Lesson 2: Generating Equivalent Expressions

Student Outcomes

- Students generate equivalent expressions using the fact that addition and multiplication can be done in any order (commutative property) and any grouping (associative property).
- Students recognize how *any order, any grouping* can be applied in a subtraction problem by using additive inverse relationships (adding the opposite) to form a sum and likewise with division problems by using the multiplicative inverse relationships (multiplying by the reciprocal) to form a product.
- Students recognize that "any order" does not apply for expressions mixing addition and multiplication, leading to the need to follow the order of operations.

Classwork

Opening Exercises (5 minutes)

Students complete the table in the Opening Exercise that scaffolds the concept of opposite expressions from the known concept of opposite numbers to find the opposite of the expression $3x - 7$.

MP.8

Opening Exercise

Additive inverses have a sum of zero. Fill in the center column of the table with the opposite of the given number or expression, then show the proof that they are opposites. The first row is completed for you.

Expression	Opposite	Proof of Opposites
$\mathbf{1}$	$\mathbf{-1}$	$\mathbf{1 + (-1) = 0}$
$\mathbf{3}$	$\mathbf{-3}$	$\mathbf{3 + (-3) = 0}$
$\mathbf{-7}$	$\mathbf{7}$	$\mathbf{-7 + 7 = 0}$
$\mathbf{-\frac{1}{2}}$	$\mathbf{\frac{1}{2}}$	$\mathbf{-\frac{1}{2} + \frac{1}{2} = 0}$
$\boldsymbol{x}$	$\boldsymbol{-x}$	$\boldsymbol{x + (-x) = 0}$
$\boldsymbol{3x}$	$\boldsymbol{-3x}$	$\boldsymbol{3x + (-3x) = 0}$
$\boldsymbol{x + 3}$	$\boldsymbol{-x + (-3)}$	$\boldsymbol{(x+3) + (-x + (-3))}$ $\boldsymbol{(x + (-x)) + (3 + (-3)) = 0}$
$\boldsymbol{3x - 7}$	$\boldsymbol{-3x + 7}$	$\boldsymbol{(3x - 7) + (-3x + 7)}$ $\boldsymbol{3x + (-7) + (-3x) + 7}$ $\boldsymbol{(3x + (-3x)) + ((-7) + 7) = 0}$

Encourage students to provide their answers aloud. When finished, discuss the following:

- In the last two rows, explain how the given expression and its opposite compare.
  - *Recall that the opposite of a number, say a, satisfies the equation $a + (-a) = 0$. We can use this equation to recognize when two expressions are opposites of each other. For example, since $(x+3) + \big(-x + (-3)\big) = 0$, we conclude that $-x + (-3)$ must be the opposite of $x + 3$. This is because when either $-(x+3)$ or $-x + (-3)$ are substituted into the blank in $(x+3) +$ ______ $= 0$, the resulting equation is true for every value of x. Therefore, the two expressions must be equivalent: $-(x+3) = -x + (-3)$.*

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- Since the opposite of x is $-x$ and the opposite of 3 is -3, what can we say about the opposite of the sum of x and 3?
  - *We can say that the opposite of the sum $x + 3$ is the sum of its opposites $(-x) + (-3)$.*
- Is this relationship also true for the last example $3x - 7$?
  - *Yes, because opposites have a sum of zero, so $(3x - 7) +$ ________ $= 0$. If the expression $-3x + 7$ is substituted in the blank, the resulting equation is true for every value of x. The opposite of $3x$ is $-3x$, the opposite of (-7) is 7, and the sum of these opposites is $-3x + 7$; therefore, it is true that the opposite of the sum $3x + (-7)$ is the sum of its opposites $-3x + 7$.*

$$\overbrace{\big(3x + (-7)\big)}^{\text{sum}} + \overbrace{\big((-3x) + 7\big)}^{\text{opposite}} = 0, \textit{ so } -\big(3x + (-7)\big) = -3x + 7.$$

- Can we generalize a rule for the opposite of a sum?
  - *"The opposite of a sum is the sum of its opposites."*

Tell students that we can use this property as justification for converting the opposites of sums as we work to rewrite expressions in standard form.

Example 1 (5 minutes): Subtracting Expressions

Student and teacher investigate the process for subtracting expressions where the subtrahend is a grouped expression containing two or more terms.

- Subtract the expressions in Example 1(a) first by changing subtraction of the expression to adding the expression's opposite.

> *Scaffolding:*
>
> Finding the opposite (or inverse) of an expression is just like finding the opposite of a mixed number; remember that the opposite of a sum is equal to the sum of its opposites:
>
> $$-\left(2\frac{3}{4}\right) = (-2) + \left(-\frac{3}{4}\right);$$
>
> $$-(2 + x) = -2 + (-x).$$

Example 1: Subtracting Expressions

a. Subtract: $(40 + 9) - (30 + 2)$.

Opposite of a sum is the sum of its opposites	*Order of operations*
$40 + 9 + \big(-(30 + 2)\big)$	$(40 + 9) - (30 + 2)$
$40 + 9 + (-30) + (-2)$	$(49) - (32)$
$49 + (-30) + (-2)$	17
$19 + (-2)$	
17	

- Next, subtract the expressions using traditional order of operations. Does the difference yield the same number in each case?
  - *Yes. (See above right.)*
- Which of the two methods seems more efficient and why?
  - *Answers may vary, but students will likely choose the second method as they are more familiar with it.*
- Which method will have to be used in Example 1(b) and why?
  - *We must add the opposite expression because the terms in parentheses are not like terms, so they cannot be combined as we did with the sum of numbers in Example 1(a).*

b. Subtract: $(3x + 5y - 4) - (4x + 11)$.

