Lesson 19: Writing, Evaluating, and Finding Equivalent Expressions with Rational Numbers

Student Outcomes

- Students create equivalent forms of expressions in order to see structure, reveal characteristics, and make connections to context.
- Students compare equivalent forms of expressions and recognize that there are multiple ways to represent the context of a word problem.
- Students write and evaluate expressions to represent real-world scenarios.

Lesson Notes

Students should round to the nearest penny whenever necessary. Make sure this is emphasized throughout the lesson.

Classwork

Example 1 (10 minutes): Tic-Tac-Toe Review

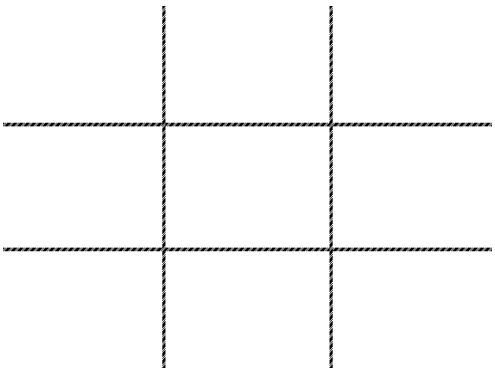
Begin by having students play an equivalent expression Tic-Tac-Toe game. Have students randomly fill in the 9 spots on their game boards with an expression from the student list of 10. Once students have their game boards filled in, show them an expression from the teacher list. Have students then find and mark (with an X) all equivalent expressions on their game boards. A student wins the game by getting 3 in a row.

Suggestion: Go through all of the expressions for practice even if the game is won before the end. The expression $1(x + 2) + 2(x - 2)$ from the teacher's list is equivalent to $3x - 2$, which is not on the students' game board. Discuss with students why and how $3x - 2$ is not the same as $3(x - 2)$.

Example 1: Tic-Tac-Toe Review

Fill in the 9 spaces with one expression from the list below. Use one expression per space. You will use 9 of the expressions:

$12 - 4x$
$8x + 4 - 12x$
$8\left(\frac{1}{2}x - 2\right)$
$12 - 6x + 2x$
$-4x + 4$
$x - 2 + 2x - 4$
$4x - 12$
$4(x - 4)$
$3(x - 2)$
$0.1(40x) - \frac{1}{2}(24)$



Teacher List

$2x + 2(x - 6)$

$4x - 16$

$1(x + 2) + 2(x - 2)$

$4(3 - x)$

$4(2x + 1) - 12x$

$3x - 6$

Example 2 (12 minutes)

Students complete the first row by using their knowledge of percents and discounts to find the discount amount and new price when the original price is given. Students then write a numerical and/or equivalent expression to find the new price of different items whose original price is given. The teacher leads the discussion in showing students how the problem can be solved both by arithmetic, as well as visually, using a tape diagram. Students extend this by creating expressions that combine discounts (and include sales tax using whichever approach they prefer).

Example 2

Original Price (100%)	Discount Amount (20%) Off	New Price (Pay 80%)	Expression
100	$100(0.20) = 20$	$100 - 20 = 80$	$100 - 100(0.20)$ $100(1 - 0.20)$ $100(0.80)$
50	$50(0.20) = 10$	$50 - 10 = 40$	$50 - 50(0.20)$ $50(1 - 0.20)$ $50(0.80)$
28	$28(0.20) = 5.60$	$28 - 5.60 = 22.40$	$28 - 28(0.20)$ $28(1 - 0.20)$ $28(0.80)$
14.50	$14.50(0.20) = 2.90$	$14.50 - 2.90 = 11.60$	$14.50 - 14.50(0.20)$ $14.50(1 - 0.20)$ $14.50(0.80)$
x	$x(0.20) = 0.20x$	$x - 0.20x$	$x - 0.20x$ $x(1 - 0.20)$ $x(0.80)$

Discussion

A discount is an amount that is subtracted from the original price.

