Lesson 12: Division of Integers

Student Outcomes

- Students recognize that division is the reverse process of multiplication, and that integers can be divided provided the divisor is not zero. If p and q are integers, then $-\left(\frac{p}{q}\right) = \frac{-p}{q} = \frac{p}{-q}$.
- Students understand that every quotient of integers (with a non-zero divisor) is a rational number and divide signed numbers by dividing their absolute values to get the absolute value of the quotient. The quotient is positive if the divisor and dividend have the same signs and negative if they have opposite signs.

Classwork

Exercise 1 (5 minutes): Recalling the Relationship Between Multiplication and Division

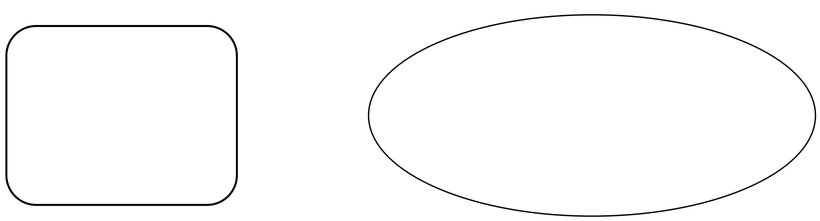
MP.2

The teacher gives each student a card with a whole number multiplication or division math fact on it. Students move around the room in search of other students who have related math facts. (If the class size does not allow for exact multiples of 4, then extra cards may be placed on desk tops for students to find.) Four cards will make a "match" (e.g., $6 \times 4 = 24$, $4 \times 6 = 24$, $24 \div 6 = 4$, and $24 \div 4 = 6$). After four students locate each other, they sit down together and record the equations from their cards into their student materials as indicated below. The teacher circulates among students as a facilitator, guiding those who are having trouble. Once all groups are formed and each group has shared its related facts with the class, the teacher collects the fact cards and directs students back to their original seats.

Exercise 1: Recalling the Relationship Between Multiplication and Division

Record equations from Exercise 1 on the left.

Equations **Integers**



Example 1 (15 minutes): Transitioning from Integer Multiplication Rules to Integer Division Rules

Students make an "integer multiplication facts bubble" by expanding upon the four related math facts they wrote down.

Step 1: Students construct three similar integer multiplication problems, two problems using one negative number as a factor and one with both negative numbers as factors. Students may use the commutative property to extend their three equations to 6.

Scaffolding:

- Provide an example of a completed integer bubble for students who are struggling with the task.

Example 1: Transitioning from Integer Multiplication Rules to Integer Division Rules

Record your group's number sentences in the space on the left below.

Equations

$4 \times 6 = 24$

$6 \times 4 = 24$

$24 \div 4 = 6$

$24 \div 6 = 4$

Integers

$-6 \times 4 = -24$ *or* $4 \times -6 = -24$

$-4 \times 6 = -24$ *or* $6 \times -4 = -24$

$-4 \times (-6) = 24$ *or* $-6 \times -4 = -24$

Step 2: Students use the integer multiplication facts in their integer bubble to create six related integer division facts. Group members should discuss the inverse relationship and the resulting division fact that must be true based on each multiplication equation.

Equations

$4 \times 6 = 24$

$6 \times 4 = 24$

$24 \div 4 = 6$

$24 \div 6 = 4$

Integers

$-6 \times 4 = -24 \rightarrow -24 \div (-6) = 4$

$-24 \div 4 = -6$

$-4 \times 6 = -24 \rightarrow -24 \div (-4) = 6$

$-24 \div 6 = -4$

$-4 \times (-6) = 24 \rightarrow 24 \div (-4) = -6$

$24 \div (-6) = -4$

MP.8

Step 3: Students use the equations in their integer bubble and the patterns they observed to answer the following questions.

a. **List examples of division problems that produced a quotient that is a negative number.**

$-24 \div 4 = -6$; $-24 \div 6 = -4$; $24 \div (-4) = -6$; $24 \div (-6) = -4$

b. **If the quotient is a negative number, what must be true about the signs of the dividend and divisor?**

The quotient is a negative number when the signs of the dividend and divisor are not the same; one is positive and one is negative.

c. **List your examples of division problems that produced a quotient that is a positive number.**

$-24 \div (-4) = 6$; $-24 \div (-6) = 4$; $24 \div 4 = 6$; $24 \div 6 = 4$

d. **If the quotient is a positive number, what must be true about the signs of the dividend and divisor?**

The quotient is a positive number when the signs of the dividend and the divisor are the same in each case.