$3x + 5y + (-4) + \big(-(4x + 11)\big)$	*Subtraction as adding the opposite*
$3x + 5y + (-4) + (-4x) + (-11)$	*Opposite of a sum is the sum of its opposites*
$3x + (-4x) + 5y + (-4) + (-11)$	*Any order, any grouping*
$-x + 5y + (-15)$	*Combining like terms*
$-x + 5y - 15$	*Subtraction replaces adding the opposite*

Have students check the equivalency of the expressions by substituting 2 for x and 6 for y.

$(3x + 5y - 4) - (4x + 11)$	$-x + 5y - 15$
$(3(2) + 5(6) - 4) - (4(2) + 11)$	$-(2) + 5(6) - 15$
$(6 + 30 - 4) - (8 + 11)$	$-2 + 30 + (-15)$
$(36 - 4) - (19)$	$28 + (-15) = 13$
$32 - 19 = 13$	

The expressions yield the same number (13) *for* $x = 2$ *and* $y = 6$.

- When writing the difference as adding the expression's opposite in Example 1(b), what happens to the grouped terms that are being subtracted?
  - *When the subtraction is changed to addition, every term in the parentheses that follows must be converted to its opposite.*

Example 2 (5 minutes): Combining Expressions Vertically

Students combine expressions by vertically aligning like terms.

- Any order, any grouping allows us to write sums and differences as vertical math problems. If we want to combine expressions vertically, we align their like terms vertically.

Example 2: Combining Expressions Vertically

a. **Find the sum by aligning the expressions vertically.**

$(5a + 3b - 6c) + (2a - 4b + 13c)$

$\big(5a + 3b + (-6c)\big) + (2a + (-4b) + 13c)$ *Subtraction as adding the opposite*

$$\begin{array}{r} 5a + 3b + (-6c) \\ \underline{+2a + (-4b) + 13c} \\ 7a + (-b) + 7c \end{array}$$

Align like terms vertically and combine by addition

$7a - b + 7c$ *Adding the opposite is equivalent to subtraction*

b. **Find the difference by aligning the expressions vertically.**

$(2x + 3y - 4) - (5x + 2)$

$\big(2x + 3y + (-4)\big) + \big(-5x + (-2)\big)$ *Subtraction as adding the opposite*

$$\begin{array}{r} 2x + 3y + (-4) \\ \underline{+(-5x) + (-2)} \\ -3x + 3y + (-6) \end{array}$$

Align like terms vertically and combine by addition

$-3x + 3y - 6$ *Adding the opposite is equivalent to subtraction*

Students should recognize that the subtracted expression in Example 1(b) did not include a term containing the variable y, so the $3y$ from the first grouped expression remains unchanged in the answer.

Example 3 (3 minutes): Using Expressions to Solve Problems

Students write an expression representing an unknown real-world value, rewrite as an equivalent expression, and use the equivalent expression to find the unknown value.

Example 3: Using Expressions to Solve Problems

A stick is x meters long. A string is 4 times as long as the stick.

a. **Express the length of the string in terms of x.**

The length of the stick in meters is x meters, so the string is $4 \cdot x$, or $4x$, meters long.

b. **If the total length of the string and the stick is 15 meters long, how long is the string?**

The length of the stick and the string together in meters can be represented by $x + 4x$, or $5x$. If the length of the stick and string together is 15 meters, the length of the stick is 3 meters, and the length of the string is 12 meters.

Example 4 (4 minutes): Expressions from Word Problems

Students write expressions described by word problems and rewrite the expressions in standard form.

Example 4: Expressions from Word Problems

It costs Margo a processing fee of $\$3$ to rent a storage unit, plus $\$17$ per month to keep her belongings in the unit. Her friend Carissa wants to store a box of her belongings in Margo's storage unit and tells her that she will pay her $\$1$ toward the processing fee and $\$3$ for every month that she keeps the box in storage. Write an expression in standard form that represents how much Margo will have to pay for the storage unit if Carissa contributes. Then, determine how much Margo will pay if she uses the storage unit for 6 months.

Let m represent the number of months that the storage unit is rented.

$(17m+3)-(3m+1)$	*Original expression*
$17m+3+\left(-(3m+1)\right)$	*Subtraction as adding the opposite*
$17m+3+(-3m)+(-1)$	*Opposite of the sum is the sum of its opposites*
$17m+(-3m)+3+(-1)$	*Any order, any grouping*
$14m+2$	*Combined like terms*

This means that Margo will have to pay only $\$2$ of the processing fee and $\$14$ per month that the storage unit is used.

$14(6)+2$

$84+2=86$ *Margo will pay $\$86$ toward the storage unit rental for 6 months of use.*

If time allows, encourage students to calculate their answer in other ways and compare their answers.

Example 5 (7 minutes): Extending Use of the Inverse to Division

Students connect the strategy of using the additive inverse to represent a subtraction problem as a sum and using the multiplicative inverse to represent a division problem as a product so that the associative and commutative properties can then be used.