- If you know the original cost of an item, how do you find the discount amount by using a picture and by using arithmetic?
  - *Answers will vary.*

The intent is for students to complete the first row, before the teacher leads a discussion on how to find the discount with both a picture and arithmetic. After that, students may use whichever method they prefer. Some students may choose to calculate 10% of the total and then double it to find 20%.

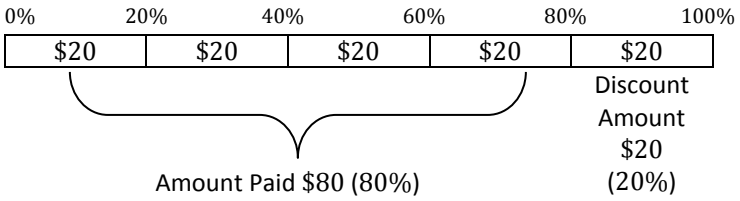
- Picture: 20% off of $\$100$.

$\frac{20}{100} = \frac{1}{5}$

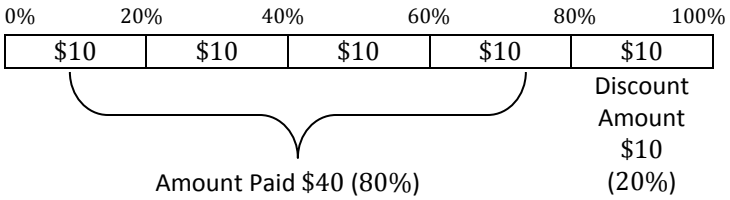
Make a tape diagram and break the whole into 5 parts, each part representing 20%.

Then divide the total amount of money into 5 parts. The discount is the amount represented in one of the parts; the amount paid is the remaining parts.

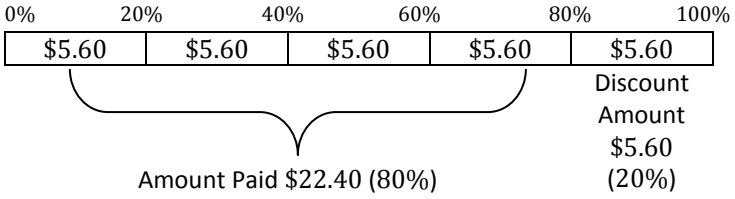
$\$100$



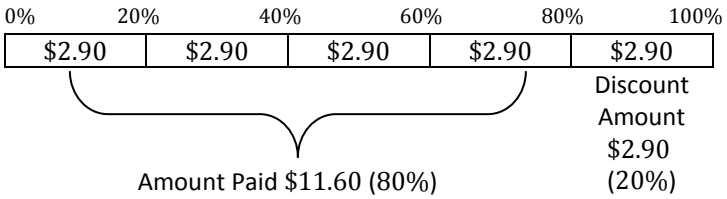
$\$50$



$\$28$



$\$14.50$



Arithmetic: Calculate the amount of discount that corresponds to the discount % using any method. Then subtract this value from the original amount.