Step 4: Whole-group discussion. Students share answers from Step 3 with the class. The class comes to a consensus and realizes that since multiplication and division are related* (inverse operations), the integer rules for these operations are related. Students summarize the rules for division, which are stated in the Lesson Summary of the student materials. (*Reminder: The rules apply to all situations except dividing by zero.)

Rules for Dividing Two Integers:

- **A quotient is negative if the divisor and the dividend have** ***opposite*** **signs.**
- **A quotient is positive if the divisor and the dividend have** ***the same*** **signs.**

Exercise 2 (8 minutes): Is the Quotient of Two Integers Always an Integer?

MP.3

Students explore the question above by coming up with an example to prove or refute their position.

Allow 3–5 minutes for students to create a math example or *counter example*, along with a written response to support their position. Students present their cases to the class.

Exercise 2: Is the Quotient of Two Integers Always an Integer?

Is the quotient of two integers always an integer? Use the work space below to create quotients of integers. Answer the question and use examples or a counterexample to support your claim.

Work Space:

$-24 \div 6 = -4$ *Example of an integer quotient*

$6 \div (-24) = \frac{6}{-24} = \frac{1}{-4} = -\frac{1}{4}$ *Counterexample: has a non-integer quotient*

Answer:

No, quotients of integers are not always integers. In my first example above, $-24 \div 6$ yields an integer quotient -4. However, when I switched the divisor and dividend, that quotient divides a number with a smaller absolute value by a number with a greater absolute value, making the quotient a rational number between -1 and 1. In dividing $6 \div (-24)$, the quotient is $\frac{6}{-24} = \frac{1}{-4}$. Of course $-\frac{1}{4}$ is not an integer, but is the opposite value of the fraction $\frac{1}{4}$. This counterexample shows that quotients of integers are not always integers.

Conclusion: Every quotient of two integers is always a rational number, but not always an integer.

Once students have disproved the statement with a counterexample (where the quotient is a decimal or fraction), ask students to determine what must be true of two integers if their quotient *is* an integer. Students may need some time to study the examples where the quotient is an integer to determine that the quotient of two integers, $\frac{A}{B}$, $B \neq 0$, is an integer when either $B = 1$ or $A = kB$ for any integer k.

Exercise 3 (5 minutes): Different Representations of the Same Quotient

Students are given the three different representations below and must determine the answers. Are the answers the same or different? Why or why not? Allow time for students to answer with their groups or learning partner before addressing this in the form of a whole-group discussion.

Exercise 3: Different Representation of the Same Quotient

Are the answers to the three quotients below the same or different? Why or why not?

a. $-14 \div 7$

$-14 \div 7 = -2$

b. $14 \div (-7)$

$14 \div (-7) = -2$

c. $-(14 \div 7)$

$-(14 \div 7) = -(2) = -2$

The answers to the problems are the same: -2. In part (c), the negative in front of the parentheses changes the value inside the parentheses to its opposite. The value in the parentheses is 2, and the opposite of 2 is -2.

Scaffolding:

- For part (c), remind students to think about how a negative sign in front of a number or expression means the opposite.

Fluency Exercise (2 minutes): Integer Division

MP.7

(See attached hand-out.) Students answer as many questions as possible in one minute. One minute is allocated to going over the answers and recognizing achievements.