- Why do we convert differences into sums using opposites?
  - *The commutative and associative properties do not apply to subtraction; therefore, we convert differences to sums of the opposites so that we can use the any order, any grouping property with addition.*

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- We have seen that the any order, any grouping property can be used with addition or with multiplication. If you consider how we extended the property to subtraction, can we use the any order, any grouping property in a division problem? Explain.
  - *Dividing by a number is equivalent to multiplying by the number's multiplicative inverse (reciprocal), so division can be converted to multiplication of the reciprocal, similar to how we converted the subtraction of a number to addition using its additive inverse. After converting a quotient to a product, use of the any order, any grouping property is allowed.*

Example 5: Extending Use of the Inverse to Division

Multiplicative inverses have a product of 1. Find the multiplicative inverses of the terms in the first column. Show that the given number and its multiplicative inverse have a product of 1. Then, use the inverse to write each corresponding expression in standard form. The first row is completed for you.

Given	Multiplicative Inverse	Proof—Show that their product is 1.	Use each inverse to write its corresponding expression below in standard form.
3	$\frac{1}{3}$	$3 \cdot \frac{1}{3}$ $\frac{3}{1} \cdot \frac{1}{3}$ $\frac{3}{3} = 1$	$12 \div 3$ $12 \cdot \frac{1}{3}$ 4
5	$\frac{1}{5}$	$5 \cdot \frac{1}{5}$ $\frac{5}{1} \cdot \frac{1}{5}$ $\frac{5}{5} = 1$	$65 \div 5$ $65 \cdot \frac{1}{5}$ 13
-2	$-\frac{1}{2}$	$-2 \cdot \left(-\frac{1}{2}\right)$ $-\frac{2}{1} \cdot \left(-\frac{1}{2}\right)$ $\frac{2}{2} = 1$	$18 \div (-2)$ $18 \cdot \left(-\frac{1}{2}\right)$ $18 \cdot (-1) \cdot \left(\frac{1}{2}\right)$ $-18 \cdot \frac{1}{2} = -9$
$-\frac{3}{5}$	$-\frac{5}{3}$	$-\frac{3}{5} \cdot \left(-\frac{5}{3}\right)$ $\frac{15}{15} = 1$	$6 \div \left(-\frac{3}{5}\right)$ $6 \cdot \left(-\frac{5}{3}\right)$ $6 \cdot (-1) \cdot \frac{5}{3}$ $-6 \cdot \frac{5}{3}$ $-2 \cdot 5 = -10$
x	$\frac{1}{x}$	$x \cdot \frac{1}{x}$ $\frac{x}{1} \cdot \frac{1}{x}$ $\frac{x}{x} = 1$	$5x \div x$ $5x \cdot \frac{1}{x}$ $5 \cdot \frac{x}{x}$ $5 \cdot 1 = 5$
$2x$	$\frac{1}{2x}$	$2x \cdot \left(\frac{1}{2x}\right)$ $2 \cdot x \cdot \left(\frac{1}{2} \cdot \frac{1}{x}\right)$ $2 \cdot \frac{1}{2} \cdot x \cdot \frac{1}{x}$ $1 \cdot 1 = 1$	$12x \div 2x$ $12x \cdot \frac{1}{2x}$ $\frac{12x}{2x}$ $\frac{12}{2} \cdot \frac{x}{x}$ $6 \cdot 1 = 6$

MP.8

- How do we know that two numbers are multiplicative inverses (reciprocals)?
  - *Recall that the multiplicative inverse of a number, a, satisfies the equation $a \cdot \frac{1}{a} = 1$. We can use this equation to recognize when two expressions are multiplicative inverses of each other.*

- Since the reciprocal of x is $\frac{1}{x}$, and the reciprocal of 2 is $\frac{1}{2}$, what can we say about the reciprocal of the product of x and 2?
  - *We can say that the reciprocal of the product $2x$ is the product of its factor's reciprocals:* $\frac{1}{2} \cdot \frac{1}{x} = \frac{1}{2x}$.

MP.8

- What is true about the signs of reciprocals? Why?
  - *The signs of reciprocals are the same because their product must be 1. This can only be obtained when the two numbers in the product have the same sign.*

Tell students that because the reciprocal is not complicated by the signs of numbers as in opposites, we can justify converting division to multiplication of the reciprocal by simply stating "multiplying by the reciprocal."

Sprint (8 minutes): Generating Equivalent Expressions

Students complete a two-round Sprint exercise (Sprints and answer keys provided at end of lesson) where they practice their knowledge of combining like terms by addition and/or subtraction. Provide one minute for each round of the Sprint. Refer to the Sprints and Sprint Delivery Script sections in the Module Overview for directions to administer a Sprint. Be sure to provide any answers not completed by the students. (If there is a need for further guided division practice, consider using the division portion of the Problem Set, or other division examples, in place of the provided Sprint exercise.)

Closing (3 minutes)

- Why can't we use any order, any grouping directly with subtraction? With division?
  - *Subtraction and division are not commutative or associative.*
- How can we use any order, any grouping in expressions where subtraction or division are involved?
  - *Subtraction can be rewritten as adding the opposite (additive inverse), and division can be rewritten as multiplying by the reciprocal (multiplicative inverse).*

Relevant Vocabulary

AN EXPRESSION IN EXPANDED FORM: An expression that is written as sums (and/or differences) of products whose factors are numbers, variables, or variables raised to whole number powers is said to be in *expanded form.* A single number, variable, or a single product of numbers and/or variables is also considered to be in expanded form. Examples of expressions in expanded form include: 324, $3x$, $5x + 3 - 40$, $x + 2x + 3x$, etc.

