



Lesson 10: Angle Problems and Solving Equations

Student Outcomes

- Students use vertical angles, adjacent angles, angles on a line, and angles at a point in a multistep problem to write and solve simple equations for an unknown angle in a figure.

Lesson Notes

In Lessons 10 and 11, students apply their understanding of equations to unknown angle problems. The geometry topic is a natural context within which they apply algebraic skills. Students understand that the unknown angle is an actual, measureable angle; they simply need to find the value that makes each equation true. They set up the equations based on the angle facts they learned in Grade 4. The problems presented are not as simple as in Grade 4 because diagrams incorporate angle facts in combination, rather than in isolation. Encourage students to verify their answers by measuring relevant angles in each diagram—all diagrams are drawn to scale.

Classwork

Opening (5 minutes)

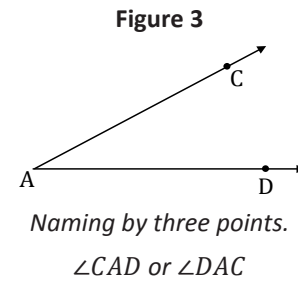
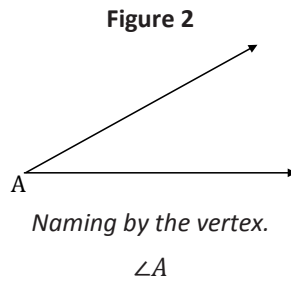
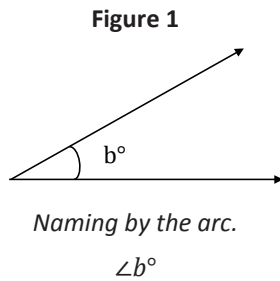
Discuss the ways in which angles are named and notated.

- What do you notice about the three figures below? What is the same about all three figures; what is different?
 - There are three angles that appear to be the same measurement but are notated differently.*
- What is a likely implication of the three different kinds of notation?
 - They indicate the different ways of labeling or identifying the angle.*

Students are familiar with addressing Figure 1 as b and having a measurement of b° and addressing Figure 2 as angle A . Elicit this from students and say that in a case like Figure 1, the angle is named by the measure of the arc, and in a case like Figure 2, the angle is named by the single letter.

- In a case like Figure 3, we use three letters when we name the angle. Why use three points to name an angle?
 - In a figure where several angles share the same vertex, naming a particular angle by the vertex point is not sufficient information to distinguish that angle. Two additional points, one belonging to each side of the intended angle, are necessary to identify it.*

Encourage students to use both multiple forms of angle notation in the table to demonstrate each angle relationship.



Recall the definitions of adjacent and vertical and the facts regarding angles on a line and angles at a point. If an abbreviation exists, students should include the abbreviation of the angle fact under the name of each relationship. In the Angle Fact column, students write the definitions and practice the different angle notations when describing the relationship in the angle fact.

Note: The *angles on a line* fact applies to two or more angles.

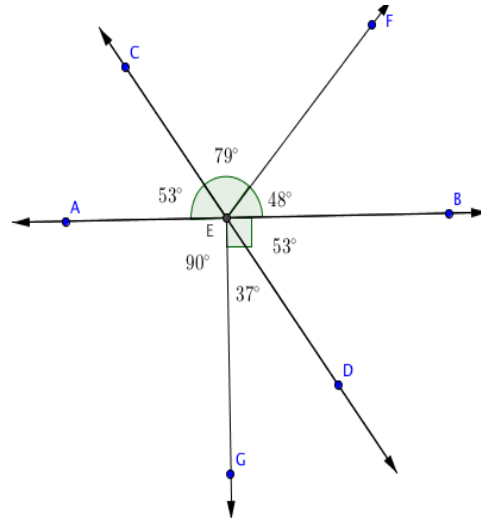
Angle Facts and Definitions		
Name of Angle Relationship	Angle Fact	Diagram
Adjacent Angles	<p>Two angles, $\angle BAC$ and $\angle CAD$ with a common side \overrightarrow{AC}, are adjacent angles if C belongs to the interior of $\angle BAD$.</p> <p>Angles a and b are adjacent angles; $\angle BAC$ and $\angle CAD$ are adjacent angles.</p>	
Vertical Angles (vert. \angle s)	<p>Two angles are vertical angles (or vertically opposite angles) if their sides form two pairs of opposite rays.</p> <p>$a = b$</p> <p>$m\angle DCF = m\angle GCE$</p>	
Angles on a Line (\angle s on a line)	<p>The sum of the measures of two angles that share a ray and form a line is 180°.</p> <p>$a + b = 180$</p> <p>$m\angle ABC + m\angle CBD = 180^\circ$</p>	
Angles at a Point (\angle s at a point)	<p>The measure of all angles formed by three or more rays with the same vertex is 360°.</p> <p>$a + b + c = 360$</p> <p>$m\angle BAC + m\angle CAD + m\angle DAB = 360^\circ$</p>	

