



## Lesson 20: Investments—Performing Operations with Rational Numbers

### Student Outcomes

- Students perform various calculations involving rational numbers to solve a problem related to the change in an investment's balance over time.
- Students recognize and use mathematics as a tool to solve real-life problems.

### Classwork

#### Mathematical Modeling Exercise (25 minutes): College Investments

Students are given records for an investment fund over the past 5 years. The records include beginning balance, semi-annual statements, and additional fees. Students have a 4-part task:

1. Determine the balance at the end of 5 years.
2. Determine the annual gain or loss and the overall 5-year gain or loss.
3. Analyze the result and write a comparative conclusion, defending or refuting their conclusion.
4. Answer questions related to the investment data.

Familiarize students with the format of the register. Discuss how to complete the register by completing the first six months as a class together. Then have students individually complete the rest of the register for the remaining time.

Suggestion: Allow students to use calculators to assist in the arithmetic.

#### Mathematical Modeling Exercise: College Investments

Justin and Adrienne deposited \$20,000 into an investment account for 5 years. They hoped the money invested and the money made on their investment would amount to at least \$30,000 to help pay for their daughter's college tuition and expenses. The account they chose has several benefits and fees associated with it. Every 6 months, a summary statement is sent to Justin and Adrienne. The statement includes the amount of money either gained or lost. Below are semi-annual (twice a year) statements for a period of 5 years. In addition to the statements, the following information is needed to complete the task:

- Every statement, there is an administrative fee of \$15 to cover costs such as secretarial work, office supplies, postage, etc.
- If there is a withdrawal made, a broker's fee is deducted from the account. The amount of the broker's fee is 2% of the transaction amount.

**TASK:** Using the above information, semi-annual statements, register, and beginning balance, do the following:

1. Record the beginning balance and all transactions from the account statements into the register.
2. Determine the annual gain or loss as well as the overall 5-year gain/loss.
3. Determine if there is enough money in the account after 5 years to cover \$30,000 of college expenses for Justin and Adrienne's daughter. Write a summary to defend your answer. Be sure to indicate how much money is in excess, or the shortage that exists.
4. Answer the related questions that follow.

(Note: This activity may be adapted for use with spreadsheet software.)

**College Investment Fund Semi-Annual Statement**

January 1, 2008 – June 30, 2008

Investment Gain/(Loss): 700.00

**College Investment Fund Semi-Annual Statement**

July 1, 2008 – December 31, 2008

Investment Gain/(Loss): 754.38

**College Investment Fund Semi-Annual Statement**

January 1, 2009 – June 30, 2009

Investment Gain/(Loss): (49.88)

**College Investment Fund Semi-Annual Statement**

July 1, 2009 – December 31, 2009

Withdrawal: 500.00  
Investment Gain/(Loss): (17.41)