- How do you set up a tape diagram for a percent that is not a factor of 100? For example 30%.
  - *Determine the greatest common factor of the percent and 100. Divide 100 by the greatest common factor and that will determine into how many parts to break the tape diagram.*
  - *Since 30 is not a factor of 100, find the greatest common factor of 30 and 100. The greatest common factor of 30 and 100 is 10. Therefore, when 100 is divided by the greatest common factor of 10, the result of 10 indicates how many parts into which to break the tape diagram.*
- What is the process to find a percent of a number without using a tape diagram?
  - *Multiply the whole by the percent as a fraction out of 100, or multiply the whole by the percent, written as a decimal.*
- Under what circumstances would you prefer to use a tape diagram to help you calculate the percent of a number?
  - *Finding the percent of a number using arithmetic is sometimes quicker than using a tape diagram. Using a tape diagram would be most beneficial when the percent and 100 have a greatest common factor and when the GCF isn't so small that it divides the tape diagram into numerous parts.*
- When the original price is not known, how can an expression be used to represent the new price?
  - *When the original price is unknown, it can be represented by a variable such as x. To write an expression that represents the new price, the discount amount must be subtracted from the original amount. The expression can then be written as an equivalent expression.*
- When a discount of 20% is being deducted, what percent is being paid? How do you know?
  - *The amount being paid would be 80%. We know this because an item not on sale represents 100%. If there is a discount of 20%, then the overall price would be 20% less than the original 100%. To find this, subtract 20% from 100% and the difference is the percent that is paid.*
- How is $x - 0.2x = 0.8x$?
  - *When the expression $x - 0.2x$ is written as an equivalent expression, you know that x represents $1x$ and when you subtract $0.2x$ from $1x$ the result is $0.8x$.*
- Describe the meaning of $x - 0.2x = 0.8x$ in the context of the problem.
  - *The original price of the item is unknown, represented by x. If the item is on sale for 20% then the percent that is paid is 80%. $x - 0.20x$ represents the original price less the discount amount which will equal the new price. The new price is the price that is paid, which is 80% of the original cost, which is represented as $0.8x$.*

Example 3 (5 minutes)

Example 3

An item that has an original price of x dollars is discounted 33%.

a. Write an expression that represents the amount of the discount.

$0.33x$

b. Write two equivalent expressions that represent the new, discounted price.

$x - 0.33x$

$x(1 - 0.33)$

$x(0.67)$

c. Use one of your expressions to calculate the new, discounted price if the original price was $\$56$.

$0.67x$

$0.67(56)$

37.52

d. How would the expressions you created in parts (a) and (b) have to change if the item's price had increased by 33% instead of decreased by 33%?

Instead of subtracting $0.33x$, you would have to add for the increase. The expression would be

$x + 0.33x$

$1.33x$

Example 4 (10 minutes)

Discussion

Generate a classroom discussion about a new concept — the concept of sales tax. Discuss what it is, the purpose of it, and how it is calculated.

MP.1 & MP.2

Once the students have a general understanding that the sales tax is a number added to the cost of an item and it is found by finding the sales rate (%) of the item and added to the cost, lead students through the second chart, which is an extension of the first.

Example 4

Original Price (100%)	Discount (20%) off	Amount Pay (pay 80%)	Expression	New Price	Sales Tax (8%)	Overall Cost	Expression
100	20	80	$100 - 100(0.20) = 100(0.80)$				
50	10	40	$50 - 50(0.20) = 50(0.80)$				
28	5.60	22.40	$28 - 28(0.20) = 28(0.80)$				
14.50	2.90	11.60	$14.50 - 14.50(0.20) = 14.50(0.80)$				
x	$0.20x$	$x - 0.20x$	$x - 0.20x = 0.80x$				

- If a tape diagram were used to model the sales tax, into how many parts would the tape diagram need to be broken? Explain how you knew that.
  - *Since the GCF of 8 and 100 is 4, the tape diagram would need to broken into $\frac{100}{4} = 25$ parts. This is not the easiest or most efficient way of finding the sales tax.*

- What is 1% of 80?
  - 0.80
- If you can find 1% of 80 easily, how can you use that answer to find 8% of 80?
  - *Multiply by* 8*, because* 1% *multiplied by* 8 *will give* 8%*.*

    $0.80(8) = 6.40$
  - *Arithmetic:*

    $80 \cdot 0.08 = 6.40$

    $80 + 6.40 = 86.40$

Overall expression:

