Lesson 17: Comparing Tape Diagram Solutions to Algebraic Solutions

Student Outcomes

- Students use tape diagrams to solve equations of the form $px + q = r$ and $p(x + q) = r$, (where p, q, and r are *small positive* integers), and identify the sequence of operations used to find the solution.
- Students translate word problems to write and solve algebraic equations using tape diagrams to model the steps they record algebraically.

Lesson Notes

In Lesson 17, students relate their algebraic steps in solving an equation to the steps they take to arrive at the solution arithmetically. They refer back to the use of tape diagrams to conceptually understand the algebraic steps taken to solve an equation. It is not until Lesson 21 that students use the Properties of Equality to formally justify performing the same operation to both sides of the equation. Teachers need to have scenarios ready for each group. There should be enough copies of each scenario for each person in the group so they can bring this information with them to the next group.

Classwork

Exploratory Challenge (30 minutes): Expenses on Your Family Vacation

MP.1 & MP.4

Divide students into seven groups. Each group is responsible for one of the seven specific expense scenarios. In these groups, students write algebraic equations and solve by modeling the problem using a tape diagram. Then have groups collaborate to arrive at the sequence of operations used to find the solution. Lastly, challenge the students to show an algebraic solution to the same problem. Once groups work on an individual scenario, mix up the groups so that each group now has seven students (i.e., one student to represent each of the seven expenses). Within each group, students present their specific scenario to the other members of the group: the solution and model used to find the solution, the sequence of operations used and a possible algebraic solution. After all scenarios have been shared and each student completes the summary sheet, have students answer the questions regarding total cost for several different combinations.

Scaffolding:

Review how to set up a tape diagram when given the parts and total.

Exploratory Challenge: Expenses on Your Family Vacation

John and Ag are summarizing some of the expenses of their family vacation for themselves and their three children, Louie, Missy, and Bonnie. Write an algebraic equation, create a model to determine how much each item will cost using all of the given information, and answer the questions that follow.

Expenses:

Car and insurance fees: $\$400$	**Airfare and insurance fees:** $\$875$	**Motel and tax:** $\$400$
Baseball game and hats: $\$103.83$	**Movies for one day:** $\$75$	**Soda and pizza:** $\$37.95$
	Sandals and t-shirts: $\$120$	

Your Group's Scenario Solution:

Scenario 1

During one rainy day on the vacation, the entire family decided to go watch a matinee movie in the morning and a drive-in movie in the evening. The price for a matinee movie in the morning is different than the cost of a drive-in movie in the evening. The tickets for the matinee morning movie cost $6 each. How much did each person spend that day on movie tickets if the ticket cost for each family member was the same? What was the cost for a ticket for the drive-in movie in the evening?

Algebraic Equation & Solution

Morning matinee movie: $\$6$ *each*

Evening Drive-In Movie: e each

$$5(e+6)=75$$
$$5e+30=75$$
$$5e+30-30=75-30$$
$$5e+0=45$$
$$\left(\frac{1}{5}\right)5e=45\left(\frac{1}{5}\right)$$
$$1e=9$$
$$e=9$$

OR

$$5(e+6)=75$$
$$\left(\frac{1}{5}\right)5(e+6)=75\left(\frac{1}{5}\right)$$
$$e+6=15$$
$$e+6-6=15-6$$
$$e=9$$

Tape Diagram

$$\overbrace{\hspace{10em}}^{75}$$

$e+6$	$e+6$	$e+6$	$e+6$	$e+6$
John	*Ag*	*Louie*	*Missy*	*Bonnie*

$5e$

$5(6)=30$

$75-30=45$

$45\div5=9$

The total each person spent on movies in one day was $\$15$. *The evening drive-in movie costs* $\$9$ *each.*

Scenario 2

For dinner one night, the family went to the local pizza parlor. The cost of a soda was $3. If each member of the family had a soda and one slice of pizza, how much did one slice of pizza cost?