TERM: Each summand of an expression in expanded form is called a *term.* For example, the expression $2x + 3x + 5$ consists of 3 terms: $2x$, $3x$, and 5.

COEFFICIENT OF THE TERM: The number found by multiplying just the numbers in a term together is called the *coefficient*. For example, given the product $2 \cdot x \cdot 4$, its equivalent term is $8x$. The number 8 is called the coefficient of the term $8x$.

AN EXPRESSION IN STANDARD FORM: An expression in expanded form with all its like terms collected is said to be in *standard form. For example, $2x + 3x + 5$ is an expression written in expanded form; however, to be written in standard form, the like terms $2x$ and $3x$ must be combined. The equivalent expression $5x + 5$ is written in standard form.*

Lesson Summary

- **Rewrite subtraction as adding the opposite before using any order, any grouping.**
- **Rewrite division as multiplying by the reciprocal before using any order, any grouping.**
- **The opposite of a sum is the sum of its opposites.**
- **Division is equivalent to multiplying by the reciprocal.**

Exit Ticket (5 minutes)

Name ____ Date____________________

Lesson 2: Generating Equivalent Expressions

Exit Ticket

1. Write the expression in standard form.
   $(4f - 3 + 2g) - (-4g + 2)$

2. Find the result when $5m + 2$ is subtracted from $9m$.

3. Rewrite the expression in standard form.
   $27h \div 3h$

Exit Ticket Sample Solutions

1. Write the expression in standard form.

   $(4f - 3 + 2g) - (-4g + 2)$

   $4f + (-3) + 2g + \big(-(-4g + 2)\big)$ *Subtraction as adding the opposite*

   $4f + (-3) + 2g + 4g + (-2)$ *Opposite of a sum is the sum of its opposites*

   $4f + 2g + 4g + (-3) + (-2)$ *Any order, any grouping*

   $4f + 6g + (-5)$ *Combined like terms*

   $4f + 6g - 5$ *Subtraction as adding the opposite*

2. Find the result when $5m + 2$ is subtracted from $9m$.

   $9m - (5m + 2)$ *Original expression*

   $9m + \big(-(5m + 2)\big)$ *Subtraction as adding the opposite*

   $9m + (-5m) + (-2)$ *Opposite of a sum is the sum of its opposites*

   $4m + (-2)$ *Combined like terms*

   $4m - 2$ *Subtraction as adding the opposite*

3. Rewrite the expression in standard form.

   $27h \div 3h$

   $27h \cdot \frac{1}{3h}$ *Multiplying by the reciprocal*

   $\frac{27h}{3h}$ *Multiplication*

   $\frac{27}{3} \cdot \frac{h}{h}$ *Any order, any grouping*

   $9 \cdot 1$

   9

Problem Set Sample Solutions

1. Write each expression in standard form. Verify that your expression is equivalent to the one given by evaluating each expression using $x = 5$.

a. $3x + (2 - 4x)$ $-x + 2$ $-5 + 2 = -3$ $3(5) + (2 - 4(5))$ $15 + (2 + (-20))$ $15 + (-18) = -3$	b. $3x + (-2 + 4x)$ $7x - 2$ $7(5) - 2$ $35 - 2 = 33$ $3(5) + (-2 + 4(5))$ $15 + (-2 + 20)$ $15 + 18 = 33$	c. $-3x + (2 + 4x)$ $x + 2$ $5 + 2 = 7$ $-3(5) + (2 + 4(5))$ $-15 + (2 + 20)$ $-15 + 22 = 7$
d. $3x + (-2 - 4x)$ $-x - 2$ $-5 - 2 = -7$ $3(5) + (-2 - 4(5))$ $15 + (-2 + (-4(5)))$ $15 + (-2 + (-20))$ $15 + (-22) = -7$	e. $3x - (2 + 4x)$ $-x - 2$ $-5 - 2 = -7$ $3(5) - (2 + 4(5))$ $15 - (2 + 20)$ $15 - 22$ $15 + (-22) = -7$	f. $3x - (-2 + 4x)$ $-x + 2$ $-5 + 2 = -3$ $3(5) - (-2 + 4(5))$ $15 - (-2 + 20)$ $15 - (18)$ $15 + (-18) = -3$
g. $3x - (-2 - 4x)$ $7x + 2$ $7(5) + 2$ $35 + 2 = 37$ $3(5) - (-2 - 4(5))$ $15 - (-2 + (-4(5)))$ $15 - (-2 + (-20))$ $15 - (-22)$ $15 + 22 = 37$	h. $3x - (2 - 4x)$ $7x - 2$ $7(5) - 2$ $35 - 2 = 33$ $3(5) - (2 - 4(5))$ $15 - (2 + (-4(5)))$ $15 - (2 + (-20))$ $15 - (-18)$ $15 + 18 = 33$	i. $-3x - (-2 - 4x)$ $x + 2$ $5 + 2 = 7$ $-3(5) - (-2 - 4(5))$ $-15 - (-2 + (-4(5)))$ $-15 - (-2 + (-20))$ $-15 - (-22)$ $-15 + 22 = 7$

j. In problems (a)–(d) above, what effect does addition have on the terms in parentheses when you removed the parentheses?