Opening Exercise (4 minutes)

Opening Exercise

Use the diagram to complete the chart.

Name the angles that are ...	
Vertical	$\angle AEC$ and $\angle BED$, $\angle CEB$ and $\angle DEA$
Adjacent	Answers include: $\angle AEC$ and $\angle CEF$ $\angle CEF$ and $\angle FEB$
Angles on a line	Answers include: $\angle BED$, $\angle DEG$, and $\angle GEA$ $\angle AEC$, $\angle CEF$, and $\angle FEB$
Angles at a point	$\angle AEC$, $\angle CEF$, $\angle FEB$, $\angle BED$, $\angle DEG$, $\angle GEA$



Example 1 (4 minutes)

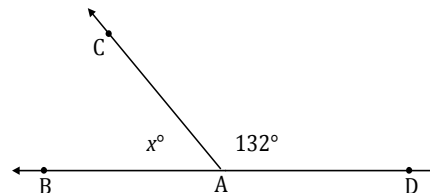
Students describe the angle relationship in the diagram and set up and solve an equation that models it. Have students verify their answers by measuring the unknown angle with a protractor.

Example 1

Estimate the measurement of x . _____

In a complete sentence, describe the angle relationship in the diagram.

$\angle BAC$ and $\angle CAD$ are angles on a line and have a sum of 180° .



Write an equation for the angle relationship shown in the figure and solve for x . Then, find the measures of $\angle BAC$ and confirm your answers by measuring the angle with a protractor.

$$\begin{aligned} x + 132 &= 180 \\ x + 132 - 132 &= 180 - 132 \\ x &= 48 \end{aligned}$$

$$m\angle BAC = 48^\circ$$

Exercise 1 (4 minutes)

Students describe the angle relationship in the diagram and set up and solve an equation that models it. Have students verify their answers by measuring the unknown angle with a protractor.

Exercise 1

In a complete sentence, describe the angle relationship in the diagram.

$\angle BAC$, $\angle CAD$, and $\angle DAE$ are angles on a line and have a sum of 180° .

Find the measurements of $\angle BAC$ and $\angle DAE$.

$$3x + 90 + 2x = 180$$

$$5x + 90 = 180$$

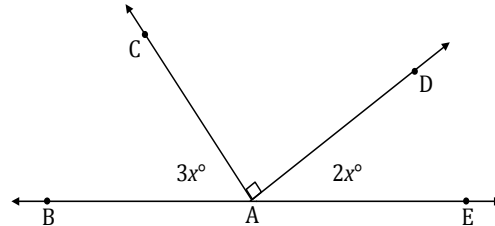
$$5x + 90 - 90 = 180 - 90$$

$$\left(\frac{1}{5}\right)(5x) = \left(\frac{1}{5}\right)(90)$$

$$x = 18$$

$$m\angle BAC = 3(18^\circ) = 54^\circ$$

$$m\angle DAE = 2(18^\circ) = 36^\circ$$

**Example 2 (4 minutes)**

Students describe the angle relationship in the diagram and set up and solve an equation that models it. Have students verify their answers by measuring the unknown angle with a protractor.

Example 2

In a complete sentence, describe the angle relationship in the diagram.

$\angle AEL$ and $\angle LEB$ are angles on a line and have a sum of 180° . $\angle AEL$ and $\angle KEB$ are vertical angles and are of equal measurement.

Write an equation for the angle relationship shown in the figure and solve for x and y . Find the measurements of $\angle LEB$ and $\angle KEB$.