Algebraic Equation & Solution

One Soda: $\$3$

Slice of Pizza: p *dollars*

$$5(p+3)=37.95$$
$$5p+15=37.95$$
$$5p+15-15=37.95-15$$
$$5p+0=22.95$$
$$\left(\frac{1}{5}\right)5p=22.95\left(\frac{1}{5}\right)$$
$$1p=4.59$$
$$p=4.59$$

OR

$$5(p+3)=37.95$$
$$\left(\frac{1}{5}\right)5(p+3)=(37.95)\left(\frac{1}{5}\right)$$
$$p+3=7.59$$
$$p+3-3=7.59-3$$
$$p=4.59$$

Tape Diagram

$$\overbrace{\hspace{10em}}^{37.95}$$

$p+3$	$p+3$	$p+3$	$p+3$	$p+3$
John	*Ag*	*Louie*	*Missy*	*Bonnie*

$5(3)=15$

$5p$

$37.95-15=22.95$

$22.95\div5=4.59$

One slice of pizza costs $\$4.59$.

Scenario 3

One night, John, Louie and Bonnie went to the see the local baseball team play a game. They each bought a game ticket and a hat that cost $\$10$. How much was each ticket to enter the ballpark?

Algebraic Equation & Solution

Ticket: t dollars

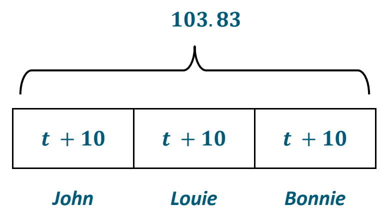
Hat: $\$10$

$$3(t + 10) = 103.83$$
$$3t + 30 = 103.83$$
$$3t + 30 - 30 = 103.83 - 30$$
$$3t + 0 = 73.83$$
$$\left(\frac{1}{3}\right)3t = 73.83\left(\frac{1}{3}\right)$$
$$1t = 24.61$$
$$t = 24.61$$

OR

$$3(t + 10) = 103.83$$
$$\left(\frac{1}{3}\right)3(t + 10) = (103.83)\left(\frac{1}{3}\right)$$
$$t + 10 = 34.61$$
$$t + 10 - 10 = 34.61 - 10$$
$$t = 24.61$$

Tape Diagram



$3(10) = 30$ $\quad 103.83 - 30 = 73.83$

$73.83 \div 3 = 24.61$

One ticket costs $\$24.61$.

Scenario 4

While John, Louie, and Bonnie went to see the baseball game, Ag and Missy went shopping. They bought a t-shirt for each member of the family and bought two pairs of sandals that cost $\$10$ a pair. How much was each t-shirt?

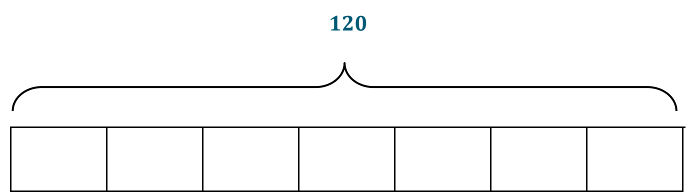
Algebraic Equation & Solution

T-Shirt: t dollars

Sandals: $2 \times \$10 = \$20$

$$5t + 20 = 120$$
$$5t + 20 - 20 = 120 - 20$$
$$5t + 0 = 100$$
$$\left(\frac{1}{5}\right)5t = 100\left(\frac{1}{5}\right)$$
$$1t = 20$$
$$t = 20$$

Tape Diagram



$2(10) = 20$ $\quad 120 - 20 = 100$ $\quad 100 \div 5 = 20$

One t-shirt costs $\$20$.

Scenario 5

The family was going to fly in an airplane to their vacation destination. Each person needs to have their own ticket for the plane, and also pay $\$25$ in insurance fees per person. What was the cost of one ticket?

Algebraic Equation & Solution

One ticket: t dollars

Insurance: $\$25$ per person

$$5(t + 25) = 875$$
$$5t + 125 - 125 = 875 - 125$$
$$5t + 0 = 750$$
$$5t = 750\left(\frac{1}{5}\right)$$
$$t = 150$$

OR

$$5(t + 25) = 875$$
$$\left(\frac{1}{5}\right)5(t + 25) = \left(\frac{1}{5}\right)(875)$$
$$t + 25 = 175$$
$$t + 25 - 25 = 175 - 25$$
$$t = 150$$

Tape Diagram

$$\overbrace{\qquad\qquad\qquad\qquad}^{875}$$

$t + 25$	$t + 25$	$t + 25$	$t + 25$	$t + 25$
John	*Ag*	*Missy*	*Louie*	*Bonnie*

$$5(25) = 125$$
$$875 - 125 = 750$$
$$750 \div 5 = 150$$

One ticket costs $\$150$.