By the any grouping property, the terms remained the same with or without the parentheses.

k. In problems (e)–(i), what effect does subtraction have on the terms in parentheses when you removed the parentheses?

The opposite of a sum is the sum of the opposites; each term within the parentheses is changed to its opposite.

2. Write each expression in standard form. Verify that your expression is equivalent to the one given by evaluating each expression for the given value of the variable.

a. $4y-(3+y);\ y=2$ $3y-3$ $3(2)-3$ $6-3=3$ $4(2)-(3+2)$ $8-5$ $8+(-5)=3$	b. $(2b+1)-b;\ b=-4$ $b+1$ $-4+1=-3$ $(2(-4)+1)-(-4)$ $(-8+1)+4$ $(-7)+4=-3$	c. $(6c-4)-(c-3);\ c=-7$ $5c-1$ $5(-7)-1$ $-35-1=-36$ $(6(-7)-4)-(-7-3)$ $(-42-4)-(-10)$ $-42+(-4)+(10)$ $-46+10=-36$
d. $(d+3d)-(-d+2);$ $d=3$ $5d-2$ $5(3)-2$ $15-2=13$ $\big(3+3(3)\big)-(-3+2)$ $(3+9)-(-1)$ $12+1=13$	e. $(-5x-4)-(-2-5x);$ $x=3$ -2 $(-5(3)-4)-\big(-2-5(3)\big)$ $(-15-4)-(-2-15)$ $(-19)-(-17)$ $(-19)+17=-2$	f. $11f-(-2f+2);\ f=\frac{1}{2}$ $13f-2$ $13\left(\frac{1}{2}\right)-2$ $\frac{13}{2}-2$ $6\frac{1}{2}-2=4\frac{1}{2}$ $11\left(\frac{1}{2}\right)-\left(-2\left(\frac{1}{2}\right)+2\right)$ $\frac{11}{2}-(-1+2)$ $\frac{11}{2}-1$ $\frac{11}{2}+\left(-\frac{2}{2}\right)=\frac{9}{2}=4\frac{1}{2}$
g. $-5g+(6g-4);\ g=-2$ $g-4$ $-2-4=-6$ $-5(-2)+(6(-2)-4)$ $10+(-12-4)$ $10+\big(-12+(-4)\big)$ $10+(-16)=-6$	h. $(8h-1)-(h+3);$ $h=-3$ $7h-4$ $7(-3)-4$ $-21-4=-25$ $(8(-3)-1)-(-3+3)$ $(-24-1)-(0)$ $(-25)-0=-25$	i. $(7+w)-(w+7);\ w=-4$ 0 $\big(7+(-4)\big)-(-4+7)$ $3-3$ $3+(-3)=0$
j. $(2g+9h-5)-(6g-4h+2);\ g=-2$ and $h=5$ $-4g+13h-7$ $-4(-2)+13(5)-7$ $8+65+(-7)$ $73+(-7)=66$	$(2(-2)+9(5)-5)-(6(-2)-4(5)+2)$ $(-4+45-5)-\big(-12+(-4(5))+2\big)$ $(41-5)-(-12+(-20)+2)$ $\big(41+(-5)\big)-(-32+2)$ $36-(-30)$ $36+30=66$	

3. Write each expression in standard form. Verify that your expression is equivalent to the one given by evaluating both expressions for the given value of the variable.

a. $-3(8x);\ x=\frac{1}{4}$ $-24x$ $-24\left(\frac{1}{4}\right)$ $-\frac{24}{4}=-6$ $-3\left(8\left(\frac{1}{4}\right)\right)$ $-3(2)=-6$	b. $5\cdot k\cdot(-7);\ k=\frac{3}{5}$ $-35k$ $-35\left(\frac{3}{5}\right)$ $-\frac{105}{5}=-21$ $5\left(\frac{3}{5}\right)(-7)$ $3(-7)=-21$	c. $2(-6x)\cdot 2;\ x=\frac{3}{4}$ $-24x$ $-24\left(\frac{3}{4}\right)$ $-\frac{72}{4}=-18$ $2\left(-6\left(\frac{3}{4}\right)\right)\cdot 2$ $2\left(-3\left(\frac{3}{2}\right)\right)\cdot 2$ $2(-3)\left(\frac{3}{2}\right)(2)$ $-6(3)=-18$
d. $-3(8x)+6(4x);\ x=2$ 0 $-3(8(2))+6(4(2))$ $-3(16)+6(8)$ $-48+48=0$	e. $8(5m)+2(3m);\ m=-2$ $46m$ $46(-2)=-92$ $8(5(-2))+2(3(-2))$ $8(-10)+2(-6)$ $-80+(-12)=-92$	f. $-6(2v)+3a(3);\ v=\frac{1}{3};$ $a=\frac{2}{3}$ $-12v+9a$ $-12\left(\frac{1}{3}\right)+9\left(\frac{2}{3}\right)$ $-\frac{12}{3}+\frac{18}{3}$ $-4+6=2$ $-6\left(2\left(\frac{1}{3}\right)\right)+3\left(\frac{2}{3}\right)(3)$ $-6\left(\frac{2}{3}\right)+2(3)$ $-4+6=2$

4. Write each expression in standard form. Verify that your expression is equivalent to the one given by evaluating both expressions for the given value of the variable.