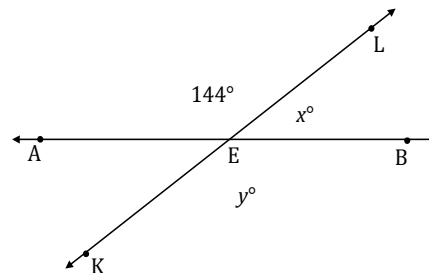
$$y = 144^\circ; m\angle KEB = 144^\circ \text{ (or vert. } \angle\text{s are =)}$$

$$x + 144 = 180$$

$$x + 144 - 144 = 180 - 144$$

$$x = 36$$

$$m\angle LEB = 36^\circ$$



Exercise 2 (4 minutes)

Students describe the angle relationship in the diagram and set up and solve an equation that models it. Have students verify their answers by measuring the unknown angle with a protractor.

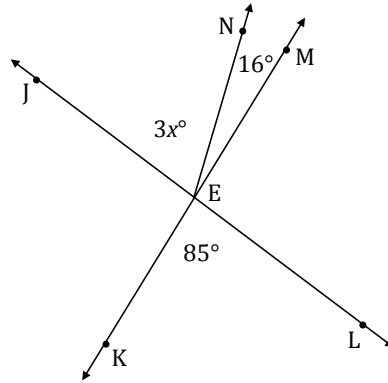
Exercise 2

In a complete sentence, describe the angle relationships in the diagram.

$\angle JEN$ and $\angle NEM$ are adjacent angles and, when added together, are the measure of $\angle JEM$; $\angle JEM$ and $\angle KEL$ are vertical angles and are of equal measurement.

Write an equation for the angle relationship shown in the figure and solve for x .

$$\begin{aligned} 3x + 16 &= 85 \\ 3x + 16 - 16 &= 85 - 16 \\ 3x &= 69 \\ \left(\frac{1}{3}\right) 3x &= 69 \left(\frac{1}{3}\right) \\ x &= 23 \end{aligned}$$

**Example 3 (4 minutes)**

Students describe the angle relationship in the diagram and set up and solve an equation that models it. Have students verify their answers by measuring the unknown angle with a protractor.

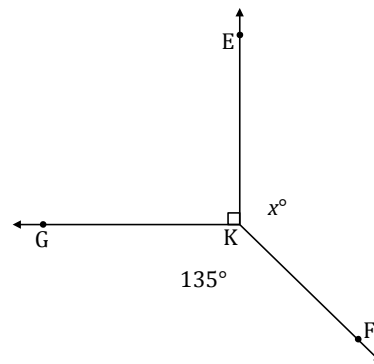
Example 3

In a complete sentence, describe the angle relationships in the diagram.

$\angle GKE$, $\angle EKF$, and $\angle GKF$ are angles at a point and have a sum of 360° .

Write an equation for the angle relationship shown in the figure and solve for x . Find the measurement of $\angle EKF$ and confirm your answers by measuring the angle with a protractor.

$$\begin{aligned} x + 90 + 135 &= 360 \\ x + 225 &= 360 \\ x + 225 - 225 &= 360 - 225 \\ x &= 135 \\ m\angle EKF &= 135^\circ \end{aligned}$$



Exercise 3 (4 minutes)

Students describe the angle relationship in the diagram and set up and solve an equation that models it. Have students verify their answers by measuring the unknown angle with a protractor.

Exercise 3

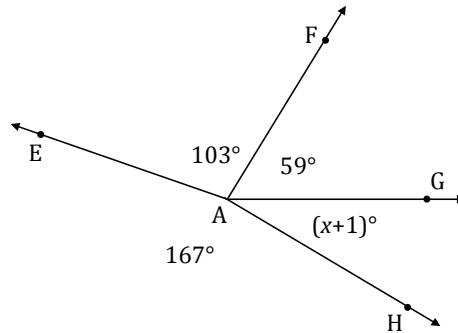
In a complete sentence, describe the angle relationships in the diagram.

$\angle EAG$, $\angle GAH$, $\angle GAF$, and $\angle FAE$ are angles at a point and sum to 360° .