Scenario 6

While on vacation, the family rented a car to get them to all the places they wanted to see for five days. The car costs a certain amount each day, plus a one-time insurance fee of $\$50$. How much was the daily cost of the car (not including the insurance fees)?

Algebraic Equation & Solution

Daily fee: d dollars

Insurance fee: $\$50$

$$5d + 50 = 400$$
$$5d + 50 - 50 = 400 - 50$$
$$5d + 0 = 350$$
$$\left(\frac{1}{5}\right)5d = 350\left(\frac{1}{5}\right)$$
$$1d = 70$$
$$d = 70$$

Tape Diagram

$$\overbrace{\qquad\qquad\qquad\qquad}^{400}$$

d	d	d	d	d	50
Day 1	*Day* 2	*Day* 3	*Day* 4	*Day* 5	*Insurance*

$$1(50) = 50$$
$$5d$$

$$400 - 50 = 350$$
$$350 \div 5 = 70$$

One day costs $\$70$.

Scenario 7

The family decided to stay in a motel for four nights. The motel charges a nightly fee plus $\$60$ in state taxes. What is the nightly charge with no taxes included?

Algebraic Equation & Solution

Nightly charge: n *dollars*

taxes: $\$60$

$$4n + 60 = 400$$
$$4n + 60 - 60 = 400 - 60$$
$$4n + 0 = 340$$
$$\left(\frac{1}{4}\right)4n = 340\left(\frac{1}{4}\right)$$
$$1n = 85$$
$$n = 85$$

Tape Diagram

400

n	n	n	n	60
Day 1	*Day* 2	*Day* 3	*Day* 4	*Taxes*

$4n$ $\quad 400 - 60 = 340 \quad 340 \div 4 = 85$

One night costs $\$85$.

Once students have completed their group scenario to determine the cost of the item, and the groups have been mixed so students have seen the problems and solutions to each expense, have them complete the summary chart and answer the questions that follow.

After collaborating with all of the groups, summarize the findings in the table below.

Cost of Evening Movie	$\$9$
Cost of 1 Slice of Pizza	$\$4.59$
Cost of the admission ticket to the baseball game	$\$24.61$
Cost of 1 T-Shirt	$\$20$
Cost of 1 Airplane Ticket	$\$150$
Daily Cost for Car Rental	$\$70$
Nightly Charge for Motel	$\$85$

Using the results, determine the cost of the following:

1. **A slice of pizza, 1 plane ticket, 2 nights in the motel, and 1 evening movie.**

   $4.59 + 150 + 2(85) + 9 = 333.59$

2. **One t-shirt, 1 ticket to the baseball game, and 1 day of the rental car.**

   $20 + 24.61 + 70 = 114.61$

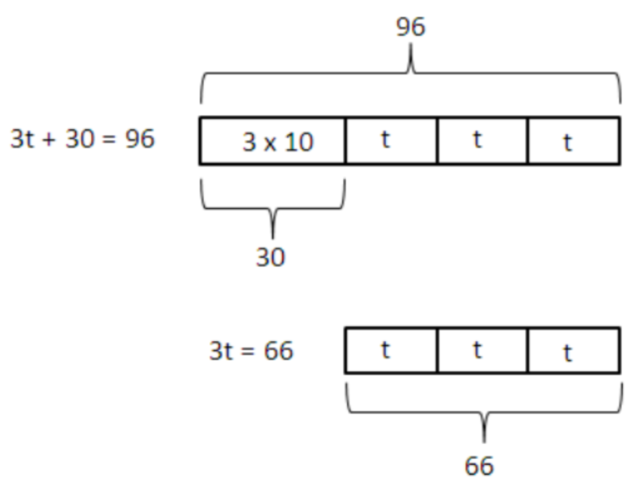
Discussion/Lesson Questions for Algebraic Approach

The importance of undoing addition and multiplication to get 0 and 1's (i.e., using the additive inverse undoes addition to get 0 and multiplicative inverse undoes multiplication by a non-zero number to get 1) should be stressed.

> *Scaffolding:*
> Review from Grade 6 solving 1-step and 2-step equations algebraically as well as the application of the distributive property.

