Lesson 11: Develop Rules for Multiplying Signed Numbers

Student Outcomes

- Students understand the rules for multiplication of integers and that multiplying the absolute values of integers results in the absolute value of the product. The sign, or absolute value, of the product is positive if the factors have the same sign and negative if they have opposite signs.
- Students realize that $(-1)(-1) = (1)$ and see that it can be proven mathematically using the distributive property and the additive inverse.
- Students use the rules for multiplication of signed numbers and give real-world examples.

Classwork

Example 1 (17 minutes): Extending Whole Number Multiplication to the Integers

Part A: Students complete only the right half of the table in the student materials. They do this by calculating the total change to a player's score using the various sets of matching cards. Students complete the table with these values to reveal patterns in multiplication.

Students describe, using Integer Game scenarios, what the right quadrants of the table represent and record this in the student materials.

Example 1: Extending Whole Number Multiplication to the Integers

Part A: Complete quadrants I and IV of the table below to show how sets of matching integer cards will affect a player's score in the Integer Game. For example, three 2's would increase a player's score by $0 + 2 + 2 + 2 = 6$ points.

Quadrant II Quadrant I

What does this quadrant represent?
____
____
____
____

What does this quadrant represent?
____
____
____
____

					5					
					4					
					3					
					2			6		
					1					
						1	2	3	4	5
					-1					
					-2					
					-3					
					-4					
					-5					

← Number of matching cards

What does this quadrant represent?
____
____
____
____

What does this quadrant represent?
____
____
____
____

Quadrant III Quadrant IV

↑ Integer card values

Quadrant II Quadrant I

What does this quadrant represent? *Removing positive value cards.* (Quadrant II)

What does this quadrant represent? *Picking up positive value cards.* (Quadrant I)

-25	-20	-15	-10	-5	5	5	10	15	20	25
-20	-16	-12	-8	-4	4	4	8	12	16	20
-15	-12	-9	-6	-3	3	3	6	9	12	15
-10	-8	-6	-4	-2	2	2	4	6	8	10
-5	-4	-3	-2	-1	1	1	2	3	4	5
-5	-4	-3	-2	-1	0	1	2	3	4	5
5	4	3	2	1	-1	-1	-2	-3	-4	-5
10	8	6	4	2	-2	-2	-4	-6	-8	-10
15	12	9	6	3	-3	-3	-6	-9	-12	-15
20	16	12	8	4	-4	-4	-8	-12	-16	-20
25	20	15	10	5	-5	-5	-10	-15	-20	-25

← Number of matching cards

What does this quadrant represent? *Removing negative value cards.* (Quadrant III)

What does this quadrant represent? *Picking up negative value cards.* (Quadrant IV)

Quadrant III Quadrant IV

↑ Integer card values

a. **What patterns do you see in the right half of the table?**

The products in quadrant I are positive and the products in quadrant IV are negative. When looking at the absolute values of the products, quadrants I and IV are a reflection of each other with respect to the middle row.

b. **Enter the missing integers in the left side of the middle row and describe what they represent.**

The numbers represent how many matching cards are being discarded or removed.

-5	-4	-3	-2	-1	0

Part B: Students complete quadrant II of the table.
Students describe, using an Integer Game scenario, what quadrant II of the table represents and record this in the student materials.

Part B: Complete quadrant II of the table.

Quadrant II

What does this quadrant represent?

Removing positive value cards.

-25	-20	-15	-10	-5	5
-20	-16	-12	-8	-4	4
-15	-12	-9	-6	-3	3
-10	-8	-6	-4	-2	2
-5	-4	-3	-2	-1	1
-5	-4	-3	-2	-1	0

Students answer the following questions:

MP.7

c. **What relationships or patterns do you notice between the produtcs (values) in quadrant II and the products (values) in quadrant I?**

The products in quadrant II are all negative values. Looking at the absolute values of the products, quadrant I and II are a reflection of each other with respect to the center column.

d. **What relationships or patterns do you notice between the products (values) in quadrant II and the products (values) in quadrant IV?**

The products in quadrants II and IV are all negative values. Each product of integers in quadrant II is equal to the product of their opposites in quadrant IV.

e. **Use what you know about the products (values) in quadrants I, II, and IV to describe what quadrant III will look like when its products (values) are entered.**

The reflection symmetry of quadrant I to quadrants II and IV suggests that there should be similar relationships between quadrant II, III, and IV. The number patterns in quadrants II and IV also suggest that the products in quadrant III are positive values.

Part C: Discuss the following question, then instruct students to complete the final quadrant of the table.

- In the Integer Game, what happens to a player's score when he removes a matching set of cards with negative values from his hand?
  - *His score increases because the negative cards that cause the score to decrease are removed.*

Students describe, using an Integer Game scenario, what quadrant III of the table represents and complete the quadrant in the student materials.

Scaffolding:

- Create an "anchor poster" showing the quadrants with the new rules for multiplying integers.

Part C: Complete the quadrant III of the table.

Refer to the completed table to help you answer the following questions:

What does this quadrant represent?

Removing negative value cards.

-5	-4	-3	-2	-1	0
5	4	3	2	1	-1
10	8	6	4	2	-2
15	12	9	6	3	-3
20	16	12	8	4	-4
25	20	15	10	5	-5

Quadrant III

Students refer to the completed table to answer parts (f) and (g).

f. **Is it possible to know the sign of a product of two integers just by knowing in which quadrant each integer is located? Explain.**

Yes, it is possible to know the sign of a product of two integers just by knowing each integer's quadrant because the signs of the values in each of the quadrants are consistent. Two quadrants contain positive values, and the other two quadrants contain negative values.

g. **Which quadrants contain which values? Describe an integer game scenario represented in each quadrant.**

Quadrants I and III contain all positive values. Picking up three 4's is represented in quadrant I and increases your score. Removing three -4's is represented in quadrant III and also increases your score. Quadrants II and IV contain all negative values. Picking up three -4's is represented in quadrant IV and decreases your score. Removing three 4's is represented in quadrant II and also decreases your score.