**College Investment Fund Semi-Annual Statement**

January 1, 2010 – June 30, 2010

Investment Gain/(Loss): 676.93

**College Investment Fund Semi-Annual Statement**

July 1, 2010 – December 31, 2010

Investment Gain/(Loss): 759.45

**College Investment Fund Semi-Annual Statement**

January 1, 2011 – June 30, 2011

Deposit: 1,500.00  
Investment Gain/(Loss): 880.09

**College Investment Fund Semi-Annual Statement**

July 1, 2011 – December 31, 2011

Investment Gain/(Loss): 922.99

**College Investment Fund Semi-Annual Statement**

January 1, 2012 – June 30, 2012

Deposit: 800.00  
Investment Gain/(Loss): 942.33

**College Investment Fund Semi-Annual Statement**

July 1, 2012 – December 31, 2012

Investment Gain/(Loss): 909.71

## 5. Register

DATE	DESCRIPTION OF TRANSACTION	WITHDRAWAL	DEPOSIT	BALANCE	EXPRESSION
	Beginning Balance	---	---	\$20,000.00	\$20,000.00
Jan. – June: 2008	Gain on investment		700.00	20,700.00	$20,000 + 700$
	Administrative fee	15.00		20,685.00	$20,700 - 15$
July – Dec.: 2008	Gain on investment		754.38	21,439.38	$20,685 + 754.38$
	Administrative fee	15.00		21,424.38	$21,439.38 - 15$
Jan. – June: 2009	Loss on investment	49.88		21,374.50	$21,424.38 - 49.88$
	Administrative fee	15.00		21,359.50	$21,374.50 - 15$
July – Dec.: 2009	Withdrawal	500.00		20,859.50	$21,359.50 - 500$
	Broker fee	10.00		20,849.50	$20,859.50 - 10$
	Loss on investment	17.41		20,832.09	$20,849.50 - 17.41$
	Administrative fee	15.00		20,817.09	$20,832.09 - 15$
Jan. – June: 2010	Gain on investment		676.93	21,494.02	$20,817.09 + 676.93$
	Administrative fee	15.00		21,479.02	$21,494.02 - 15$
July – Dec.: 2010	Gain on investment		759.45	22,238.47	$21,479.02 + 759.45$
	Administrative fee	15.00		22,223.47	$22,238.47 - 15$
Jan. – June: 2011	Deposit		1,500.00	23,723.47	$22,223.47 + 1500$
	Gain on investment		880.09	24,603.56	$23,723.47 + 880.09$
	Administrative fee	15.00		24,588.56	$24,603.56 - 15$
July – Dec.: 2011	Gain on investment		922.99	25,511.55	$24,588.56 + 922.99$
	Administrative fee	15.00		25,496.55	$25,511.55 - 15$
Jan. – June: 2012	Deposit		800.00	26,296.55	$25,496.55 + 800$
	Gain on investment		942.33	27,238.88	$26,296.55 + 942.33$
	Administrative fee	15.00		27,223.88	$27,238.88 - 15$
July – Dec.: 2012	Gain on investment		909.71	28,133.59	$27,223.88 + 909.71$
	Administrative fee	15.00		28,118.59	$28,133.59 - 15$
Prediction:					
Jan. – June: 2013	Gain on investment		900.00	29,018.59	$28,118.59 + 900$
	Administrative fee	15.00		29,003.59	$29,018.59 - 15$

- Describe the process of completing the register.
  - Starting with the beginning balance, fill in the description of the transaction and the amount. If the transaction is an investment loss, withdrawal, or fee, then the amount is recorded in the payment column. If the transaction is an investment gain or deposit then the amount is recorded in the deposit column. To obtain the new balance, subtract the payment amount or add the deposit amount from the balance on the preceding line. Record the new balance and use that balance to complete the next line.

**Scaffolding:**

Discuss what a register is and how it is used to organize a series of transactions. Also, discuss how a loss can be represented by using parenthesis (e.g., (607.29)).

- Describe how to find the broker's fee.
  - The broker's fee is 2% of the transaction amount. To find the broker's fee, you must first find the total of the transaction amount. Once you have that, write the percent as a fraction out of 100 and multiply the fraction by the transaction amount. This result is the amount of the broker's fee, which is then subtracted from the preceding balance.*

Example: 2% of \$2500

$$\frac{2}{100} \times 2500 = \frac{1}{50} \times 2500 = 50$$

- Compare your register with the person next to you. Did each of you list the transactions in the same order? Does it make a difference?
  - The order is probably not the same. The order of the transactions for each 6-month period does not make a difference.*
- Continue to compare your registers. Do you both get the same balance at the end of 2012? If not, switch papers and check to see if you can find your neighbor's mistake.