MP.2

- When solving an equation with parentheses, order of operations must be followed. What property can be used to eliminate parentheses; for example, $3(a + b) = 3a + 3b$?
  - *To eliminate parentheses the distributive property must be applied.*
- Another approach to solving the problem is to eliminate the coefficient first. How would one go about eliminating the coefficient?
  - *To eliminate the coefficient you can multiply both sides by the reciprocal of the coefficient or divide both sides by the coefficient.*
- How do we *undo* multiplication?
  - *Multiply by the reciprocal of the coefficient of the variable or divide both sides of the equation by the coefficient.*
- What is the result when *undoing* multiplication in any problem?
  - *When undoing multiplication the result will always be* 1*, which is the multiplicative identity.*
- What mathematical property is being applied when *undoing* multiplication?
  - *Multiplicative inverse*
- What approach must be taken when solving for a variable in an equation and *undoing* addition is required?
  - *To undo addition you need to subtract the constant.*
- How can this approach be shown with a tape diagram?



- What is the result when "undoing" addition in any problem?
  - *The result will always be* 0*, which is the additive identity.*
- What mathematical property is being applied when "undoing" addition?
  - *Additive inverse*

- What mathematical property allows us to perform an operation (i.e., do the same thing) on both sides of the equation?
  - *Addition and multiplication properties of equality*
- How are the addition and multiplication properties of equality applied?
  - *The problem is an equation which means $A = B$. If a number is added or multiplied to both sides, then the resulting sum or product are equal to each other.*

Exercise (5 minutes)

Exercise

The cost of a babysitting service on a cruise is $\$10$ for the first hour and $\$12$ for each additional hour. If the total cost of babysitting baby Aaron was $\$58$, how many hours was Aaron at the sitter?

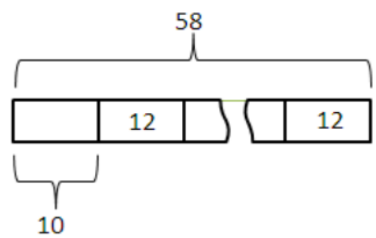
Algebraic Solution

h = number of additional hours

$$12h + 10 = 58$$
$$12h + 10 - 10 = 58 - 10$$
$$12h + 0 = 48$$
$$\left(\frac{1}{12}\right)(12h) = (48)\left(\frac{1}{12}\right)$$
$$1h = 4$$
$$h = 4$$

$$1 + 4 = 5$$

Tape Diagram



$10 + 12 + 12 = 34$ *(not enough, need 58)*

10	12	12	12

$10 + 12 + 12 + 12 = 46$ *(not enough, need 58)*

10	12	12	12	12

$10 + 12 + 12 + 12 + 12 = 58$

$58 - 10 = 48$

$48 \div 12 = 4$

Aaron was with the babysitter for 5 hours.

MP.4
- How can a tape diagram be used to model this problem?
  - *A tape diagram can be set up to show each hour and the cost associated with that hour. The total is known, so the sum of each column in the tape diagram can be calculated until the total is obtained.*

MP.1
- How is the tape diagram for this problem similar to the tape diagrams used in the previous activity?
  - *In all the problems, the total was given.*

MP.1

- How is the tape diagram for this problem different than the tape diagrams used in the Exploratory Challenge?
  - *In the previous exercise, we knew how many units there were, such as days, hours, people, etc. What was obtained was the amount for one of those units. In this tape diagram, we don't know how many units there are, but rather how much each unit represents. Therefore, to solve, we calculate the sum and increase the number of units until we obtain the given sum.*

Closing (3 minutes)

- How does modeling the sequence of operations with a tape diagram help to solve the same problem algebraically?
  - *The tape diagrams provide a visual model to demonstrate what we do when solving the problem algebraically.*
- What are the mathematical properties, and how are they used in finding the solution of a linear equation containing parenthesis?
  - *If a linear equation has parentheses, you can either solve using the distributive property or the multiplicative inverse.*

Lesson Summary

***Tape diagrams* can be used to model and identify the sequence of operations to find a solution algebraically.**

The goal in solving equations algebraically is to isolate the variable.

The process of doing this requires *undoing* addition or subtraction to obtain a 0 and *undoing* multiplication or division to obtain a 1. The additive inverse and multiplicative inverse properties are applied to get the 0 (the additive identity) and 1 (the multiplicative identity).

The addition and multiplication properties of equality are applied because in an equation, $A = B$, when a number is added or multiplied to both sides, the resulting sum or product remains equal.

Exit Ticket (7 minutes)

Complete one of the problems. Solve by modeling the solution with a tape diagram and write and solve an algebraic equation.

