## Lesson 9: Using If-Then Moves in Solving Equations

## Student Outcomes

- Students understand and use the addition, subtraction, multiplication, division, and substitution properties of equality to solve word problems leading to equations of the form $p x+q=r$ and $p(x+q)=r$, where $p, q$, and $r$ are specific rational numbers.
- Students understand that any equation can be rewritten as an equivalent equation with expressions that involve only integer coefficients by multiplying both sides by the correct number.


## Lesson Notes

This lesson is a continuation from Lesson 8. Students examine and interpret the structure between $p x+q=r$ and $p(x+q)=r$. Students will continue to write equations from word problems including distance and age problems. Also, students will play a game during this lesson which requires students to solve 1-2 problems and then arrange the answers in correct numerical order. This game can be played many different times as long as students receive different problems each time.

## Classwork

## Opening Exercise (10 minutes)

Have students work in small groups to write and solve an equation for each problem, followed by a whole group discussion.

## Opening Exercise

Heather practices soccer and piano. Each day she practices piano for 2 hours. After 5 days, she practiced both piano and soccer for a total of $\mathbf{2 0}$ hours. Assuming that she practiced soccer the same amount of time each day, how many hours per day, $h$, did Heather practice soccer?
$h$ : hours per day that soccer was practiced

$$
\begin{aligned}
5(h+2) & =20 \\
5 h+10 & =20 \\
5 h+10-10 & =20-10 \\
5 h & =10 \\
\left(\frac{1}{5}\right)(5 h) & =\left(\frac{1}{5}\right)(10) \\
h & =2
\end{aligned}
$$

Heather practiced soccer for 2 hours each day.

Over 5 days, Jake practices piano for a total of 2 hours. Jake practices soccer for the same amount of time each day. If he practiced piano and soccer for a total of $\mathbf{2 0}$ hours, how many hours, $h$, per day did Jake practice soccer?
$h$ : hours per day that soccer was practiced

$$
\begin{aligned}
5 h+2 & =20 \\
5 h+2-2 & =20-2 \\
5 h & =18 \\
\left(\frac{1}{5}\right)(5 h) & =(18)\left(\frac{1}{5}\right) \\
h & =3.6
\end{aligned}
$$

Jake practiced soccer 3.6 hours each day.

- Examine both equations. How are they similar, and how are they different?
- Both equations have the same numbers and deal with the same word problem. They are different in the set-up of the equations. The first problem includes parentheses where the second does not. This is because in the first problem, both soccer and piano were being practiced every day, so the total for each day had to be multiplied by the total number of days, five. Whereas in the second problem, only soccer was being practiced every day, and piano was only practiced a total of two hours for that time frame. Therefore, only the number of hours of soccer practice had to be multiplied by five, and not the piano time.
- Do the different structures of the equations affect the answer? Explain why or why not.
- Yes, the first problem requires students to use the distributive property, so the number of hours of soccer and piano practice are included every day. Using the distributive property changes the 2 in the equation to 10 , which is the total hours of piano practice over the entire 5 days. An if-then move of dividing both sides by 5 first could have also been used to solve the problem. The second equation does not use parentheses since piano is not practiced every day. Therefore, the 5 days are only multiplied by the number of hours of soccer practice, and not the piano time. This changes the end result.
- Which if-then moves were used in solving the equations?
- In the first equation, students may have used division of a number on both sides, subtracting a number on both sides, and multiplying a number on both sides. If the student distributed first, then only the if-then moves of subtracting a number on both sides and multiplying a non-zero number on both sides were used.
- In the second equation, the if-then moves of subtracting a number on both sides and multiplying a non-zero number on both sides were used.
- Interpret what 3.6 hours means in hours and minutes? Describe how to determine this.
- The solution 3.6 hours means 3 hours 36 minutes. Since there are 60 minutes in an hour and 0.6 is part of an hour, multiply 0.6 by 60 to get the part of the hour that 0.6 represents.


## Example 1 (8 minutes)

Lead students through the following problem.

## Example 1

Fred and Sam are a team in the local 138.2 mile bike-run-athon. Fred will compete in the bike race, and Sam will compete in the run. Fred bikes at an average speed of 8 miles per hour and Sam runs at an average speed of 4 miles per hour. The bike race begins at 6:00 a.m., followed by the run. Sam predicts he will finish the run at 2:33 a.m. the next morning.
a. How many hours will it take them to complete the entire bike-run-athon?

From 6:00 a.m. to 2:00 a.m. the following day is $\mathbf{2 0}$ hours.
33 minutes, in hours, is $\frac{33}{60}=\frac{11}{20}=0.55$ hours.
Therefore, the total time it will take to complete the entire bike-run-athon is 20.55 hours.
b. If $t$ is how long it takes Fred to complete the bike race, in hours, write an expression to find Fred's total distance.
$d=r t$
$d=8 t$
The expression of Fred's total distance is $8 t$.
c. Write an expression, in terms of $t$ to express Sam's time.

