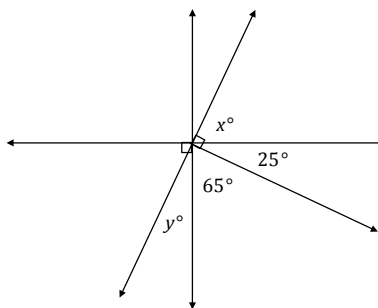
4. Find the measure of $\angle HAJ$.

Adjacent angles $x°$ and $15°$ together are vertically opposite from and are equal to angle $81°$.

$$x + 15 = 81$$
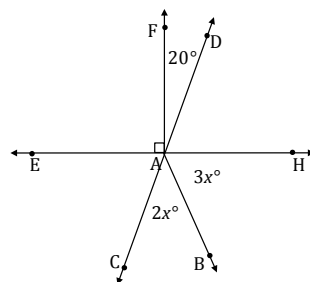$$x + 15 - 15 = 81 - 15$$
$$x = 66$$

$$m\angle HAJ = 66°$$

5. Find the measures of $\angle HAB$ and $\angle CAB$.

Adjacent angles $\angle CAB$ and $\angle HAB$ have a sum of the measurement of $\angle CAH$, which is vertically opposite from and equal to the measurement of $\angle DAE$.

$$2x + 3x + 70 = 180$$
$$5x = 110$$
$$\left(\frac{1}{5}\right) 5x = \left(\frac{1}{5}\right) 110$$
$$x = 22$$

$$m\angle HAB = 3(22°) = 66°$$
$$m\angle CAB = 2(22°) = 44°$$

6. The measure of $\angle SPT = b°$. The measure of $\angle TPR$ is five more than two times $\angle SPT$. The measure of $\angle QPS$ is twelve less than eight times $\angle SPT$. Find the measures of $\angle SPT$, $\angle TPR$, and $\angle QPS$.
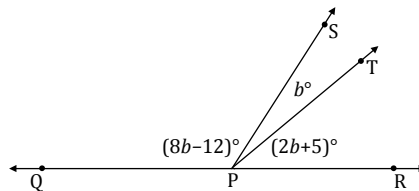
$\angle QPS$, $\angle SPT$, and $\angle TPR$ are angles on a line and have a sum of $180°$.

$$(8b - 12) + b + (2b + 5) = 180$$
$$11b - 7 = 180$$
$$11b - 7 + 7 = 180 + 7$$
$$11b = 187$$
$$\left(\frac{1}{11}\right) 11b = \left(\frac{1}{11}\right) 187$$
$$b = 17$$

$$m\angle SPT = (17°) = 17°$$
$$m\angle TPR = 2(17°) + 5° = 39°$$
$$m\angle QPS = 8(17°) - 12° = 124°$$

7. **Find the measures of ∠HQE and ∠AQG.**

∠AQG, ∠AQH, and ∠HQE are adjacent angles that have a sum of 90°.

$$2y + 21 + y = 90$$
$$3y + 21 = 90$$
$$3y + 21 - 21 = 90 - 21$$
$$3y = 69$$
$$\left(\frac{1}{3}\right)3y = \left(\frac{1}{3}\right)69$$
$$y = 23$$

$m\angle HQE = 2(23°) = 46°$

$m\angle AQG = (23°) = 23°$
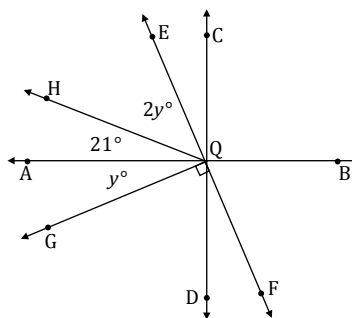
8. **The measures of three angles at a point are in the ratio of 2 : 3 : 5. Find the measures of the angles.**

Angle A = 2x, Angle B = 3x, Angle C = 5x

$$2x + 3x + 5x = 360$$
$$10x = 360$$
$$\left(\frac{1}{10}\right)10x = \left(\frac{1}{10}\right)360$$
$$x = 36$$

Angle A = 2(36°) = 72°

Angle B = 3(36°) = 108°

Angle C = 5(36°) = 180°

9. **The sum of the measures of two adjacent angles is 72°. The ratio of the smaller angle to the larger angle is 1 : 3. Find the measures of each angle.**

Angle A = x, Angle B = 3x

$$x + 3x = 72$$
$$4x = 72$$
$$\left(\frac{1}{4}\right)(4x) = \left(\frac{1}{4}\right)(72)$$
$$x = 18$$

Angle A = (18) = 18°

Angle B = 3(18) = 54°

10. **Find the measures of ∠CQA and ∠EQB.**

$$4x + 5x = 108$$
$$9x = 108$$
$$\left(\frac{1}{9}\right)9x = \left(\frac{1}{9}\right)108$$
$$x = 12$$

$m\angle CQA = 5(12) = 60°$

$m\angle EQB = 4(12) = 48°$