Find the measurement of $\angle GAH$.

$$\begin{aligned}(x + 1) + 59 + 103 + 167 &= 360 \\ x + 1 + 59 + 103 + 167 &= 360 \\ x &= 30\end{aligned}$$

$$m\angle GAH = ((30^\circ) + 1) = 31^\circ$$

**Example 4 (5 minutes)**

MP.8

- List pairs of angles whose measurements are in a ratio of 2 : 1.
 - Examples include: 90° and 45° , 60° and 30° , 150° and 75° .
- What does it mean for the ratio of the measurements of two angles to be 2 : 1?
 - The measurement of one angle is two times the measure of the other angle. If the smaller angle is defined as x° , then the larger angle is $2x^\circ$. If the larger angle is defined as x° , then the smaller angle is $\frac{1}{2}x^\circ$.
- Based on the following figure, which angle relationship(s) can be utilized to find the measure of an obtuse and acute angle?
 - Any adjacent angle pair are on a line and have a sum of 180° .

Scaffolding:

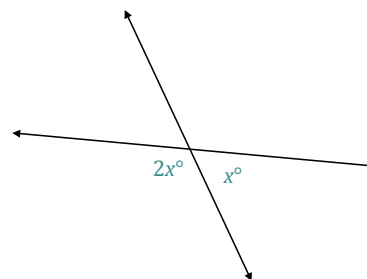
Students may find it helpful to highlight the pairs of equal vertical angles.

Students describe the angle relationship in the question and set up and solve an equation that models it. Have students verify their answers by measuring the unknown angle with a protractor.

Example 4

The following two lines intersect. The ratio of the measurements of the obtuse angle to the acute angle in any adjacent angle pair in this figure is 2 : 1. In a complete sentence, describe the angle relationships in the diagram.

The measurement of an obtuse angle is twice the measurement of an acute angle in the diagram.



Label the diagram with expressions that describe this relationship. Write an equation that models the angle relationship and solve for x . Find the measurements of the acute and obtuse angles.

$$\begin{aligned} 2x + 1x &= 180 \\ 3x &= 180 \\ \left(\frac{1}{3}\right)(3x) &= \left(\frac{1}{3}\right)(180) \\ x &= 60 \end{aligned}$$

Acute angle = 60°

Obtuse angle = $2x = 2(60) = 120^\circ$

Exercise 4 (4 minutes)

Students describe the angle relationship in the diagram and set up and solve an equation that models it. Have students verify their answers by measuring the unknown angle with a protractor.

Exercise 4

The ratio of $\angle GFH$ to $\angle EFH$ is 2 : 3. In a complete sentence, describe the angle relationships in the diagram.

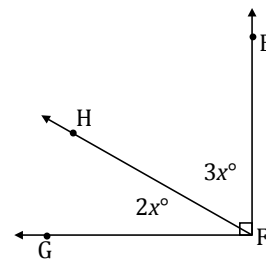
The measurement of $\angle GFH$ is $\frac{2}{3}$ the measurement of $\angle EFH$; $\angle GFH$ and $\angle EFH$ have a sum of 90° .

Find the measures of $\angle GFH$ and $\angle EFH$.

$$\begin{aligned} 2x + 3x &= 90 \\ 5x &= 90 \\ \left(\frac{1}{5}\right)(5x) &= \left(\frac{1}{5}\right)(90) \\ x &= 18 \end{aligned}$$

$m\angle GFH = 2(18^\circ) = 36^\circ$

$m\angle EFH = 3(18^\circ) = 54^\circ$



Relevant Vocabulary

ADJACENT ANGLES: Two angles $\angle BAC$ and $\angle CAD$ with a common side \overrightarrow{AC} are *adjacent angles* if C belongs to the interior of $\angle BAD$.

VERTICAL ANGLES: Two angles are *vertical angles* (or *vertically opposite angles*) if their sides form two pairs of opposite rays.

ANGLES ON A LINE: The sum of the measures of adjacent *angles on a line* is 180° .

ANGLES AT A POINT: The sum of the measures of adjacent *angles at a point* is 360° .