Since $t$ is Fred's time and 20.55 is the total time, then Sam's time would be the difference between the total time and Fred's time. The expression would be $20.55-t$.
d. Write an expression, in terms of $t$, that represents Sam's total distance.
$d=r t$
$d=4(20.55-t)$
The expressions $4(20.55-t)$ or $82.2-4 t$ is Sam's total distance.
e. Write and solve an equation using the total distance both Fred and Sam will travel.

$$
\begin{aligned}
8 t+4(20.55-t) & =138.2 \\
8 t+82.2-4 t & =138.2 \\
8 t-4 t+82.2 & =138.2 \\
4 t+82.2 & =138.2 \\
4 t+82.2-82.2 & =138.2-82.2 \\
4 t+0 & =56 \\
\left(\frac{1}{4}\right)(4 t) & =\left(\frac{1}{4}\right)(56) \\
t & =14
\end{aligned}
$$

Fred's time: $\quad t=14$ hours
Sam's time: $\quad 20.55-t=20.55-14=6.55$ hours
f. How far will Fred bike, and how much time will it take him to complete his leg of the race?

$$
8(14)=112 \text { miles, and Fred will complete the bike race in } 14 \text { hours.. }
$$

## Scaffolding:

- Refer to a clock when determining the total amount of time.
- Teachers may need to review the formula $d=r t$ from Grade 6 and Module 1.
.
g. How far will Sam run, and how much time will it take him to complete his leg of the race?

$$
\begin{aligned}
& 4(20.55-t) \\
& 4(20.55-14) \\
& 4(6.55)=26.2 \text { miles }
\end{aligned}
$$

Sam will run 26.2 miles, and it will take him 6.55 hours.

## Discussion (5 minutes)

- Why isn't the total time from 6:00 a.m. to 2:33 a.m. written as 20.33 hours?
- Time is based on 60 minutes. If the time in minutes just became the decimal, then time would have to be out of 100 because 20.33 represents 20 and 33 hundredths.
- To help determine the expression for Sam's time, work through the following chart. (This will lead to subtracting Fred's time from the total time.)

| Total Time (hours) | Fred's Time (hours) | Sam's Time (hours) |
| :---: | :---: | :---: |
| 10 | 6 | $10-6=4$ |
| 15 | 12 | $15-12=3$ |
| 20 | 8 | $20-8=12$ |
| 18.35 | 8 | $18.35-8=10.35$ |
| 20.55 | $t$ | $20.55-t$ |

- How do you find the distance traveled?
- Multiply the rate of speed by the amount of time.
- Model how to organize the problem in a distance, rate, and time chart.

|  | Rate (mph) | Time (hours) | Distance (miles) |
| :---: | :---: | :---: | :---: |
| Fred | 8 | $t$ | $8 t$ |
| Sam | 4 | $20.55-t$ | $4(20.55-t)$ <br> $82.2-4 t$ |

- Explain how to write the equation to have only integers and no decimals. Write the equation.
- Since the decimal terminates in the tenths place, if we multiply every term by 10 , the equation would result with only integer coefficients. The equation would be $40 t+822=1382$.


## Example 2 (7 minutes)

## Example 2

Shelby is seven times as old as Bonnie. If in 5 years, the sum of Bonnie's and Shelby's ages is 98, find Bonnie's present age. Use an algebraic approach.

|  | Present Age (in years) | Future Age (in years) |
| :---: | :---: | :---: |
| Bonnie | $x$ | $x+5$ |
| Shelby | $7 x$ | $7 x+5$ |

$$
\begin{aligned}
x+5+7 x+5 & =98 \\
8 x+10 & =98 \\
8 x+10-10 & =98-10 \\
8 x & =88 \\
\left(\frac{1}{8}\right)(8 x) & =\left(\frac{1}{8}\right)(88) \\
x & =11
\end{aligned}
$$

Bonnie's present age is 11 years old.

- The first step we must take is to write expressions that represent the present ages of both Bonnie and Shelby. The second step is to write expressions for future time or past time, using the present age expressions. How would the expression change if the time were in the past and not in the future?
- If the time were in the past, then the expression would be the difference between the present age and the amount of time in the past.


## Game (10 minutes)

The purpose of this game is for students to continue to practice solving linear equations when given in a contextual form. Divide students into 3 groups. There are 25 problems total, so if there are more than 25 students in the class, assign the extra students as the checkers of student work. Each group receives a puzzle (found at the end of the lesson). Depending on the size of the class, some students may receive only one card, while others may have multiple cards. Direct students to complete the problem(s) they receive. Each problem has a letter to the right. Students are to write and solve an equation unless other directions are stated. Once students get an answer, they are to locate the numerical answer under the blank and put the corresponding letter in the blank. When all problems are completed correctly, the letters in the blanks answer the riddle. Encourage students to check each other's work. This game can be replayed as many times as desired provided students receive different problems from a different set of cards. A variation to this game can be for students to arrange the answers in numerical order from least to greatest and/or greatest to least instead of the riddle or in addition to the riddle.