- What was the expression for the discount?
  - $100(0.80)$
- Using the previous expression, write an expression to determine the amount of the sales tax?
  - $\big(100(0.80)\big)(0.08)$
- Would it change the final price of the item if the sales clerk charged the sales tax first and then discounted the item? Why do you think this is the case?
  - *No the order wouldn't matter. If the sales tax was calculated first, then the discount would be calculated on both the original price of the item and on the sales tax as well.*
- Describe the process for calculating the final cost of an item, which has been discounted 20% and was sold in a state that has a sales tax of 8%.
  - *Step 1: First take the original amount and multiply by* 0.20 *to figure out the discount amount.*
  - *Step 2: Use that amount from Step 1 and subtract from the original amount.*
  - *Step 3: Use the new amount from Step 2 (original* $-$ 0.20*(original)) and multiply by* 0.08 *to figure out the sales tax amount.*
  - *Step 4: Use that new amount from Step 3 and add to the discounted price from Step 2.*

- Using the steps you described, write an expression to represent the price paid after a 20% discount and 8% sales tax if the original price was $\$100$. Describe in words what is being found at each step.

1.	$100(0.20)$	*Finding the discount amount.*
2.	$100 - 100(0.20)$	*Finding the discount price after* 20% *is deducted.*
3.	$(100 - 100(0.20))(0.08)$	*Finding the sales tax of* 8% *on the new discounted price.*
4.	$(100 - 100(0.20)) + 0.08\big(100 - 100(0.20)\big)$	*Finding the total paid after finding the discounted price, sales tax on that discounted price, and adding them together.*
5.	$1.08\big(100(0.80)\big)$	

- Using the same steps, write an expression to represent the price paid if the original price is $\$50$ with a 20% discount and 8% sales tax. Describe in words what is being found at each step.

1. $50(0.20)$	*Finding the discount amount.*
2. $50 - 50(0.20)$	*Finding the discount price after 20% is deducted.*
3. $(0.08)\big(50 - 50(0.20)\big)$	*Finding the sales tax of 8% on the new discounted price.*
4. $(50 - 50(0.20)) + (0.08)\big(50 - 50(0.20)\big)$	*Finding the total paid after finding the discounted price, sales tax on that discounted price, and adding them together.*
5. $1.08\big(50(0.80)\big)$	

- Using the same steps, write an expression to represent the price paid if the original price is $\$28$ with a 20% discount and 8% sales tax. Describe in words what is being found at each step.

1. $28(0.20)$	*Finding the discount amount.*
2. $28 - 28(0.20)$	*Finding the discount price after 20% is deducted.*
3. $(0.08)(28 - 28(0.20))$	*Finding the sales tax of 8% on the new discounted price.*
4. $\big(28 - 28(0.20)\big) + 0.08\big(28 - 28(0.20)\big)$	*Finding the total paid after finding the discounted price, sales tax on that discounted price, and adding them together.*
5. $1.08\big(28(0.80)\big)$	

- Using the same steps, write an expression to represent the price paid if the original price is $\$14.50$ with a 20% discount and 8% sales tax. Describe in words what is being found at each step.

1. $14.50(0.20)$	*Finding the discount amount.*
2. $14.50 - 14.50(0.20)$	*Finding the discount price after 20% is deducted.*
3. $0.08\big(14.50 - 14.50(0.20)\big)$	*Finding the sales tax of 8% on the new discounted price.*
4. $\big(14.50 - 14.50(0.20)\big) + 0.08\big(14.50 - 14.50(0.20)\big)$	*Finding the total paid after finding the discounted price, sales tax on that discounted price, and adding them together.*
5. $1.08\big(14.50(0.80)\big)$	

- Using the same steps, write an expression to represent the price paid if the original price is x with a 20% discount and 8% sales tax. Describe in words what is being found at each step.