a. $8x\div 2;\ x=-\frac{1}{4}$ $4x$ $4\left(-\frac{1}{4}\right)=-1$ $8\left(-\frac{1}{4}\right)\div 2$ $-2\div 2=-1$	b. $18w\div 6;\ w=6$ $3w$ $3(6)=18$ $18(6)\div 6$ $108\div 6=18$	c. $25r\div 5r;\ r=-2$ 5 $25(-2)\div(5(-2))$ $-50\div(-10)=5$
d. $33y\div 11y;\ y=-2$ 3 $33(-2)\div(11(-2))$ $(-66)\div(-22)=3$	e. $56k\div 2k;\ k=3$ 28 $56(3)\div(2(3))$ $168\div 6=28$	f. $24xy\div 6y;\ x=-2;y=3$ $4x$ $4(-2)=-8$ $24(-2)(3)\div(6(3))$ $-48(3)\div 18$ $-144\div 18=-8$

5. For each problem (a)–(e), write an expression in standard form.

   a. Find the sum of $-3x$ and $8x$.

   $-3x + 8x = 5x$

   b. Find the sum of $-7g$ and $4g + 2$.

   $-7g + (4g + 2) = -3g + 2$

   c. Find the difference when $6h$ is subtracted from $2h - 4$.

   $(2h - 4) - 6h = -4h - 4$

   d. Find the difference when $-3n - 7$ is subtracted from $n + 4$.

   $(n + 4) - (-3n - 7) = 4n + 11$

   e. Find the result when $13v + 2$ is subtracted from $11 + 5v$.

   $(11 + 5v) - (13v + 2) = -8v + 9$

   f. Find the result when $-18m - 4$ is added to $4m - 14$.

   $(4m - 14) + (-18m - 4) = -14m - 18$

   g. What is the result when $-2x + 9$ is taken away from $-7x + 2$?

   $(-7x + 2) - (-2x + 9) = -5x - 7$

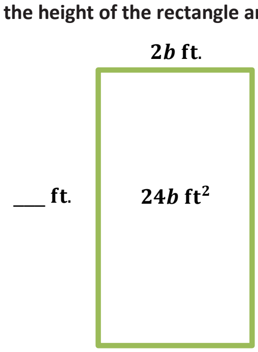
6. Marty and Stewart are stuffing envelopes with index cards. They are putting x index cards in each envelope. When they are finished, Marty has 15 stuffed envelopes and 4 extra index cards, and Stewart has 12 stuffed envelopes and 6 extra index cards. Write an expression in standard form that represents the number of index cards the boys started with. Explain what your expression means.

   They inserted the same number of index cards in each envelope, but that number is unknown x. An expression that represents Marty's index cards is $15x + 4$ because he had 15 envelopes and 4 cards left over. An expression that represents Stewart's index cards is $12x + 6$ because he had 12 envelopes and 6 left over cards. Their total number of cards together would be:

   $$15x + 4 + 12x + 6$$
   $$15x + 12x + 4 + 6$$
   $$27x + 10$$

   This means that all together, they have 27 envelopes with x index cards in each, plus another 10 left over index cards.

7. The area of the pictured rectangle below is $24b\ \text{ft}^2$. Its width is $2b$ ft. Find the height of the rectangle and name any properties used with the appropriate step.



   $24b \div 2b$

   $24b \cdot \frac{1}{2b}$ — *Multiplying the reciprocal*

   $\frac{24b}{2b}$ — *Multiplication*

   $\frac{24}{2} \cdot \frac{b}{b}$ — *Any order, any grouping in multiplication*

   $12 \cdot 1$

   12

   The height of the rectangle is 12 ft.

Number Correct: ______

Generating Equivalent Expressions—Round 1

Directions: Write each as an equivalent expression in standard form as quickly and accurately as possible within the allotted time.

1.	$1+1$		23.	$4x+6x-12x$	
2.	$1+1+1$		24.	$4x-6x+4x$	
3.	$(1+1)+1$		25.	$7x-2x+3$	
4.	$(1+1)+(1+1)$		26.	$(4x+3)+x$	
5.	$(1+1)+(1+1+1)$		27.	$(4x+3)+2x$	
6.	$x+x$		28.	$(4x+3)+3x$	
7.	$x+x+x$		29.	$(4x+3)+5x$	
8.	$(x+x)+x$		30.	$(4x+3)+6x$	
9.	$(x+x)+(x+x)$		31.	$(11x+2)-2$	
10.	$(x+x)+(x+x+x)$		32.	$(11x+2)-3$	
11.	$(x+x+x)+(x+x+x)$		33.	$(11x+2)-4$	
12.	$2x+x$		34.	$(11x+2)-7$	
13.	$3x+x$		35.	$(3x-9)+(3x+5)$	
14.	$4x+x$		36.	$(11-5x)+(4x+2)$	
15.	$7x+x$		37.	$(2x+3y)+(4x+y)$	
16.	$7x+2x$		38.	$(5x+1.3y)+(2.9x-0.6y)$	
17.	$7x+3x$		39.	$(2.6x-4.8y)+(6.5x-1.1y)$	
18.	$10x-x$		40.	$\left(\frac{3}{4}x-\frac{1}{2}y\right)+\left(-\frac{7}{4}x-\frac{5}{2}y\right)$	
19.	$10x-5x$		41.	$\left(-\frac{2}{5}x-\frac{7}{9}y\right)+\left(-\frac{7}{10}x-\frac{2}{3}y\right)$	
20.	$10x-10x$		42.	$\left(\frac{1}{2}x-\frac{1}{4}y\right)+\left(-\frac{3}{5}x+\frac{5}{6}x\right)$	
21.	$10x-11x$		43.	$\left(1.2x-\frac{3}{4}y\right)-\left(-\frac{3}{5}x+2.25x\right)$	
22.	$10x-12x$		44.	$(3.375x-8.9y)-\left(-7\frac{5}{8}x-5\frac{2}{5}y\right)$	

Generating Equivalent Expressions—Round 1 [KEY]

Directions: Write each as an equivalent expression in standard form as quickly and accurately as possible within the allotted time.