## Closing (2 minutes)

- How can an equation be written with only integer coefficients and constant terms?
- How are the addition, subtraction, multiplication, division, and substitution properties of equality used to solve algebraic equations?


## Exit Ticket (3 minutes)

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## Lesson 9: Using If-Then Moves in Solving Equations

## Exit Ticket

1. Brand A scooter has a top speed that goes 2 miles per hour faster than Brand B. If after 3 hours, Brand A scooter traveled 24 miles at its top speed, at what rate did Brand B scooter travel at its top speed if it traveled the same distance? Write an equation to determine the solution. Identify the if-then moves used in your solution.
2. At each scooter's top speed, Brand A scooter goes 2 miles per hour faster than Brand B. If after traveling at its top speed for 3 hours, Brand A scooter traveled 40.2 miles, at what rate did Brand B scooter travel if it traveled the same distance as Brand A? Write an equation to determine the solution and then write an equivalent equation using only integers.

## Exit Ticket Sample Solutions

1. Brand A scooter has a top speed that goes 2 miles per hour faster than Brand B. If after $\mathbf{3}$ hours, Brand A scooter traveled 24 miles at its top speed, at what rate did Brand B scooter travel at its top speed if it traveled the same distance? Write an equation to determine the solution. Identify the if-then moves used in your solution.
$x$ : speed, in mph, of Brand B scooter
$x+2$ : speed, in mph, of Brand A scooter
$d=r t$
$24=(x+2)(3)$
$24=3(x+2)$
Possible solution 1:
Possible solution 2:

$$
\begin{aligned}
24 & =3(x+2) \\
8 & =x+2 \\
8-2 & =x+2-2 \\
6 & =x
\end{aligned}
$$

$$
\begin{aligned}
24 & =3(x+2) \\
24 & =3 x+6 \\
24-6 & =3 x+6-6 \\
18 & =3 x+0 \\
\left(\frac{1}{3}\right)(18) & =\left(\frac{1}{3}\right)(3 x) \\
6 & =x
\end{aligned}
$$

If-then Moves: Divide both sides by 3.
If-then Moves: Subtract 6 from both sides.
Subtract 2 from both sides.

$$
\text { Multiply both sides by } \frac{1}{3} \text {. }
$$

2. At each scooter's top speed, Brand A scooter goes 2 miles per hour faster than Brand B. If after traveling at its top speed for 3 hours, Brand A scooter traveled 40.2 miles, at what rate did Brand B scooter travel if it traveled the same distance as Brand $A$ ? Write an equation to determine the solution and then write an equivalent equation using only integers.

$$
\begin{aligned}
x: & \quad \text { speed, in mph, of Brand B scooter } \\
x+2: \quad & \text { speed, in mph, of Brand A scooter } \\
d & =r t \\
40.2 & =(x+2)(3) \\
40.2 & =3(x+2)
\end{aligned}
$$

## Possible solution 1:

$$
\begin{aligned}
40.2 & =3(x+2) \\
13.4 & =x+2 \\
134 & =10 x+20 \\
134-20 & =10 x+20-20 \\
114 & =10 x \\
\left(\frac{1}{10}\right)(114) & =\left(\frac{1}{10}\right)(10 x) \\
11.4 & =x
\end{aligned}
$$

Possible solution 2:

$$
\begin{aligned}
40.2 & =3(x+2) \\
40.2 & =3 x+6 \\
402 & =30 x+60 \\
402-60 & =30 x+60-60 \\
342 & =30 x \\
\left(\frac{1}{30}\right)(342) & =\left(\frac{1}{30}\right)(30 x) \\
11.4 & =x
\end{aligned}
$$

Brand B's scooter travels at 11.4 miles per hour.