1. $x(0.20)$ — *Finding the discount amount.*
2. $x - 0.20x$ — *Finding the discount price after 20% is deducted.*
3. $0.08(x - 0.20x)$ — *Finding the sales tax of 8% on the new discounted price.*
4. $(x - 0.20x) + 0.08(x - 0.20)$ — *Finding the total paid after finding the discounted price, sales tax on that discounted price, and adding them together.*
5. $1.08(0.80x)$

Original Price (100%)	Discount (20%) off	Amount Pay (pay 80%)	Expression	New Price	Sales Tax (8%)	Overall Cost	Expression
100	20	80	$100 - 100(0.20) = 100(0.80)$	80	$80(0.08) = 6.40$	$80 + 6.40 = 86.40$	$(100 - 100(0.20)) + 0.08(100 - 100(0.20))$ *or* $1.08(100 - 100(0.20))$ *or* $1.08(100(0.80))$
50	10	40	$50 - 50(0.20) = 50(0.80)$	40	$40(0.08) = 3.20$	$40 + 3.20 = 43.20$	$(50 - 50(0.20)) + 0.08(50 - 50(0.20))$ *or* $1.08(50 - 50(0.20))$ *or* $1.08(50(0.80))$
28	5.60	22.40	$28 - 28(0.20) = 28(0.80)$	22.40	$22.40(0.08) = 1.79$	$22.40 + 1.79 = 24.19$	$(28 - 28(0.20)) + 0.08(28 - 28(0.20))$ *or* $1.08(28 - 28(0.20))$ *or* $1.08(28(0.80))$
14.50	2.90	11.60	$14.50 - 14.50(0.20) = 14.50(0.80)$	11.60	$11.60(0.08) = 0.93$	$11.60 + 0.93 = 12.53$	$(14.50 - 14.50(0.20)) + 0.08(14.50 - 14.50(0.20))$ *or* $1.08(14.50 - 14.50(0.20))$ *or* $1.08(14.50(0.80))$
x	$0.20x$	$x - 0.20x$	$x - 0.20x = 0.80x$	$x - 0.20x$	$(x - 0.20x)(0.08)$	$(x - 0.20x) + (x - 0.20x)(0.08)$	$(x - 0.20x) + (x - 0.20x)(0.08)$ *or* $1.08(x - 0.20x)$ *or* $1.08(0.80x)$

Discussion

- Describe the meaning of the expression $(x - 0.20x)$?
  - *A number reduced by 20%.*
- Describe why $\big((x - 0.20x) + 0.08(x - 0.20)\big)$ is equivalent to $1.08(x - 0.20x)$.
  - *In the first expression, $(x - 0.20x)$ gives us the discounted price of the item, and we are adding that value to 8% of the discounted price.*

- Describe why $(x - 0.20x) + 0.08(x - 0.20x)$ and $1.08(x - 0.20x)$ are equivalent to $1.08(0.80x)$.
  - *The expression gives 108% of the discounted price, which is equivalent to the discounted price of the item plus 8% of the discounted price of the item.*

Closing (3 minutes)

- Describe how to write an expression that incorporates the use of multiple percents.
- Describe how expressions with percents can be written as equivalent expressions.

Lesson Summary

- **Two expressions are equivalent if they yield the same number for every substitution of numbers for the letters in each expression.**
- **The expression that allows us to find the cost of an item after the discount has been taken and the sales tax has been added is written by representing the discount price added to the discount price multiplied by the sales tax rate.**

Exit Ticket (5 minutes)

Name _____ Date_____

Lesson 19: Writing, Evaluating, and Finding Equivalent Expressions with Rational Numbers

Exit Ticket

Write three equivalent expressions that can be used to find the final price of an item costing g dollars that is on sale for 15% off and charged 7% sales tax.

1. Using all of the expressions, determine the final price for an item that costs \$75. If necessary, round to the nearest penny.

2. If each expression yields the same final sale price, is there anything to be gained by using one over the other?

3. Describe the benefits, special characteristics, and properties of each expression.

Exit Ticket Sample Solutions

Write three equivalent expressions that can be used to find the final price of an item costing g dollars that is on sale for 15% off and charged 7% sales tax.