Exit Ticket (3 minutes)

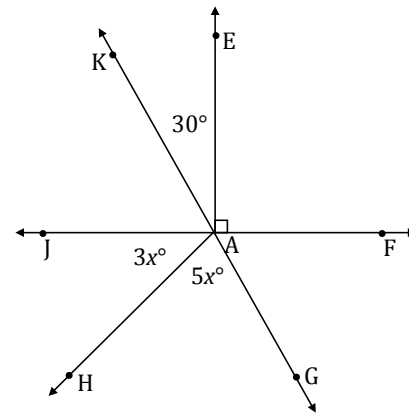
Name _____

Date _____

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Exit Ticket

In a complete sentence, describe the relevant angle relationships in the following diagram. That is, describe the angle relationships you could use to determine the value of x .



Use the angle relationships described above to write an equation to solve for x . Then, determine the measurements of $\angle JAH$ and $\angle HAG$.

Exit Ticket Sample Solutions

In a complete sentence, describe the relevant angle relationships in the following diagram. That is, describe the angle relationships you could use to determine the value of x .

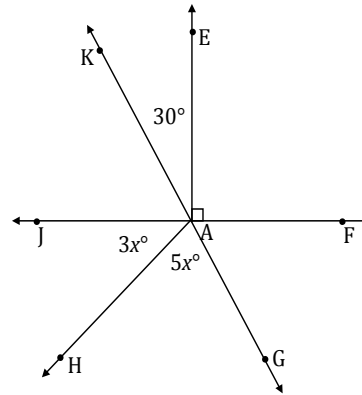
$\angle KAE$ and $\angle EAF$ are adjacent angles whose measurements are equal to $\angle KAF$; $\angle KAF$ and $\angle JAG$ are vertical angles and are of equal measurement.

Use the angle relationships described above to write an equation to solve for x . Then, determine the measurements of $\angle JAH$ and $\angle HAG$.

$$\begin{aligned} 5x + 3x &= 90 + 30 \\ 8x &= 120 \\ \left(\frac{1}{8}\right)(8x) &= \left(\frac{1}{8}\right)(120) \\ x &= 15 \end{aligned}$$

$$m\angle JAH = 3(15^\circ) = 45^\circ$$

$$m\angle HAG = 5(15^\circ) = 75^\circ$$



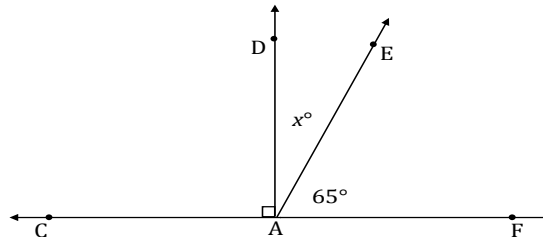
Problem Set Sample Solutions

For each question, use angle relationships to write an equation in order to solve for each variable. Determine the indicated angles. You can check your answers by measuring each angle with a protractor.

1. In a complete sentence, describe the relevant angle relationships in the following diagram. Find the measurement of $\angle DAE$.

One possible response: $\angle CAD$, $\angle DAE$, and $\angle FAE$ are angles on a line and sum to 180° .

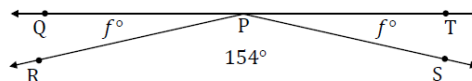
$$\begin{aligned} 90 + x + 65 &= 180 \\ x + 155 &= 180 \\ x + 155 - 155 &= 180 - 155 \\ x &= 25 \\ m\angle DAE &= 25^\circ \end{aligned}$$



2. In a complete sentence, describe the relevant angle relationships in the following diagram. Find the measurement of $\angle QPR$.

$\angle QPR$, $\angle RPS$, and $\angle SPT$ are angles on a line and sum to 180° .

$$\begin{aligned} f + 154 + f &= 180 \\ 2f + 154 &= 180 \\ 2f + 154 - 154 &= 180 - 154 \\ 2f &= 26 \\ \left(\frac{1}{2}\right)2f &= \left(\frac{1}{2}\right)26 \\ f &= 13 \\ m\angle QPR &= 13^\circ \end{aligned}$$



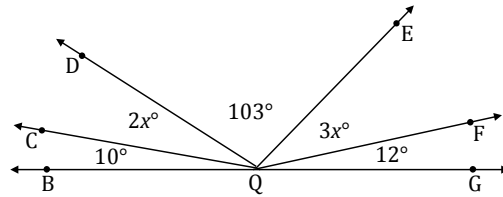
3. In a complete sentence, describe the relevant angle relationships in the following diagram. Find the measurements of $\angle CQD$ and $\angle EQF$.

