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Lesson 10: Angle Problems and Solving Equations

Student Outcomes

* Students use vertical angles, adjacent angles, angles on a line, and angles at a point in a multistep problem to write and solve simple equations for an unknown angle in a figure.

Lesson Notes

In Lessons 10 and 11, students apply their understanding of equations to unknown angle problems. The geometry topic is a natural context within which they apply algebraic skills. Students understand that the unknown angle is an actual, measureable angle; they simply need to find the value that makes each equation true. They set up the equations based on the angle facts they learned in Grade 4. The problems presented are not as simple as in Grade 4 because diagrams incorporate angle facts in combination, rather than in isolation. Encourage students to verify their answers by measuring relevant angles in each diagram―all diagrams are drawn to scale.

Classwork

Opening (5 minutes)

Discuss the ways in which angles are named and notated.

* What do you notice about the three figures below? What is the same about all three figures; what is different?
  + *There are three angles that appear to be the same measurement but are notated differently.*
* What is a likely implication of the three different kinds of notation?
  + *They indicate the different ways of labeling or identifying the angle.*

Students are familiar with addressing Figure 1 as and having a measurement of and addressing Figure 2 as angle . Elicit this from students and say that in a case like Figure 1, the angle is named by the measure of the arc, and in a case like Figure 2, the angle is named by the single letter.

* In a case like Figure 3, we use three letters when we name the angle. Why use three points to name an angle?
  + *In a figure where several angles share the same vertex, naming a particular angle by the vertex point is not sufficient information to distinguish that angle. Two additional points, one belonging to each side of the intended angle, are necessary to identify it.*

Encourage students to use both multiple forms of angle notation in the table to demonstrate each angle relationship.

|  |  |  |
| --- | --- | --- |
| **Figure 1** | **Figure 2** | **Figure 3** |
|  |  |  |
| *Naming by the arc.* | *Naming by the vertex.* | *Naming by three points.* |
|  |  | *or* |

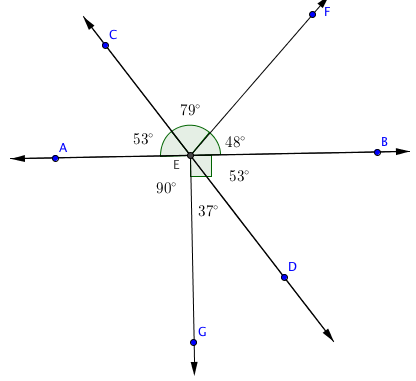
Recall the definitions of adjacent and vertical and the facts regarding angles on a line and angles at a point. If an abbreviation exists, students should include the abbreviation of the angle fact under the name of each relationship. In the Angle Fact column, students write the definitions and practice the different angle notations when describing the relationship in the angle fact.

Note: The *angles on a line* fact applies to two or more angles.

Angle Facts and Definitions

|  |  |  |
| --- | --- | --- |
| Name of Angle Relationship | Angle Fact | Diagram |
| Adjacent Angles | Two angles, and with a common side , are adjacent angles if belongs to the interior of .  Angles and are adjacent angles; and are adjacent angles. |  |
| Vertical Angles  (vert. s) | Two angles are vertical angles (or vertically opposite angles) if their sides form two pairs of opposite rays. |  |
| Angles on a Line  (s on a line) | The sum of the measures of two angles that share a ray and form a line is . |  |
| Angles at a Point  (s at a point) | The measure of all angles formed by three or more rays with the same vertex is . |  |

Opening Exercise (4 minutes)

  
Opening Exercise

Use the diagram to complete the chart.

|  |  |
| --- | --- |
| Name the angles that are … | |
| Vertical | and , and |
| Adjacent | Answers include:  and  and |
| Angles on a line | Answers include:  ,, and  ,, and |
| Angles at a point | , ,, , , |

**Example 1 (4 minutes)**

Students describe the angle relationship in the diagram and set up and solve an equation that models it. Have students verify their answers by measuring the unknown angle with a protractor.

Example 1

Estimate the measurement of .

In a complete sentence, describe the angle relationship in the diagram.

and are angles on a line and have a sum of

Write an equation for the angle relationship shown in the figure and solve for . Then, find the measures of and confirm your answers by measuring the angle with a protractor.

Exercise 1 (4 minutes)

Students describe the angle relationship in the diagram and set up and solve an equation that models it. Have students verify their answers by measuring the unknown angle with a protractor.

Exercise 1

In a complete sentence, describe the angle relationship in the diagram.

, , and are angles on a line and have a sum of .

Find the measurements of and .

Example 2 (4 minutes)

Students describe the angle relationship in the diagram and set up and solve an equation that models it. Have students verify their answers by measuring the unknown angle with a protractor.

Example 2

In a complete sentence, describe the angle relationship in the diagram.

and are angles on a line and have a sum of . and are vertical angles and are of equal measurement.

Write an equation for the angle relationship shown in the figure and solve for and . Find the measurements of and .

**;**  (or vert. s are )

Exercise 2 (4 minutes)

Students describe the angle relationship in the diagram and set up and solve an equation that models it. Have students verify their answers by measuring the unknown angle with a protractor.

Exercise 2

In a complete sentence, describe the angle relationships in the diagram.

and are adjacent angles and, when added together, are the measure of ; and are vertical angles and are of equal measurement.

Write an equation for the angle relationship shown in the figure and solve for .

Example 3 (4 minutes)

Students describe the angle relationship in the diagram and set up and solve an equation that models it. Have students verify their answers by measuring the unknown angle with a protractor.