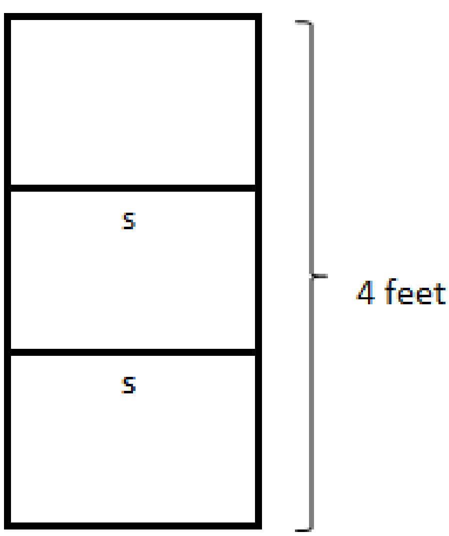
Name ____ Date____________________

Lesson 17: Comparing Tape Diagram Solutions to Algebraic Solutions

Exit Ticket

1. Eric's father works two part-time jobs, one in the morning and one in the afternoon, and works a total of 40 hours each 5-day workweek. If his schedule is the same each day, and he works 3 hours each morning, how many hours does Eric's father work each afternoon?

2. Henry is making a bookcase and has a total of 16 ft. of lumber. The left and right sides of the bookcase are each 4 ft. high. The top, bottom, and two shelves are all the same length, labeled S. How long is each shelf?



Exit Ticket Sample Solutions

1. Eric's father works two part-time jobs, one in the morning, and one in the afternoon, and works a total of 40 hours each 5-day work week. If his schedule is the same each day and he works 3 hours each morning, how many hours does Eric's father work each afternoon?

Algebraic Equation & Solution

Number of Afternoon hours: a

Number of Morning hours: 3

$$5(a+3)=40$$
$$5a+15-15=40-15$$
$$5a+0=25$$
$$\left(\frac{1}{5}\right)5a=25\left(\frac{1}{5}\right)$$
$$a=5$$

OR

$$5(a+3)=40$$
$$\left(\frac{1}{5}\right)5(a+3)=40\left(\frac{1}{5}\right)$$
$$a+3=8$$
$$a+3-3=8-3$$
$$a=5$$

Tape Diagram

$$\overbrace{\boxed{a+3}\boxed{a+3}\boxed{a+3}\boxed{a+3}\boxed{a+3}}^{40}$$

$a+3$	$a+3$	$a+3$	$a+3$	$a+3$
Day 1	*Day* 2	*Day* 3	*Day* 4	*Day* 5

$5(3)=15$ $\quad 40-15=25$ $\quad 25\div 5=5$

Eric's father works 5 *hours in the afternoon.*

2. Henry is making a bookcase and has a total of 16 ft. of lumber. The left and right sides of the bookcase are each 4 ft. high. The top, bottom, and two shelves are all the same length. How long is each shelf?

Algebraic Equation & Solution

Shelves: s *ft.*

Sides: 8 *ft.*

$$4s+8=16$$
$$4s+8-8=16-8$$
$$4s+0=8$$
$$\left(\frac{1}{4}\right)4s=8\left(\frac{1}{4}\right)$$
$$1s=2$$
$$s=2$$

Tape Diagram

$$\overbrace{\boxed{s}\boxed{s}\boxed{s}\boxed{s}\boxed{\ \ 4\ \ }\boxed{\ \ 4\ \ }}^{16}$$

$2(4)=8$ $\quad 16-8=8$ $\quad 8\div 4=2$

Each shelf is 2 *ft. long.*

Problem Set Sample Solutions

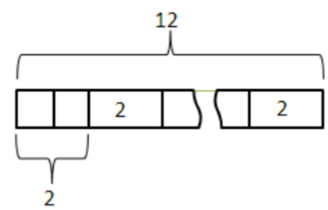
1. A taxi cab in Myrtle Beach charges $\$2$ per mile and $\$1$ for every person. If a taxi cab ride for two people costs $\$12$, how far did the taxi cab travel?

Algebraic Equation & Solution

Number of Miles: m
People: 2

$$2m + 2 = 2$$
$$2m + 2 - 2 = 12 - 2$$
$$2m + 0 = 10$$
$$\left(\frac{1}{2}\right)2m = 10\left(\frac{1}{2}\right)$$
$$1m = 5$$
$$m = 5$$

Tape Diagram



$12 - 2 = 10$ $\quad$ $10 \div 2 = 5$

The taxi cab travelled 5 *miles.*

2. Heather works as a waitress at her family's restaurant. She works 2 hours every morning during the breakfast shift and the same number of hours every evening during the dinner shift. In the last four days she worked 28 hours. How many hours did she work during each dinner shift?