$\angle BQC$, $\angle CQD$, $\angle DQE$, $\angle EQF$, and $\angle FQG$ are angles on a line and sum to 180° .

$$\begin{aligned} 10 + 2x + 103 + 3x + 12 &= 180 \\ 5x + 125 &= 180 \\ 5x + 125 - 125 &= 180 - 125 \\ 5x &= 55 \\ \left(\frac{1}{5}\right) 5x &= \left(\frac{1}{5}\right) 55 \\ x &= 11 \end{aligned}$$

$$m\angle CQD = 2(11^\circ) = 22^\circ$$

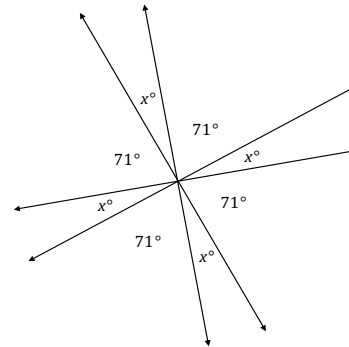
$$m\angle EQF = 3(11^\circ) = 33^\circ$$



4. In a complete sentence, describe the relevant angle relationships in the following diagram. Find the measure of x .

All of the angles in the diagram are angles at a point and sum to 360° .

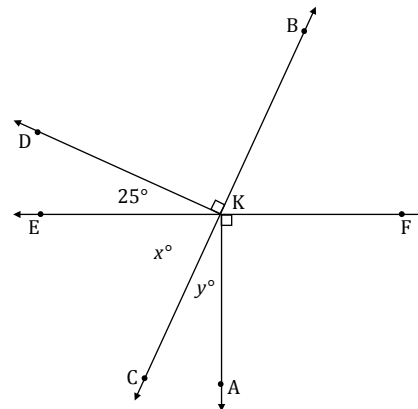
$$\begin{aligned} 4(x + 71) &= 360 \\ 4x + 284 &= 360 \\ 4x + 284 - 284 &= 360 - 284 \\ 4x &= 76 \\ \left(\frac{1}{4}\right) 4x &= \left(\frac{1}{4}\right) 76 \\ x &= 19 \end{aligned}$$



5. In a complete sentence, describe the relevant angle relationships in the following diagram. Find the measures of x and y .

$\angle CKE$, $\angle EKD$, and $\angle DKB$ are angles on a line and sum to 180° . Since $\angle FKA$ and $\angle AKE$ form a straight angle and the measurement of $\angle FKA$ is 90° , $\angle AKE$ is 90° , making $\angle CKE$ and $\angle AKC$ form a right angle and have a sum of 90° .

$$\begin{aligned} x + 25 + 90 &= 180 \\ x + 115 &= 180 \\ x + 115 - 115 &= 180 - 115 \\ x &= 65 \\ (65) + y &= 90 \\ 65 - 65 + y &= 90 - 65 \\ y &= 25 \end{aligned}$$

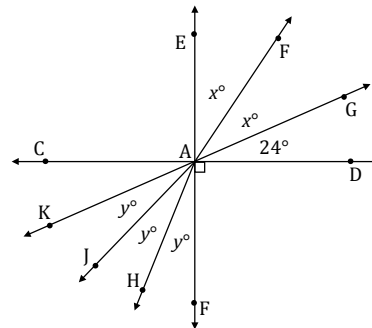


6. In a complete sentence, describe the relevant angle relationships in the following diagram. Find the measures of x and y .

$\angle EAG$ and $\angle FAK$ are vertical angles and are of equal measurement.
 $\angle EAG$ and $\angle GAD$ form a right angle and have a sum of 90° .

$$\begin{aligned} 2x + 24 &= 90 \\ 2x + 24 - 24 &= 90 - 24 \\ 2x &= 66 \\ \left(\frac{1}{2}\right) 2x &= \left(\frac{1}{2}\right) 66 \\ x &= 33 \end{aligned}$$

$$\begin{aligned} 3y &= 66 \\ \left(\frac{1}{3}\right) 3y &= \left(\frac{1}{3}\right) 66 \\ y &= 22 \end{aligned}$$



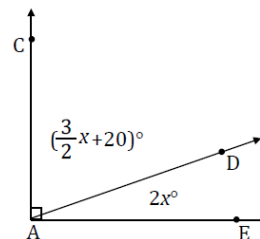
7. In a complete sentence, describe the relevant angle relationships in the following diagram. Find the measures of $\angle CAD$ and $\angle DAE$.