1.	$1+1$	$\mathbf{2}$
2.	$1+1+1$	$\mathbf{3}$
3.	$(1+1)+1$	$\mathbf{3}$
4.	$(1+1)+(1+1)$	$\mathbf{4}$
5.	$(1+1)+(1+1+1)$	$\mathbf{5}$
6.	$x+x$	$\mathbf{2x}$
7.	$x+x+x$	$\mathbf{3x}$
8.	$(x+x)+x$	$\mathbf{3x}$
9.	$(x+x)+(x+x)$	$\mathbf{4x}$
10.	$(x+x)+(x+x+x)$	$\mathbf{5x}$
11.	$(x+x+x)+(x+x+x)$	$\mathbf{6x}$
12.	$2x+x$	$\mathbf{3x}$
13.	$3x+x$	$\mathbf{4x}$
14.	$4x+x$	$\mathbf{5x}$
15.	$7x+x$	$\mathbf{8x}$
16.	$7x+2x$	$\mathbf{9x}$
17.	$7x+3x$	$\mathbf{10x}$
18.	$10x-x$	$\mathbf{9x}$
19.	$10x-5x$	$\mathbf{5x}$
20.	$10x-10x$	$\mathbf{0}$
21.	$10x-11x$	$\mathbf{-1x\ or\ -x}$
22.	$10x-12x$	$\mathbf{-2x}$

23.	$4x+6x-12x$	$\mathbf{-2x}$
24.	$4x-6x+4x$	$\mathbf{2x}$
25.	$7x-2x+3$	$\mathbf{5x+3}$
26.	$(4x+3)+x$	$\mathbf{5x+3}$
27.	$(4x+3)+2x$	$\mathbf{6x+3}$
28.	$(4x+3)+3x$	$\mathbf{7x+3}$
29.	$(4x+3)+5x$	$\mathbf{9x+3}$
30.	$(4x+3)+6x$	$\mathbf{10x+3}$
31.	$(11x+2)-2$	$\mathbf{11x}$
32.	$(11x+2)-3$	$\mathbf{11x-1}$
33.	$(11x+2)-4$	$\mathbf{11x-2}$
34.	$(11x+2)-7$	$\mathbf{11x-5}$
35.	$(3x-9)+(3x+5)$	$\mathbf{6x-4}$
36.	$(11-5x)+(4x+2)$	$\mathbf{13-x\ or\ -x+13}$
37.	$(2x+3y)+(4x+y)$	$\mathbf{6x+4y}$
38.	$(5x+1.3y)+(2.9x-0.6y)$	$\mathbf{7.9x+0.7y}$
39.	$(2.6x-4.8y)+(6.5x-1.1y)$	$\mathbf{9.1x-5.9y}$
40.	$\left(\frac{3}{4}x-\frac{1}{2}y\right)+\left(-\frac{7}{4}x-\frac{5}{2}y\right)$	$\mathbf{-x-3y}$
41.	$\left(-\frac{2}{5}x-\frac{7}{9}y\right)+\left(-\frac{7}{10}x-\frac{2}{3}y\right)$	$\mathbf{-\frac{11}{10}x-\frac{13}{9}y}$
42.	$\left(\frac{1}{2}x-\frac{1}{4}y\right)+\left(-\frac{3}{5}x+\frac{5}{6}x\right)$	$\mathbf{-\frac{1}{10}x+\frac{7}{12}y}$
43.	$\left(1.2x-\frac{3}{4}y\right)-\left(-\frac{3}{5}x+2.25x\right)$	$\mathbf{1\frac{4}{5}x+1\frac{1}{2}y}$
44.	$(3.375x-8.9y)-\left(-7\frac{5}{8}x-5\frac{2}{5}y\right)$	$\mathbf{11x-\frac{7}{2}y}$

Number Correct: ______

Generating Equivalent Expressions—Round 2

Improvement: ______

Directions: Write each as an equivalent expression in standard form as quickly and accurately as possible within the allotted time.