## Problem Set Sample Solutions

1. A company buys a digital scanner for $\$ 12,000$. The value of the scanner is $12,000\left(1-\frac{n}{5}\right)$ after $n$ years. The company has budgeted to replace the scanner when the trade-in value is $\$ 2,400$. After how many years should the company plan to replace the machine in order to receive this trade-in value?

$$
\begin{aligned}
12,000\left(1-\frac{n}{5}\right) & =2,400 \\
12,000-2,400 n & =2,400 \\
-2,400 n+12,000-12,000 & =2,400-12,000 \\
-2,400 n & =-9,600 \\
n & =4
\end{aligned}
$$

They will replace the scanner after 4 years.
2. Michael is 17 years older than John. In 4 years, the sum of their ages will be 49. Find Michael's present age.
x represents Michael's age now in years.

|  | Now | 4 years later |
| :---: | :---: | :---: |
| Michael | $x$ | $x+4$ |
| John | $x-17$ | $(x-17)+4$ |

$$
\begin{aligned}
x+4+x-17+4 & =49 \\
x+4+x-13 & =49 \\
2 x-9 & =49 \\
2 x-9+9 & =49+9 \\
2 x & =58 \\
\left(\frac{1}{2}\right)(2 x) & =\left(\frac{1}{2}\right)(58) \\
x & =29
\end{aligned}
$$

Michael's present age is 29 years old.
3. Brady rode his bike $\mathbf{7 0}$ miles in $\mathbf{4}$ hours. He rode at an average speed of $\mathbf{1 7} \mathbf{~ m p h}$ for $\boldsymbol{t}$ hours and at an average rate of speed of 22 mph for rest of the time. How long did Brady ride at the slower speed? Use the variable $t$ to represent the time, in hours, Brady rode at 17 mph .
\(\left.$$
\begin{array}{|c|c|c|c|}\hline & \begin{array}{c}\text { Rate } \\
\text { (mph) }\end{array} & \begin{array}{c}\text { Time } \\
\text { (hours) }\end{array} & \begin{array}{c}\text { Distance } \\
\text { (miles) }\end{array}
$$ <br>
\hline Brady speed 1 \& 17 \& t \& 17 t <br>
\hline Brady speed 2 \& 22 \& 4-t \& 22(4-t) <br>

\hline\end{array}\right\}\)| Total |
| :--- |
| distance |

## The total distance he rode:

$$
\begin{aligned}
& 17 t+22(4-t) \\
& 17 t+22(4-t)=70
\end{aligned}
$$

The total distance equals $\mathbf{7 0}$ miles.

$$
\begin{aligned}
17 t+22(4-t) & =70 \\
17 t+88-22 t & =70 \\
-5 t+88 & =70 \\
-5 t+88-88 & =70-88 \\
-5 t & =-18 \\
t & =3.6
\end{aligned}
$$

The time he rode at 17 mph is $\mathbf{3 . 6}$ hours.
4. Caitlan went to the store to buy school clothes. She had a store credit from a previous return in the amount of $\$ 39$. 58. If she bought 4 of the same style shirt in different colors and spent a total of $\$ 52.22$ after the store credit was taken off her total, what was the price of each shirt she bought? Write and solve an equation with integer coefficients.
$t$ : the price of one shirt

$$
\begin{aligned}
4 t-39.58 & =52.22 \\
4 t-39.58+39.58 & =52.22+39.58 \\
4 t+0 & =91.80 \\
\left(\frac{1}{4}\right)(4 t) & =\left(\frac{1}{4}\right)(91.80) \\
t & =22.95
\end{aligned}
$$

The price of one shirt was $\$ 22.95$.
5. A young boy is growing at a rate of 3.5 cm per month. He is currently 90 cm tall. At that rate, in how many months will the boy grow to a height of 132 cm ?

Let $m$ represent the number of months.

$$
\begin{aligned}
3.5 m+90 & =132 \\
3.5 m+90-90 & =132-90 \\
3.5 m & =42 \\
\left(\frac{1}{3.5}\right)(3.5 m) & =\left(\frac{1}{3.5}\right)(42) \\
m & =12
\end{aligned}
$$

The boy will grow to be 132 cm tall 12 months from now.
6. The sum of a number, $\frac{1}{6}$ of that number, $2 \frac{1}{2}$ of that number, and 7 is $2 \frac{1}{2}$. Find the number.

Let $n$ represent the given number.

$$
\begin{aligned}
n+\frac{1}{6} n+\left(2 \frac{1}{2}\right) n+7 & =12 \frac{1}{2} \\
n\left(1+\frac{1}{6}+\frac{5}{2}\right)+7 & =12 \frac{1}{2} \\
n\left(\frac{6}{6}+\frac{1}{6}+\frac{15}{6}\right)+7 & =12 \frac{1}{2} \\
n\left(\frac{22}{6}\right)+7 & =12 \frac{1}{2} \\
\frac{11}{3} n+7-7 & =12 \frac{1}{2}-7 \\
\frac{11}{3} n+0 & =5 \frac{1}{2} \\
\frac{11}{3} n & =5 \frac{1}{2} \\
\frac{3}{11} \cdot \frac{11}{3} n & =\frac{3}{11} \cdot \frac{11}{2} \\
1 n & =\frac{3}{2} \\
n & =1 \frac{1}{2}
\end{aligned}
$$

The number is $1 \frac{1}{2}$.
7. The sum of two numbers is 33 and their difference is 2 . Find the numbers.