$\angle CAD$ and $\angle DAE$ form a right angle and have a sum of 90° .

$$\begin{aligned} \left(\frac{3}{2}x + 20\right) + 2x &= 90 \\ \frac{7}{2}x + 20 &= 90 \\ \frac{7}{2}x + 20 - 20 &= 90 - 20 \\ \frac{7}{2}x &= 70 \\ \left(\frac{2}{7}\right) \frac{7}{2}x &= 70 \left(\frac{2}{7}\right) \\ x &= 20 \end{aligned}$$

$$m\angle CAD = \frac{3}{2}(20) + 20 = 50^\circ$$

$$m\angle DAE = 2(20) = 40^\circ$$

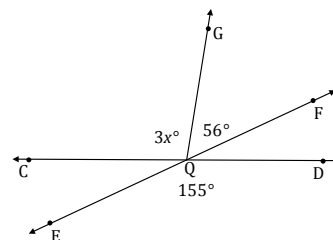


8. In a complete sentence, describe the relevant angle relationships in the following diagram. Find the measure of $\angle CQG$.

$\angle DQE$ and $\angle CQF$ are vertical angles and are of equal measurement.
 $\angle CQG$ and $\angle GQF$ are adjacent and sum to the measurement of $\angle CQF$.

$$\begin{aligned} 3x + 56 &= 155 \\ 3x + 56 - 56 &= 155 - 56 \\ 3x &= 99 \\ \left(\frac{1}{3}\right) 3x &= \left(\frac{1}{3}\right) 99 \\ x &= 33 \end{aligned}$$

$$m\angle CQG = 3(33^\circ) = 99^\circ$$



9. The ratio of the measures of a pair of adjacent angles on a line is 4 : 5.

- a. Find the measures of the two angles.

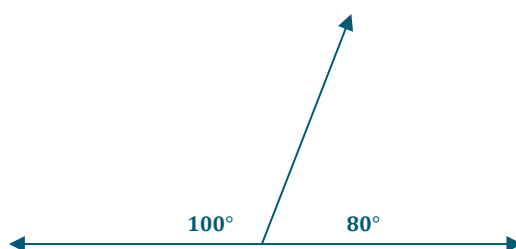
$$\text{Angle 1} = 4x, \text{Angle 2} = 5x$$

$$\begin{aligned} 4x + 5x &= 180 \\ 9x &= 180 \\ \left(\frac{1}{9}\right) 9x &= \left(\frac{1}{9}\right) 180 \\ x &= 20 \end{aligned}$$

$$\text{Angle 1} = 4(20^\circ) = 80^\circ$$

$$\text{Angle 2} = 5(20^\circ) = 100^\circ$$

- b. Draw a diagram to scale of these adjacent angles. Indicate the measurements of each angle.



10. The ratio of the measures of three adjacent angles on a line is 3 : 4 : 5.

- a. Find the measures of the three angles.

$$\text{Angle 1} = 3x, \text{Angle 2} = 4x, \text{Angle 3} = 5x$$

$$\begin{aligned} 3x + 4x + 5x &= 180 \\ 12x &= 180 \\ \left(\frac{1}{12}\right) 12x &= \left(\frac{1}{12}\right) 180 \\ x &= 15 \end{aligned}$$

$$\text{Angle 1} = 3(15^\circ) = 45^\circ$$

$$\text{Angle 2} = 4(15^\circ) = 60^\circ$$

$$\text{Angle 3} = 5(15^\circ) = 75^\circ$$

- b. Draw a diagram to scale of these adjacent angles. Indicate the measurements of each angle.

$$\text{Angle 3} = 5(15^\circ) = 75^\circ$$

- c. Draw a diagram to scale of these adjacent angles. Indicate the measurements of each angle.