1.	$1+1+1$		23.	$3x+5x-4x$	
2.	$1+1+1+1$		24.	$8x-6x+4x$	
3.	$(1+1+1)+1$		25.	$7x-4x+5$	
4.	$(1+1+1)+(1+1)$		26.	$(9x-1)+x$	
5.	$(1+1+1)+(1+1+1)$		27.	$(9x-1)+2x$	
6.	$x+x+x$		28.	$(9x-1)+3x$	
7.	$x+x+x+x$		29.	$(9x-1)+5x$	
8.	$(x+x+x)+x$		30.	$(9x-1)+6x$	
9.	$(x+x+x)+(x+x)$		31.	$(-3x+3)-2$	
10.	$(x+x+x)+(x+x+x)$		32.	$(-3x+3)-3$	
11.	$(x+x+x+x)+(x+x)$		33.	$(-3x+3)-4$	
12.	$x+2x$		34.	$(-3x+3)-5$	
13.	$x+4x$		35.	$(5x-2)+(2x+5)$	
14.	$x+6x$		36.	$(8-x)+(3x+2)$	
15.	$x+8x$		37.	$(5x+y)+(x+y)$	
16.	$7x+x$		38.	$\left(\frac{5}{2}x+\frac{3}{2}y\right)+\left(\frac{11}{2}x-\frac{3}{4}y\right)$	
17.	$8x+2x$		39.	$\left(\frac{1}{6}x-\frac{3}{8}y\right)+\left(\frac{2}{3}x-\frac{7}{4}y\right)$	
18.	$2x-x$		40.	$(9.7x-3.8y)+(-2.8x+4.5y)$	
19.	$2x-2x$		41.	$(1.65x-2.73y)+(-1.35x+3.76y)$	
20.	$2x-3x$		42.	$(6.51x-4.39y)+(-7.46x+8.11x)$	
21.	$2x-4x$		43.	$\left(0.7x-\frac{2}{9}y\right)-\left(-\frac{7}{5}x+2\frac{1}{3}x\right)$	
22.	$2x-8x$		44.	$(8.4x-2.25y)-\left(-2\frac{1}{2}x-4\frac{3}{8}y\right)$	

Generating Equivalent Expressions—Round 2 [KEY]

Directions: Write each as an equivalent expression in standard form as quickly and accurately as possible within the allotted time.

1.	$1+1+1$	$\mathbf{3}$
2.	$1+1+1+1$	$\mathbf{4}$
3.	$(1+1+1)+1$	$\mathbf{4}$
4.	$(1+1+1)+(1+1)$	$\mathbf{5}$
5.	$(1+1+1)+(1+1+1)$	$\mathbf{6}$
6.	$x+x+x$	$\mathbf{3x}$
7.	$x+x+x+x$	$\mathbf{4x}$
8.	$(x+x+x)+x$	$\mathbf{4x}$
9.	$(x+x+x)+(x+x)$	$\mathbf{5x}$
10.	$(x+x+x)+(x+x+x)$	$\mathbf{6x}$
11.	$(x+x+x+x)+(x+x)$	$\mathbf{6x}$
12.	$x+2x$	$\mathbf{3x}$
13.	$x+4x$	$\mathbf{5x}$
14.	$x+6x$	$\mathbf{7x}$
15.	$x+8x$	$\mathbf{9x}$
16.	$7x+x$	$\mathbf{8x}$
17.	$8x+2x$	$\mathbf{10x}$
18.	$2x-x$	$\mathbf{x}$ ***or*** $\mathbf{1x}$
19.	$2x-2x$	$\mathbf{0}$
20.	$2x-3x$	$\mathbf{-x}$ ***or*** $\mathbf{-1x}$
21.	$2x-4x$	$\mathbf{-2x}$
22.	$2x-8x$	$\mathbf{-6x}$

23.	$3x+5x-4x$	$\mathbf{4x}$
24.	$8x-6x+4x$	$\mathbf{6x}$
25.	$7x-4x+5$	$\mathbf{3x+5}$
26.	$(9x-1)+x$	$\mathbf{10x-1}$
27.	$(9x-1)+2x$	$\mathbf{11x-1}$
28.	$(9x-1)+3x$	$\mathbf{12x-1}$
29.	$(9x-1)+5x$	$\mathbf{14x-1}$
30.	$(9x-1)+6x$	$\mathbf{15x-1}$
31.	$(-3x+3)-2$	$\mathbf{-3x+1}$
32.	$(-3x+3)-3$	$\mathbf{-3x}$
33.	$(-3x+3)-4$	$\mathbf{-3x-1}$
34.	$(-3x+3)-5$	$\mathbf{-3x-2}$
35.	$(5x-2)+(2x+5)$	$\mathbf{7x+3}$
36.	$(8-x)+(3x+2)$	$\mathbf{10+2x}$
37.	$(5x+y)+(x+y)$	$\mathbf{6x+2y}$
38.	$\left(\frac{5}{2}x+\frac{3}{2}y\right)+\left(\frac{11}{2}x-\frac{3}{4}y\right)$	$\mathbf{8x+\frac{3}{4}y}$
39.	$\left(\frac{1}{6}x-\frac{3}{8}y\right)+\left(\frac{2}{3}x-\frac{7}{4}y\right)$	$\mathbf{\frac{5}{6}x-\frac{17}{8}y}$
40.	$(9.7x-3.8y)+(-2.8x+4.5y)$	$\mathbf{6.9x+0.7y}$
41.	$(1.65x-2.73y)+(-1.35x+3.76y)$	$\mathbf{0.3x+1.03y}$
42.	$(6.51x-4.39y)+(-7.46x+8.11x)$	$\mathbf{-0.95x+3.72y}$
43.	$\left(0.7x-\frac{2}{9}y\right)-\left(-\frac{7}{5}x+2\frac{1}{3}x\right)$	$\mathbf{-\frac{21}{10}x-2\frac{5}{9}y}$
44.	$(8.4x-2.25y)-\left(-2\frac{1}{2}x-4\frac{3}{8}y\right)$	$\mathbf{10\frac{9}{10}x+2\frac{1}{8}y}$