If I let $x$ represent the first number, then $33-x$ represents the other number since their sum is 33 .

$$
\begin{aligned}
x-(33-x) & =2 \\
x+(-(33-x)) & =2 \\
x+(-33)+x & =2 \\
2 x+(-33) & =2 \\
2 x+(-33)+33 & =2+33 \\
2 x+0 & =35 \\
2 x & =35 \\
\frac{1}{2} \cdot 2 x & =\frac{1}{2} \cdot 35 \\
1 x & =\frac{35}{2} \\
x & =17 \frac{1}{2}
\end{aligned}
$$

$33-x$
$33-\left(17 \frac{1}{2}\right)=15 \frac{1}{2}$
$\left\{17 \frac{1}{2}, 15 \frac{1}{2}\right\}$
8. Aiden refills three token machines in an arcade. He puts twice the number of tokens in machine $A$ as in machine $B$, and in machine $C$, he puts $\frac{3}{4}$ of what he put in machine $A$. The three machines took a total of 18,324 tokens. How many did each machine take?

Let $A$ represent the number of tokens in machine $A$. Then $\frac{1}{2}$ A represents the number of tokens in machine $B$, and $\frac{3}{4}$ A represents the number of tokens in machine $C$.

$$
\begin{aligned}
A+\frac{1}{2} A+\frac{3}{4} A & =18,324 \\
\frac{9}{4} A & =18,324 \\
A & =8,144
\end{aligned}
$$

Machine A took 8, 144 tokens, machine B took 4, 072 tokens, and machine C took 6, 108 tokens.
9. Paulie ordered 250 pens and 250 pencils to sell for a theatre club fundraiser. The pens cost $\mathbf{1 1}$ cents more than the pencils. If Paulie's total order costs $\$ 42.50$, find the cost of each pen and pencil.

Let $l$ represent the cost of a pencil in dollars. Then, the cost of a pen in dollars is $l+\mathbf{0 . 1 1}$.

$$
\begin{aligned}
250(l+l+0.11) & =42.5 \\
250(2 l+0.11) & =42.5 \\
500 l+27.5 & =42.5 \\
(500 l+27.5)+(-27.5) & =42.5+(-27.5) \\
500 l+[27.5+(-27.5)] & =15 \\
500 l+0 & =15 \\
500 l & =15 \\
\frac{500 l}{500} & =\frac{15}{500} \\
l & =\$ 0.03
\end{aligned}
$$

A pencil costs $\$ \mathbf{0 . 0 3}$, and a pen costs $\$ 0.14$.
10. A family left their house in two cars at the same time. One car traveled an average of 7 miles per hour faster than the other. When the first car arrived at the destination after $5 \frac{1}{2}$ hours of driving, both cars had driven a total of 599.5 miles. If the second car continues at the same average speed, how much time, to the nearest minute, will it take before the second car arrives?

If I let represent the speed in miles per hour of the faster car, then $r-7$ represents the speed in miles per hour of the slower car.

$$
\begin{aligned}
5 \frac{1}{2}(r)+5 \frac{1}{2}(r-7) & =599.5 \\
5 \frac{1}{2}(r+r-7) & =599.5 \\
5 \frac{1}{2}(2 r-7) & =599.5 \\
\frac{11}{2}(2 r-7) & =599.5 \\
\frac{2}{11} \cdot \frac{11}{2}(2 r-7) & =\frac{2}{11} \cdot 599.5 \\
1 \cdot(2 r-7) & =\frac{1199}{11} \\
2 r-7 & =109 \\
2 r-7+7 & =109+7 \\
2 r+0 & =116 \\
2 r & =116 \\
\frac{1}{2} \cdot 2 r & =\frac{1}{2} \cdot 116 \\
1 r & =58 \\
r & =58
\end{aligned}
$$

The average speed of the faster car is 58 miles per hour, so the average speed of the slower car is $\mathbf{5 1}$ miles per hour.

$$
\begin{aligned}
\text { distance } & =\text { rate } \cdot \text { time } \\
d & =51 \cdot 5 \frac{1}{2} \\
d & =51 \cdot \frac{11}{2} \\
d & =280.5
\end{aligned}
$$

The slower car traveled 280.5 miles in $5 \frac{1}{2}$ hours.

$$
\begin{aligned}
& d=58 \cdot 5 \frac{1}{2} \\
& d=58 \cdot \frac{11}{2} \\
& d=319
\end{aligned}
$$

OR
$599.5-280.5=319$

The faster car traveled 319 miles in $5 \frac{1}{2}$ hours.
The slower car traveled 280.5 miles in $5 \frac{1}{2}$ hours. The remainder of their trip is 38.5 miles because $319-280.5=38.5$.

$$
\begin{aligned}
\text { distance } & =\text { rate } \cdot \text { time } \\
38.5 & =51(t) \\
\frac{1}{51}(38.5) & =\frac{1}{51}(51)(t) \\
\frac{38.5}{51} & =1 t \\
\frac{77}{102} & =t
\end{aligned}
$$

This time is in hours. To convert to minutes, multiply by 60 minutes per hour.

$$
\frac{77}{102} \cdot 60=\frac{77}{51} \cdot 30=\frac{2310}{51} \approx 45 \text { minutes }
$$

The slower car will arrive approximately 45 minutes after the first.
11. Emily counts the triangles and parallelograms in an art piece and determines that all together, there are 42 triangles and parallelograms. If there are $\mathbf{1 5 0}$ total sides, how many triangles and parallelograms are there?