#### 6. Annual Gain/Loss Summary

Year	Total Gain/(Loss)	Numerical Expression
2008	1,424.38	21,424.38 – 20,000
2009	(607.29)	20,817.09 – 21,424.38
2010	1,406.38	22,223.47 – 20,817.09
2011	3,273.08	25,496.55 – 22,223.47
2012	2,622.04	28,118.59 – 25,496.55
5-Year Gain/Loss	8,118.59	28,118.59 – 20,000

- How do you find the annual gain or loss for each year?
  - Subtract the ending balance from the beginning balance. If the beginning balance is smaller than the ending balance, then the difference is a gain. If the beginning balance is larger than the ending balance, then the difference is a loss and written within ( ).*
- In what year was the greatest gain? In what year was the greatest loss?
  - The greatest gain was in 2011. The greatest loss was in 2009, because of the withdrawal of \$500 and the losses on the investment.*
- How does knowing the overall gain or loss assist in writing the comparative conclusion?
  - Once you know the gain or loss, you can find the ending balance. Since the ending balance was not at least \$30,000, there was not enough money to cover the college expenses.*
- Using the 5-year total gain or loss figures, write an expression using positive and negative rational numbers that can be solved to find the total gain or loss.

$$\begin{aligned}
 &1,424.38 + (-607.29) + 1,406.38 + 3,273.08 + 2,622.04 \\
 &= -607.29 + 1,424.38 + 1,406.38 + 3,273.08 + 2,622.04
 \end{aligned}$$

## 7. Summary

*There is not enough money in the account at the end of 5 years to cover the college expenses, but it is close. They needed at least \$30,000 in the account to cover the expenses, and there was \$28,118.59, leaving a shortage of \$1,881.41.*

## 8. Related Questions

- a. For the first half of 2009, there was a \$700 gain on the initial investment of \$20,000. Represent the gain as a percentage of the initial investment.

$$\frac{x}{100} = \frac{700}{20,000} \quad \text{The gain was 3.5\% of \$20,000.}$$

- b. Based on the gains and losses on their investment during this 5-year period, over what period of time was their investment not doing well? How do you know? What factors might contribute to this?

*The investment was not doing well in 2009. There were losses on the investment for both halves of the year, and \$500 was taken out of the account. It could be because the economy was doing badly, and a recession affected the investment's performance.*

- c. In math class, Jaheim and Frank were working on finding the total amount of the investment after 5 years. As a final step, Jaheim subtracted \$150 for administrative fees from the balance he arrived at after adding in all the deposits and subtracting out the one withdrawal and Broker's fee. For every semi-annual statement, Frank subtracted \$15 from the account balance for the administrative fee. Both boys arrived at the same ending 5-year balance. How is this possible? Explain.

*Jaheim took the \$15 fee and multiplied it by 10, since there were 10 statements, and deducted the \$150 total. Frank subtracted \$15 from the account balance for each statement. That was 10 times. So, both ways produce the same result: reducing the account balance by \$150, overall.*

- d. Based on the past statements for their investment account, predict what activity you might expect to see on Adrienne and Justin's January–June 2013 account statement. Then record it in the register to arrive at the balance as of June 30, 2013.

*I predict the account will continue to produce gains. The gains have been around \$900 for the past four statements, so I predict it will be about \$900 again, since it decreased by a little bit the last time, and there was a \$909.71 gain the last time. If I take away \$15 for the administrative fee, the balance would go up by \$885 and it would be \$29,003.59.*

- e. Using the answer from part (d), if their daughter's college bill is due in September of 2013, how much money do you estimate will be in their investment account at the end of August 2013 before the college bill is paid? Support your answer.

*Their investment could gain more money for July and August. Right now, it is gaining about \$900 per statement. If I divide that by 6, it equals \$150 (which is the average gain per month). So, for July and August I estimate that it will earn about another \$300 (including the \$15 fee), so there might be \$29,333.59 in the account.*

## Scaffolding:

- Review or reiterate that the operation associated with payments is subtraction, and the operation associated with deposits is addition.

**Exercise (10 minutes)****MP.2**

Students are given a transaction log of a business entertainment account. The transactions are completed for the students and the ending balance is given as well. Students are required to work “backwards” to find the beginning balance.