Fluency Exercise: Integer Division

1.	$-56 \div (-7) = 8$	15.	$-28 \div (-7) = 4$	29.	$-14 \div (-7) = 2$
2.	$-56 \div (-8) = 7$	16.	$-28 \div (-4) = 7$	30.	$-14 \div (-2) = 7$
3.	$56 \div (-8) = -7$	17.	$28 \div 4 = 7$	31.	$14 \div (-2) = -7$
4.	$-56 \div 7 = -8$	18.	$-28 \div 7 = -4$	32.	$-14 \div 7 = -2$
5.	$-40 \div (-5) = 8$	19.	$-20 \div (-5) = 4$	33.	$-10 \div (-5) = 2$
6.	$-40 \div (-4) = 10$	20.	$-20 \div (-4) = 5$	34.	$-10 \div (-2) = 5$
7.	$40 \div (-4) = -10$	21.	$20 \div (-4) = -5$	35.	$10 \div (-2) = -5$
8.	$-40 \div 5 = -8$	22.	$-20 \div 5 = -4$	36.	$-10 \div 5 = -2$
9.	$-16 \div (-4) = 4$	23.	$-8 \div (-4) = 2$	37.	$-4 \div (-4) = 1$
10.	$-16 \div (-2) = 8$	24.	$-8 \div (-2) = 4$	38.	$-4 \div (-1) = 4$
11.	$16 \div (-2) = -8$	25.	$8 \div (-2) = -4$	39.	$4 \div (-1) = -4$
12.	$-16 \div 4 = -4$	26.	$-8 \div 4 = -2$	40.	$-4 \div 1 = -4$
13.	$-3 \div (-4) = 0.75$	27.	$4 \div (-8) = -0.5$	41.	$1 \div (-4) = -0.25$
14.	$-3 \div 4 = -0.75$	28.	$-4 \div 8 = -0.5$	42.	$-1 \div 4 = -0.25$

Closing (5 minutes)

- How are the rules for multiplying integers and dividing integers related?
  - *The rules for multiplying integers and dividing integers are the same, as long as the divisor is not zero.*
- If I have a negative quotient, what must be true about the signs of the dividend and/or divisor?
  - *If the quotient is negative, the dividend and divisor must have opposite signs.*
- If I have a positive quotient, what must be true about the signs of the dividend and/or divisor?
  - *If the quotient is positive, the dividend and divisor must have the same sign.*

Lesson Summary

The rules for dividing integers are similar to the rules for multiplying integers (when the divisor is not zero). The quotient is positive if the divisor and dividend have the same signs and negative if they have opposite signs.

The quotient of any 2 integers (with a non-zero divisor) will be a rational number. If p and q are integers, then $-\left(\frac{p}{q}\right) = \frac{-p}{q} = \frac{p}{-q}$.

Exit Ticket (5 minutes)

MP.3

Students determine whether or not various representations of the quotient of two integers are equivalent.

Name ____ Date____________________

Lesson 12: Division of Integers

Exit Ticket

1. Mrs. McIntire, a seventh grade math teacher, is grading papers. Three students gave the following responses to the same math problem:

   Student one: $\frac{1}{-2}$

   Student two: $-\left(\frac{1}{2}\right)$

   Student three: $-\frac{1}{2}$

   On Mrs. McIntire's answer key for the assignment, the correct answer is -0.5. Which student answer(s) is (are) correct? Explain.

2. Complete the table below. Provide an answer for each integer division problem and write a related equation using integer multiplication.

Integer Division Problem	Related Equation Using Integer Multiplication
$-36 \div (-9) =$ ________	
$24 \div (-8) =$ ________	
$-50 \div 10 =$ ________	
$42 \div 6 =$ ________	

Exit Ticket Sample Solutions

1. **Mrs. McIntire, a seventh grade math teacher, is grading papers. Three students gave the following responses to the same math problem:**

   Student one: $\frac{1}{-2}$

   Student two: $-\left(\frac{1}{2}\right)$

   Student three: $-\frac{1}{2}$

   On Mrs. McIntire's answer key for the assignment, the correct answer is -0.5. Which student answer(s) is (are) correct? Explain.

   All student answers are correct, since they are all equivalent to -0.5.

   For student one: $\frac{1}{-2}$ means 1 divided by -2. When dividing a positive 1 by a negative 2, the answer will be negative five-tenths or -0.5.

   For student two: $-\left(\frac{1}{2}\right)$ means the opposite of $\frac{1}{2}$. One-half is equivalent to five-tenths, and the opposite is negative five-tenths or -0.5.

   For student three: $-\frac{1}{2}$ means -1 divided by 2. When dividing a negative 1 by a positive 2, the answer will be negative five-tenths or -0.5.