If $t$ represents the number of triangles that Emily counted, then 42 -t represents the number of parallelograms that she counted.

$$
\begin{aligned}
3 t+4(42-t) & =150 \\
3 t+4(42+(-t)) & =150 \\
3 t+4(42)+4(-t) & =150 \\
3 t+168+(-4 t) & =150 \\
3 t+(-4 t)+168 & =150 \\
-t+168 & =150 \\
-t+168-168 & =150-168 \\
-t+0 & =-18 \\
-t & =-18 \\
-1 \cdot(-t) & =-1 \cdot(-18) \\
1 t & =18 \\
t & =18
\end{aligned}
$$

There are 18 triangles and 24 parallelograms.

Note to the Teacher: Problems 12-14 are more difficult and may not be suitable to assign to all students to solve independently.
12. Stefan is three years younger than his sister Katie. The sum of Stefan's age 3 years ago and $\frac{2}{3}$ of Katie's age at that time is 12 . How old is Katie now?

If I let $s$ represent Stefan's age in years, then $s+3$ represents Katie's current age, $s-3$ represents Stefan's age 3 years ago, and salso represents Katie's age 3 years ago.

$$
\begin{aligned}
(s-3)+\left(\frac{2}{3}\right) s & =12 \\
s+(-3)+\frac{2}{3} s & =12 \\
s+\frac{2}{3} s+(-3) & =12 \\
\frac{3}{3} s+\frac{2}{3} s+(-3) & =12 \\
\frac{5}{3} s+(-3) & =12 \\
\frac{5}{3} s+(-3)+3 & =12+3 \\
\frac{5}{3} s+0 & =15 \\
\frac{5}{3} s & =15 \\
\frac{3}{5} \cdot \frac{5}{3} s & =\frac{3}{5} \cdot 15 \\
1 s & =3 \cdot 3 \\
s & =9
\end{aligned}
$$

Stefan's current age is $\mathbf{9}$ years, so Katie is currently 12 years old.
13. Lucas bought a certain weight of oats for his horse at a unit price of $\$ \mathbf{0 . 2 0}$ per pound. The total cost of the oats left him with $\$ 1$. He wanted to buy the same weight of enriched oats instead, but at $\$ \mathbf{0 . 3 0}$ per pound, he would have been $\$ 2$ short of the total amount due. How much money did Lucas have to buy oats?

The difference in the costs is $\$ 3.00$ for the same weight in feed.
Let $w$ represent the weight in pounds of feed.

$$
\begin{aligned}
0.3 w-0.2 w & =3 \\
0.1 w & =3 \\
\frac{1}{10} w & =3 \\
10 \cdot \frac{1}{10} w & =10 \cdot 3 \\
1 w & =30 \\
w & =30
\end{aligned}
$$

Lucas bought 30 pounds of oats.
Cost $=$ unit price $\times$ weight (pounds)
Cost $=(\$ 0.20$ per pound $) \cdot(30$ pounds $)$
Cost $=\$ 6.00$
Lucas paid $\$ 6$ for 30 pounds of oats. Lucas had \$1 left after his purchase, so he started with \$7.

Group 1: Where can you buy a ruler that is $\mathbf{3}$ feet long?

| 3 | $4 \frac{1}{2}$ | 3.5 | -1 | -2 | 19 | 18.95 | 4.22 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |


| What value(s) of $z$ makes the equation $\frac{7}{6} z+\frac{1}{3}=-\frac{5}{6}$ true; $z=-1, z=2, z=1$, or $z=-\frac{36}{63}$ ? | D |
| :---: | :---: |
| Find the smaller of 2 consecutive integers if the sum of the smaller and twice the larger is -4 . | $S$ |
| Twice the sum of a number and -6 is -6 . Find the number. | Y |
| Logan is 2 years older than Lindsey. Five years ago, the sum of their ages was 30. Find Lindsey's current age. | A |
| The total charge for a taxi ride in NYC includes an initial fee of $\$ 3.75$ plus $\$ 1.25$ for every $\frac{1}{2}$ mile traveled. Jodi took a taxi, and the ride cost her exactly $\$ 12.50$. How many miles did she travel in the taxi? | $R$ |
| The perimeter of a triangular garden with 3 equal sides is 12.66 feet. What is the length of each side of the garden? | $E$ |
| A car travelling at 60 mph leaves Ithaca and travels west. Two hours later, a truck travelling at 55 mph leaves Elmira and travels east. All together, the car and truck travel 407.5 miles. How many hours does the car travel? | A |
| The Cozo family has 5 children. While on vacation, they went to a play. They bought 5 tickets at the child's price of $\$ 10.25$ and 2 tickets at the adult's price. If they spent a total of $\$ 89.15$, how much was the price of each adult ticket? | $L$ |