**Exercise**

Below is a transaction log of a business entertainment account. The transactions are completed and the ending balance in the account is \$525.55. Determine the beginning balance.

DATE	DESCRIPTION OF TRANSACTION	PAYMENT	DEPOSIT	BALANCE
	Beginning Balance	---	---	801.02
12/1/10	Bargain Electronic (I-Pod)	199.99		601.03
12/5/10	Lenny's Drive-Up (Gift Certificate)	75.00		526.03
12/7/10	Check from Customer: Reynolds		200.00	726.03
12/15/10	Pasta House (Dinner)	285.00		441.03
12/20/10	Refund from Clear's Play House		150.00	591.03
12/22/10	Gaffney's Tree Nursery	65.48		525.55

- When the beginning balance was given, the transactions were subtracted from the balance if a payment was made and the deposits were added to the balance if a deposit was made. How does that process change when the ending balance is given and the challenge is to find the beginning balance?
  - *If the ending balance is given and a payment was made then you need to add the payment to the ending balance to get the beginning balance. Likewise, if the ending balance is given and a deposit was made, then you need to subtract the payment from the ending balance to get the beginning balance.*
- Model the process described in the previous question by writing and solving an equation for the deposit made from the refund from Clear's Play House. Assume the preceding balance was  $x$ .
  - *Refund from Clear's Play House:*

$$x + 150 = 591.03$$

$$x + 150 - 150 = 591.03 - 150$$

$$x = 441.03$$
- What happens if the deposit amount is greater than the ending balance? How can this be written?
  - *If the deposit is greater than the ending balance then the beginning balance would be less than 0 and written as a negative number. This negative number indicates owing money.*

**Closing (5 minutes)**

Answer any remaining questions.

- What role do rational numbers play in solving real-world problems?
  - *Answers will vary, but students will most likely talk about recording investments and transactions.*

**Lesson Summary**

- Calculations with rational numbers are used when recording investment transactions.
- Deposits are added to an account balance; money is deposited into the account.
- Gains are added to an account balance; they are positive returns on the investment.
- Withdrawals are subtracted from an account balance; money is taken out of the account.
- Losses are subtracted from an account balance; they are negative returns on the investment.
- Fees are subtracted from an account balance; the bank or financial company is charging you for a service.

**Exit Ticket (5 minutes)**

Name \_\_\_\_\_

Date \_\_\_\_\_

## Lesson 20: Investments—Performing Operations with Rational Numbers

### Exit Ticket

1. Using the incomplete register below, work forward and backward to determine the beginning and ending balances after the series of transactions listed.

DATE	DESCRIPTION OF TRANSACTION	PAYMENT	DEPOSIT	BALANCE
	Beginning Balance	---	---	
1/31/12	Paycheck		350.55	
2/1/12	Gillian's Chocolate Factory (Candy)	32.40		685.26
2/4/12	Main Street Jeweler's	425.30		
2/14/12	Saratoga Steakhouse	125.31		

2. Write an expression to represent the balance after the paycheck was deposited on 1/31/12. Let  $x$  represent the beginning balance.
3. Write a numerical expression to represent the balance after the transaction for Main Street Jeweler's was made.



## Exit Ticket Sample Solutions

1. Using the incomplete register below, work forwards and backwards to determine the beginning and ending balances after the series of transactions listed.

DATE	DESCRIPTION OF TRANSACTION	PAYMENT	DEPOSIT	BALANCE
	Beginning Balance	---	---	367.11
1/31/12	Paycheck		350.55	717.66
2/1/12	Gillian's Chocolate Factory (Candy)	32.40		685.26
2/4/12	Main Street Jeweler's	425.30		259.96
2/14/12	Saratoga Steakhouse	125.31		134.65

2. Write an expression to represent the balance after the paycheck was deposited on 1/31/12. Let  $x$  represent the beginning balance.

$$x + 350.55$$

3. Write a numerical expression to represent the balance after the transaction for Main Street Jeweler's was made.

$$685.26 - 425.30$$

## Problem Set Sample Solutions

1. You are planning a fundraiser for your student council. The fundraiser is a Glow in the Dark Dance. Solve each entry below and complete the transaction log to determine the ending balance in the student account.