Example 3

In a complete sentence, describe the angle relationships in the diagram.

, , and are angles at a point and have a sum of .

Write an equation for the angle relationship shown in the figure and solve for . Find the measurement of and confirm your answers by measuring the angle with a protractor.

Exercise 3 (4 minutes)

Students describe the angle relationship in the diagram and set up and solve an equation that models it. Have students verify their answers by measuring the unknown angle with a protractor.

**Exercise 3**

In a complete sentence, describe the angle relationships in the diagram.

, , , and are angles at a point and sum to .

**Find the** measurement **of .**

Example 4 (5 minutes)

* List pairs of angles whose measurements are in a ratio of .

*Scaffolding:*

Students may find it helpful to highlight the pairs of equal vertical angles.

* + *Examples include: and , and , and*
* What does it mean for the ratio of the measurements of two angles to be ?

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* + *The measurement of one angle is two times the measure of the other angle. If the smaller angle is defined as , then the larger angle is . If the larger angle is defined as , then the smaller angle is .*
* Based on the following figure, which angle relationship(s) can be utilized to find the measure of an obtuse and acute angle?
  + *Any adjacent angle pair are on a line and have a sum of .*

Students describe the angle relationship in the question and set up and solve an equation that models it. Have students verify their answers by measuring the unknown angle with a protractor.

Example 4

The following two lines intersect. The ratio of the measurements of the obtuse angle to the acute angle in any adjacent angle pair in this figure is   
. In a complete sentence, describe the angle relationships in the diagram.

The measurement of an obtuse angle is twice the measurement of an acute angle in the diagram.

Label the diagram with expressions that describe this relationship. Write an equation that models the angle relationship and solve for . Find the measurements of the acute and obtuse angles.

Acute angle

Obtuse angle

Exercise 4 (4 minutes)

Students describe the angle relationship in the diagram and set up and solve an equation that models it. Have students verify their answers by measuring the unknown angle with a protractor.

**Exercise 4**

The ratio of to is . In a complete sentence, describe the angle relationships in the diagram.

The measurement of is the measurement of ; and have a sum of .

**Find the measures of and .**

Relevant Vocabulary

Adjacent Angles: Two angles and with a common side are *adjacent angles* if belongs to the interior of .

Vertical Angles: Two angles are *vertical angles* (or *vertically opposite angles)* if their sides form two pairs of opposite rays.

Angles on a Line: The sum of the measures of adjacent *angles on a line* is .

Angles at a Point: The sum of the measures of adjacent *angles at a point* is .

Exit Ticket (3 minutes)

Name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Lesson 10: Angle Problems and Solving Equations

Exit Ticket

In a complete sentence, describe the relevant angle relationships in the following diagram. That is, describe the angle relationships you could use to determine the value of .

Use the angle relationships described above to write an equation to solve for . Then, determine the measurements of and

Exit Ticket Sample Solutions



In a complete sentence, describe the relevant angle relationships in the following diagram. That is, describe the angle relationships you could use to determine the value of .

and are adjacent angles whose measurements are equal to ; and are vertical angles and are of equal measurement.

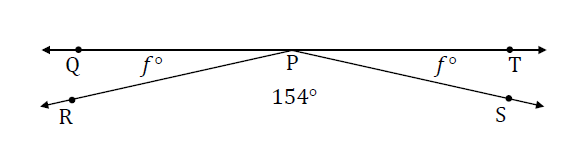
Use the angle relationships described above to write an equation to solve for . Then, determine the measurements of and .

Problem Set Sample Solutions

For each question, use angle relationships to write an equation in order to solve for each variable. Determine the indicated angles. You can check your answers by measuring each angle with a protractor.

1. In a complete sentence, describe the relevant angle relationships in the following diagram. Find the measurement of.

One possible response: , , and are angles on a line and sum to .

1. In a complete sentence, describe the relevant angle relationships in the following diagram. Find the measurement of .

, , and are angles on a line and sum to .

1. In a complete sentence, describe the relevant angle relationships in the following diagram. Find the measurements of and .

, , , , and are angles on a line and sum to .

1. In a complete sentence, describe the relevant angle relationships in the following diagram. Find the measure of.

All of the angles in the diagram are angles at a point and sum to .

1. In a complete sentence, describe the relevant angle relationships in the following diagram. Find the measures of and .

, , and are angles on a line and sum to . Since and form a straight angle and the measurement of is , is , making and form a right angle and have a sum of .

1. In a complete sentence, describe the relevant angle relationships in the following diagram. Find the measures of and .

and are vertical angles and are of equal measurement. and form a right angle and have a sum of .

1. In a complete sentence, describe the relevant angle relationships in the following diagram. Find the measures of and .

 and form a right angle and have a sum of .

1. In a complete sentence, describe the relevant angle relationships in the following diagram. Find the measure of .

and are vertical angles and are of equal measurement. and are adjacent and sum to the measurement of .

1. The ratio of the measures of a pair of adjacent angles on a line is .
   1. Find the measures of the two angles.

Angle 1 , Angle 2

Angle 1

Angle 2

* 1. Draw a diagram to scale of these adjacent angles. Indicate the measurements of each angle.

1. The ratio of the measures of three adjacent angles on a line is .
   1. Find the measures of the three angles.

Angle 1 , Angle 2 , Angle 3

Angle 1

Angle 2

Angle 3

* 1. Draw a diagram to scale of these adjacent angles. Indicate the measurements of each angle.

Angle 3

* 1. Draw a diagram to scale of these adjacent angles. Indicate the measurements of each angle.