Group 1 Sample Solutions
YARD SALE

| What value(s) of $z$ makes the equation $\frac{7}{6} z+\frac{1}{3}=-\frac{5}{6}$ true; $z=-1, z=2, z=1$, or $z=-\frac{36}{63}$ ? | -1 |
| :---: | :---: |
| Find the smaller of 2 consecutive integers if the sum of the smaller and twice the larger is -4 . | -2 |
| Twice the sum of a number and -6 is -6 . Find the number. | 3 |
| Logan is 2 years older than Lindsey. Five years ago, the sum of their ages was 30. Find Lindsey's current age. | 19 |
| The total charge for a taxi ride in NYC includes an initial fee of $\$ 3.75$ plus $\$ 1.25$ for every $\frac{1}{2}$ mile traveled. Jodi took a taxi, and the ride cost her exactly $\$ 12.50$. How many miles did she travel in the taxi? | 3.5 mi . |
| The perimeter of a triangular garden with 3 equal sides is 12.66 feet. What is the length of each side of the garden? | 4.22 feet |
| A car travelling at 60 mph leaves Ithaca and travels west. Two hours later, a truck travelling at 55 mph leaves Elmira and travels East. All together, the car and truck travel 407.5 miles. How many hours does the car travel? | $4 \frac{1}{2}$ hours |
| The Cozo family has 5 children. While on vacation, they went to a play. They bought 5 tickets at the child's price of $\$ 10.25$ and 2 tickets at the adult's price. If they spent a total of $\$ 89.15$, how much was the price of each adult ticket? | \$18.95 |

Group 2: Where do fish keep their money?

| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $-\ldots$ | - |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | -1 | 10 | 8 | 2 | -6 | 5 | 50 | $\frac{1}{8}$ |


| What value of $z$ makes the equation $\frac{2}{3} z-\frac{1}{2}=-\frac{5}{12}$ true; $z=-1, z=2, z=\frac{1}{8}$, or $z=-\frac{1}{8}$ ? | K |
| :---: | :---: |
| Find the smaller of 2 consecutive even integers if the sum of twice the smaller integer and the larger integers is -16 . | $B$ |
| Twice the difference of a number and -3 is 4 . Find the number. | I |
| Brooke is 3 years younger than Samantha. In five years, the sum of their ages will be 29. Find Brooke's age. | $E$ |
| Which of the following equations is equivalent to $4.12 x+5.2=8.23$ ? <br> (1) $412 x+52=823$ <br> (2) $412 x+520=823$ <br> (3) $9.32 x=8.23$ <br> (4) $0.412 x+0.52=8.23$ | $R$ |
| The length of a rectangle is twice the width. If the perimeter of the rectangle is 30 units, find the area of the garden. | $N$ |
| A car traveling at 70 miles per hour traveled one hour longer than a truck traveling at 60 miles per hour. If the car and truck traveled a total of 330 miles, for how many hours did the car and truck travel all together? | A |
| Jeff sold half of his baseball cards then bought sixteen more. He now has 21 baseball cards. How many cards did he begin with? | V |

Group 2 Sample Solutions
RIVER BANK

| What value of $z$ makes the equation $\frac{2}{3} z-\frac{1}{2}=-\frac{5}{12}$ true; $z=-1, z=2, z=\frac{1}{8}$, or $z=-\frac{1}{8}$ ? | $\frac{1}{8}$ |
| :---: | :---: |
| Find the smaller of 2 consecutive even integers if the sum of twice the smaller integer and the larger integer is -16 . | -6 |
| Twice the difference of a number and -3 is 4 . Find the number. | -1 |
| Brooke is 3 years younger than Samantha. In 5 years, the sum of their ages will be 29. Find Brooke's age. | 8 |
| Which of the following equations is equivalent to $4.12 x+5.2=8.23$ ? <br> (1) $412 x+52=823$ <br> (2) $412 x+520=823$ <br> (3) $9.32 x=8.23$ <br> (4) $0.412 x+0.52=8.23$ | 2 |
| The length of a rectangle is twice the width. If the perimeter of the rectangle is 30 units, find the area of the garden. | 50 units |
| A car traveling at 70 miles per hour traveled one hour longer than a truck traveling at 60 miles per hour. If the car and truck traveled a total of 330 miles, for how many hours did the car and truck travel all together? | 5 hours |
| Jeff sold half of his baseball cards then bought 16 more. He now has 21 baseball cards. How many cards did he begin with? | 10 |