- a. The cost of admission to the dance is \$7 per person. Write an expression to represent the total amount of money collected for admission. Evaluate the expression if 250 people attended the dance.

$$p = \text{number of people attending dance}$$

$$7p$$

$$7(250) = 1,750$$

- b. The following expenses were necessary for the dance, and checks were written to each company.

- DJ for the dance – "Music Madness DJ" costs \$200
- Glow Sticks for "Glow World Inc." for the first 100 entrants. Cost of glow sticks were \$0.75 each plus 8% sales tax.

Complete the transaction log below based on this information

DATE	DESCRIPTION OF TRANSACTION	PAYMENT	DEPOSIT	BALANCE
	Beginning Balance	---	---	1,243.56
	Dance Admission		1,750.00	2,993.56
	DJ Music Madness	200.00		2,793.56
	Glow Sticks from Glow World	81.00		2,712.56

- c. Write a numerical expression to determine the cost of the glow sticks.

$$\frac{8}{100} \times 0.75 = 0.06$$

$$\text{cost} = 0.75 + 0.06 = 0.81 \text{ each}$$

$$100(0.81) = \$81$$

Analyze the results

- d. Write an algebraic expression to represent the profit earned from the fundraiser. (Profit is the amount of money collected in admissions minus all expenses.)

$$7p - 200 - 81$$

$$7p + (-200) + (-81)$$

$$7p + (-281) \text{ or } 7p - 281$$

- e. Evaluate the expression to determine the profit if 250 people attended the dance. Use the variable  $p$  to represent the number of people attending the dance (from part (a)).

$$7p + (-281)$$

$$7(250) + (-281)$$

$$1,750 + (-281)$$

$$1,469$$

The profit is \$1,469.

- f. Using the transaction log above, what was the amount of the profit earned?

$$2,712.56 - 1,243.56 = 1,469 \quad \text{The profit is \$1,469.}$$

2. The register below shows a series of transactions made to an investment account. Vinnie and Anthony both completed the register in hopes of finding the beginning balance. As you can see, they do not get the same answer. Who was correct? What mistake did the other person make? What was the monthly gain or loss?

Original Register

DATE	DESCRIPTION OF TRANSACTION	PAYMENT	DEPOSIT	BALANCE
	Beginning Balance	---	---	
3/1/11	Broker's Fee	250.00		
3/10/11	Loan Withdrawal	895.22		
3/15/11	Refund – Misc. Fee		50.00	
3/31/11	Investment Results		2,012.22	18,917.00

## Vinnie's Work

DATE	DESCRIPTION OF TRANSACTION	PAYMENT	DEPOSIT	BALANCE
	Beginning Balance	---	---	18,000.00
3/1/11	Broker's Fee	250.00		17,750.00
3/10/11	Loan Withdrawal	895.22		16,854.78
3/15/11	Refund – Misc. Fee		50.00	16,904.78
3/31/11	Investment Results		2,012.22	18,917.00

## Anthony's Work

DATE	DESCRIPTION OF TRANSACTION	PAYMENT	DEPOSIT	BALANCE
	Beginning Balance	---	---	19,834.00
3/1/11	Broker's Fee	250.00		20,084.00
3/10/11	Loan Withdrawal	895.22		20,979.22
3/15/11	Refund – Misc. Fee		50.00	20,929.22
3/31/11	Investment Results		2,012.22	18,917.00

*The correct register is Vinnie's.*

*Anthony made the mistake of using the operations for moving forward. He added the deposits and subtracted the payments, but since he was working backward in the problem, he needed to do just the opposite.*

*The monthly gain was \$917. This was a gain because the ending balance was greater than the beginning balance, and the amount of the gain was calculated by  $18,917 - 18,000 = 917$ .*