2. **Complete the table below. Provide an answer for each integer division problem and write a related equation using integer multiplication.**

Integer Division Problem	Related Equation Using Integer Multiplication
$-36 \div (-9) =$ $\underline{4}$	$-9 \times 4 = -36$ *or* $4 \times (-9) = -36$
$24 \div (-8) =$ $\underline{-3}$	$-8 \times (-3) = 24$ *or* $-3 \times (-8) = 24$
$-50 \div 10 =$ $\underline{-5}$	$-5 \times 10 = -50$ *or* $10 \times (-5) = -50$
$42 \div 6 =$ $\underline{7}$	$6 \times 7 = 42$ *or* $7 \times 6 = 42$

Problem Set Sample Solutions

1. **Find the missing values in each column.**

Column A	Column B	Column C	Column D
$48 \div 4 = 12$	$24 \div 4 = 6$	$63 \div 7 = 9$	$21 \div 7 = 3$
$-48 \div (-4) = 12$	$-24 \div (-4) = 6$	$-63 \div (-7) = 9$	$-21 \div (-7) = 3$
$-48 \div 4 = -12$	$-24 \div 4 = -6$	$-63 \div 7 = -9$	$-21 \div 7 = -3$
$48 \div (-4) = -12$	$24 \div (-4) = -6$	$63 \div (-7) = -9$	$21 \div (-7) = -3$

2. **Describe the pattern you see in each column's answers in Problem 1, relating it to the problems' divisors and dividends. Why is this so?**

   The pattern in the columns' answers is the same two positive values followed by the same two negative values. This is so for the first two problems because the divisor and the dividend have the same signs and absolute values, which yields a positive quotient. This is so for the second two problems because the divisor and dividend have different signs but the same absolute values, which yields a negative quotient.

3. **Describe the pattern you see between the answers for Columns A and B in Problem 1 (e.g., compare the first answer in Column A to the first answer in Column B; compare the second answer in Column A to the second answer in Column B). Why is this so?**

   The answers in Column B are each one-half of the corresponding answers in Column A. That is because the dividend of 48 in Column A is divided by 4, and the dividend of 24 in Column B is divided by 4 (and so on with the same order and same absolute values but different signs). Since 24 is half of 48, the quotient (answer) in Column B will be one-half of the quotient in Column A.

4. **Describe the pattern you see between the answers for Columns C and D in Problem 1. Why is this so?**

   The answers in Column D are each one-third of the corresponding answers in Column C. That is because the dividend of 63 in Column C is divided by 7, and the dividend of 21 in Column D is divided by 7 (and so on with the same order and same absolute values but different signs). Since 21 is one-third of 63, the quotient (answer) in Column D will be one-third of the quotient in Column C.

Fluency Exercise: Integer Division

1.	$-56 \div (-7) =$	15.	$-28 \div (-7) =$	29.	$-14 \div (-7) =$
2.	$-56 \div (-8) =$	16.	$-28 \div (-4) =$	30.	$-14 \div (-2) =$
3.	$56 \div (-8) =$	17.	$28 \div 4 =$	31.	$14 \div (-2) =$
4.	$-56 \div 7 =$	18.	$-28 \div 7 =$	32.	$-14 \div 7 =$
5.	$-40 \div (-5) =$	19.	$-20 \div (-5) =$	33.	$-10 \div (-5) =$
6.	$-40 \div (-4) =$	20.	$-20 \div (-4) =$	34.	$-10 \div (-2) =$
7.	$40 \div (-4) =$	21.	$20 \div (-4) =$	35.	$10 \div (-2) =$
8.	$-40 \div 5 =$	22.	$-20 \div 5 =$	36.	$-10 \div 5 =$
9.	$-16 \div (-4) =$	23.	$-8 \div (-4) =$	37.	$-4 \div (-4) =$
10.	$-16 \div (-2) =$	24.	$-8 \div (-2) =$	38.	$-4 \div (-1) =$
11.	$16 \div (-2) =$	25.	$8 \div (-2) =$	39.	$4 \div (-1) =$
12.	$-16 \div 4 =$	26.	$-8 \div 4 =$	40.	$-4 \div 1 =$
13.	$-3 \div (-4) =$	27.	$4 \div (-8) =$	41.	$1 \div (-4) =$
14.	$-3 \div 4 =$	28.	$-4 \div 8 =$	42.	$-1 \div 4 =$