Group 3: The more you take, the more you leave behind. What are they?

| - | - | - | - | - | - | - | - | - |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 8 | 11.93 | 368 | $1 \frac{5}{6}$ | 10.50 | $2 \frac{1}{2}$ | $3 \frac{5}{6}$ | 21 | 4 |


| An apple has 80 calories. This is 12 less than $\frac{1}{4}$ the number of calories in a package of candy. How many calories are in the candy? | 0 |
| :---: | :---: |
| The ages of 3 brothers are represented by consecutive integers. If the oldest brother's age is decreased by twice the youngest brother's age, the result is $\mathbf{- 1 9}$. How old is the youngest brother? | P |
| A carpenter uses 3 hinges on every door he hangs. He hangs 4 doors on the first floor and $x$ doors on the second floor. If he uses 36 hinges total, how many doors did he hang on the second floor? | F |
| Kate has $12 \frac{1}{2}$ pounds of chocolate. She gives each of her 5 friends $x$ pounds each and has $3 \frac{1}{3}$ pounds left over. How much did she give each of her friends? | $T$ |
| A room is 20 feet long. If a couch that is $12 \frac{1}{3}$ feet long is to be centered in the room, how big of a table can be placed on either side of the couch? | $E$ |
| Which equation is equivalent to $\frac{1}{4} x+\frac{1}{5}=2$ ? <br> (1) $4 x+5=\frac{1}{2}$ <br> (2) $\frac{2}{9} x=2$ <br> (3) $5 x+4=18$ <br> (4) $5 x+4=40$ | $S$ |
| During a recent sale, the first movie purchased cost $\$ 29$, and each additional movie purchased costs $m$ dollars. If Jose buys 4 movies and spends a total of $\$ 64.80$, how much did each additional movie cost? | 0 |
| The Hipster Dance company purchases 5 bus tickets that cost $\$ 150$ each, and they have 7 bags that cost $b$ dollars each. If the total bill is $\$ 823.50$, how much does each bag cost? | $S$ |
| The weekend before final exams, Travis studied 1.5 hours for his science exam, $2 \frac{1}{4}$ hours for his math exam, and $h$ hours each for Spanish, English, and social studies. If he spent a total of $11 \frac{1}{4}$ hours studying, how much time did he spend studying for Spanish? | $T$ |

## Group 3 Sample Solutions

FOOTSTEPS

| An apple has 80 calories. This is 12 less than $\frac{1}{4}$ the number of calories in a package of candy. How many calories are in the candy? | 368 calories |
| :---: | :---: |
| The ages of 3 brothers are represented by consecutive integers. If the oldest brother's age is decreased by twice the youngest brother's age, the result is $\mathbf{- 1 9}$. How old is the youngest brother? | 21 |
| A carpenter uses 3 hinges on every door he hangs. He hangs 4 doors on the first floor and $x$ doors on the second floor. If he uses 36 hinges total, how many doors did he hang on the second floor? | 8 |
| Kate has $12 \frac{1}{2}$ pounds of chocolate. She gives each of her 5 friends $x$ pounds each and has $3 \frac{1}{3}$ pounds left over. How much did she give each of her friends? | $1 \frac{5}{6} \text { pounds }$ |
| A room is 20 feet long. If a couch that is $12 \frac{1}{3}$ feet long is to be centered in the room, how big of a table can be placed on either side of the couch? | $3 \frac{5}{6} \text { feet }$ |
| Which equation is equivalent to $\frac{1}{4} x+\frac{1}{5}=2$ ? <br> (1) $4 x+5=\frac{1}{2}$ <br> (2) $\frac{2}{9} x=2$ <br> (3) $5 x+4=18$ <br> (4) $5 x+4=40$ | 4 |
| During a recent sale, the first movie purchased cost \$29, and each additional movie purchased costs $m$ dollars. If Jose buys 4 movies and spends a total of $\$ 64.80$, how much did each additional movie cost? | \$11.93 |
| The Hipster Dance company purchases 5 bus tickets that cost $\$ 150$ each, and they have 7 bags that cost $b$ dollars each. If the total bill is $\$ 823.50$, how much does each bag cost? | \$10.50 |
| The weekend before final exams, Travis studied 1.5 hours for his science exam, $2 \frac{1}{4}$ hours for his math exam, and $h$ hours each for Spanish, English, and social studies. If he spent a total of $11 \frac{1}{4}$ hours studying, how much time did he spend studying for Spanish? | $2 \frac{1}{2}$ hours |