Algebraic Equation & Solution

Number of Morning hours: 2

Number of Evening hours: e

$$4(e + 2) = 28$$
$$4e + 8 - 8 = 28 - 8$$
$$4e + 0 = 20$$
$$\left(\frac{1}{4}\right)4e = 20\left(\frac{1}{4}\right)$$
$$1e = 5$$
$$e = 5$$

OR

$\left(\frac{1}{4}\right)4(e$
$+ 2)$
$= 28\left(\frac{1}{4}\right)$
e
$+ 2$
$= 7$

Tape Diagram

$$\overbrace{\qquad\qquad\qquad\qquad}^{28}$$

$e + 2$	$e + 2$	$e + 2$	$e + 2$
Day 1	*Day* 2	*Day* 3	*Day* 4

$4e$ $\quad$ $28 - 8 = 20$

$4(2) = 8$ $\quad$ $20 \div 4 = 5$

Heather worked 5 *hours in the evening.*

3. Jillian exercises 5 times a week. She runs 3 miles each morning and bikes in the evening. If she exercises a total of 30 miles for the week, how many miles does she bike each evening?

Algebraic Equation & Solution

Run: 3 *mi.*

Bikes: b *mi.*

$$5(b+3)=30$$
$$5b+15-15=30-15$$
$$5b+0=15$$
$$5b=15\left(\frac{1}{5}\right)$$
$$1b=3$$
$$b=3$$

OR

$\left(\frac{1}{5}\right)5(b$
$+3)$
$=30\left(\frac{1}{5}\right)$
b
$+3$
$=6$

Tape Diagram

30

$b+3$	$b+3$	$b+3$	$b+3$	$b+3$
Day 1	*Day* 2	*Day* 3	*Day* 4	*Day* 5

$5(3)=15$ $\quad$ $30-15=15$

$15\div 5=3$

Jillian bikes 3 *miles every evening.*

4. Marc eats an egg sandwich for breakfast and a big burger for lunch every day. The egg sandwich has 250 calories. If Marc has $5,250$ calories for breakfast and lunch for the week in total, how many calories are in one big burger?

Algebraic Equation & Solution

Egg Sandwich: 250 *cal.*

Hamburger: m *cal.*

$$7(m+250)=5,250$$
$$7m+1,750-1750=5250-1750$$
$$7m+0=3,500$$
$$\left(\frac{1}{7}\right)7m=3,500\left(\frac{1}{7}\right)$$
$$1m=500$$
$$m=500$$

OR

$$\left(\frac{1}{7}\right)7(m+250)=5,250$$
$$m+250=750$$
$$m+250-250=750$$
$$-250$$
$$m=500$$

Tape Diagram

$5,250$

$m+250$	$m+250$	$m+250$	$m+250$	$m+250$	$m+250$	$m+250$
Day 1	*Day* 2	*Day* 3	*Day* 4	*Day* 5	*Day* 6	*Day* 7

$7(250)=1,750$

$5,250-1,750=3,500$

$7m$

$3,500\div 7=500$

Each hamburger has 500 *calories.*

5. Jackie won tickets playing the bowling game at the local arcade. The first time, she won 60 tickets. The second time she won a bonus, which was 4 times the number of tickets of the original second prize. All together she won 200 tickets. How many tickets was the original second prize?

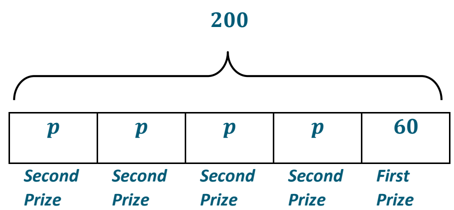
Algebraic Equation & Solution

First Prize: 60 *tickets*

Second Prize: p *tickets*

$$4p + 60 = 200$$
$$4p + 60 - 60 = 200 - 60$$
$$4p + 0 = 140$$
$$\left(\frac{1}{4}\right)4p = 140\left(\frac{1}{4}\right)$$
$$1p = 35$$
$$p = 35$$

Tape Diagram



$4p$

$200 - 60 = 140$

$140 \div 4 = 35$

The original second prize was 35 *tickets.*