$(x - 0.15x) + 0.07(x - 0.15x)$ $\quad$ $1.07(x - 0.15x)$ $\quad$ $1.07(0.85x) = 0.85(1.07)x$

1. **Using all of the expressions, determine the final price for an item that costs \$75. If necessary, round to the nearest penny.**

$x = \$75$	$(x - 0.15x) + 0.07(x - 0.15x)$	$1.07(x - 0.15x)$	$1.07(0.85x) = 0.85(1.07)x$
	$(75 - 0.15(75)) + 0.07(75 - 0.15(75))$	$1.07(75 - 0.15 * 75))$	$1.07(0.85 * 75)$
	$63.75 + 0.07(63.75)$	$1.07(63.75)$	$1.07(63.75)$
	$63.75 + 4.46$	68.21	68.21
	68.21		

2. **If each expression yields the same final sale price, is there anything to be gained by using one over the other?**

   Using the final two expressions makes the problem shorter and offers fewer areas to make errors. However, all three expressions are correct.

3. **Describe the benefits, special characteristics, and properties of each expression.**

   The second and third expressions collect like terms. The third expression can be written either way using the commutative property of multiplication. The first and second expressions find the discount price first, where the third expression is written in terms of percent paid.

Problem Set Sample Solutions

Solve the following problems. If necessary, round to the nearest penny.

1. **A family of 12 went to the local Italian restaurant for dinner. Every family member ordered a drink and meal, 3 ordered an appetizer, and 6 people ordered cake for dessert.**

   a. **Write an expression that can be used to figure out the cost of the bill. Include the definitions for the variables the server used.**

   d = drink

   m = meal

   a = appetizer

   c = cake

   $12d + 12m + 3a + 6c$

   b. **The waitress wrote on her ordering pad the following expression: $3(4d + 4m + a + 2c)$.**

   Was she correct? Explain why or why not.

   Yes, she was correct because her expression is equivalent to expression from part (a). If the distributive property is applied, the expressions would be exact.

c. What is the cost of the bill if a drink costs $\$3$, a meal costs $\$20$, an appetizer costs $\$5.50$, and a slice of cake costs $\$3.75$?

$12d + 12m + 3a + 6c$

$12(3) + 12(20) + 3(5.50) + 6(3.75)$

$36 + 240 + 16.50 + 22.50$

$\$315$

d. Suppose the family had a 10% discount coupon for the entire check and then left an 18% tip. What is the total?

$(315 - 315(0.10)) + 0.18(315 - 315(0.10))$

$1.18(315 - 315(0.10))$

$1.18(315(0.90))$

$\$334.53$

2. Sally designs web pages for customers. She charges $\$135.50$ per web page; however, she must pay a monthly rental fee of $\$650$ for her office. Write an expression to determine her take-home pay after expenses. If Sally designed 5 web pages last month, what was her take-home pay after expenses?

w = number of webpages Sally's designs

$135.50w - 650$

$135.50(5) - 650$

$\$27.50$

3. While shopping, Megan and her friend Rylie find a pair of boots on sale for 25% off of the original price. Megan calculates the final cost of the boots by first deducting the 25%, and then adding the 6% sales tax. Rylie thinks Megan will pay less if she pays the 6% sales tax first and then takes the 25% discount.

a. Write an expression to represent each girl's scenario if the original price of the boots was x dollars.

Megan	*Rylie*
$(x - 0.25x) + 0.06(x - 0.25x)$	$(x + 0.06x) - 0.25(x + 0.06x)$
$1.06(x - 0.25x)$	$0.75(x + 0.06x)$
$1.06(0.75x)$	$0.75(1.06x)$

b. Evaluate each expression if the boots originally cost $\$200$.

Megan	*Rylie*
$1.06(0.75x)$	$0.75(1.06x)$
$1.06(0.75(200))$	$0.75(1.06(200))$
$\$159$	$\$159$

c. Who was right? Explain how you know.

Neither girl was right. They both pay the same amount.

d. Explain how both girls' expressions are equivalent.

Two expressions are equivalent if they yield the same number for every substitution of numbers for the variables in each expression. Since multiplication is commutative, the order of the multiplication can be reversed and the result will remain the same.