Example 2 (10 minutes): Using Properties of Arithmetic to Explain Multiplication of Negative Numbers

Teacher guides students to verify their conjecture that the product of two negative integers is positive using the distributive property and the additive inverse property.

- We have used the Integer Game to explain that adding a number multiple times has the same effect as removing the opposite value the same number of times. What is $(-1) \times (-1)$?
  - *Removing a -1 card is the same as adding a 1 card. So, $(-1) \times (-1) = 1$.*
- Why are 1 and -1 called additive inverses? Write an equation that shows this property.
  - *The sum of 1 and -1 is 0; $1 + (-1) = 0$.*

We are now going to show $-1 \times (-1) = 1$ using properties of arithmetic.

- We know $1 + (-1) = 0$ is true.
- We will show that $(-1) \times (-1)$ is the additive inverse of -1 which is 1.

If $-1 \times 0 = 0$	By the zero product property,
then $-1 \times \big(1 + (-1)\big) = 0$	By substitution of $\big(1 + (-1)\big)$ for 0
$(-1 \times 1) + \big(-1 \times (-1)\big) = 0$	Distributive property
$-1 + \big(-1 \times (-1)\big) = 0$	Multiplication by 1

- Since -1 and $\big(-1 \times (-1)\big)$ have a sum of zero, they are additive inverses of each other; but, the additive inverse of -1 is 1.
- Because $\big(-1 \times (-1)\big)$ is the additive inverse of -1, we conclude that $(-1) \times (-1) = 1$. This fact can be used to show that $-1 \times a = -a$ for any integer and that $-a \times b = -(a \times b)$ for any integers a and b.

Scaffolding:
- Use color or highlight steps to help students organize and understand the manipulations.

Exercise 1 (8 minutes): Multiplication of Integers in the Real World

Students create real-world scenarios for expressions given in the student materials. Students may use an Integer Game scenario as a reference.

Exercise 1: Multiplication of Integers in the Real World

Generate real-world situations that can be modeled by each of the following multiplication problems. Use the Integer Game as a resource.

a. -3×5

I lost three $\$5$ bills, and now I have $-\$15$.

b. $-6 \times (-3)$

I removed six -3's from my hand in the Integer Game, and my score increased 18 points.

c. $4 \times (-7)$

If I lose 7 lb. per month for 4 months, my weight will change -28 lb. total.

Scaffolding:
- For ELL students, create teacher/student T-chart on which the teacher writes a real-world situation that corresponds with a product, and students write similar situations using different numbers.

Closing (5 minutes)

- How do we determine if the product of two signed numbers will be positive or negative?
  - *If the factors have the same sign, the product will be positive, and if the factors have opposite signs, the product will be negative.*
- Why does the product of two negative values result in a positive value? Explain using the Integer Game.
  - *The product of two negative numbers represents removing negative cards during the Integer Game. When negative cards are removed from someone's hand their score increases, therefore, the product is positive.*

Lesson Summary

To multiply signed numbers, multiply the absolute values to get the absolute value of the product. The sign of the product is positive if the factors have the same sign and negative if they have opposite signs.

Exit Ticket (5 minutes)

Name ____ Date____________________

Lesson 11: Develop Rules for Multiplying Signed Numbers

Exit Ticket

1. Create a real-life example that can be modeled by the expression -2×4, and then state the product.

2. Two integers are multiplied and their product is a positive number. What must be true about the two integers?

Exit Ticket Sample Solutions

1. **Create a real-life example that can be modeled by the expression -2×4, and then state the product.**

   Answers will vary. Tobi wants to lose 2 lb. each week for four weeks. Write an integer to represent Tobi's weight change after four weeks. Tobi's weight changes by -8 lb. after four weeks.

2. **Two integers are multiplied and their product is a positive number. What must be true about the two integers?**

   Both integers must be positive numbers, or both integers must be negative numbers.

Problem Set Sample Solutions

1. **Complete the problems below. Then, answer the question that follows.**

$3 \times 3 = 9$	$3 \times 2 = 6$	$3 \times 1 = 3$	$3 \times 0 = 0$	$3 \times (-1) = -3$	$3 \times (-2) = -6$
$2 \times 3 = 6$	$2 \times 2 = 4$	$2 \times 1 = 2$	$2 \times 0 = 0$	$2 \times (-1) = -2$	$2 \times (-2) = -4$
$1 \times 3 = 3$	$1 \times 2 = 2$	$1 \times 1 = 1$	$1 \times 0 = 0$	$1 \times (-1) = -1$	$1 \times (-2) = -2$
$0 \times 3 = 0$	$0 \times 2 = 0$	$0 \times 1 = 0$	$0 \times 0 = 0$	$0 \times (-1) = 0$	$0 \times (-2) = 0$
$-1 \times 3 = -3$	$-1 \times 2 = -2$	$-1 \times 1 = -1$	$-1 \times 0 = 0$	$-1 \times (-1) = 1$	$-1 \times (-2) = 2$
$-2 \times 3 = -6$	$-2 \times 2 = -4$	$-2 \times 1 = -2$	$-2 \times 0 = 0$	$-2 \times (-1) = 2$	$-2 \times (-2) = 4$
$-3 \times 3 = -9$	$-3 \times 2 = -6$	$-3 \times 1 = -3$	$-3 \times 0 = 0$	$-3 \times (-1) = 3$	$-3 \times (-2) = 6$

   Which row shows the same pattern as the outlined column? Are the problems similar or different? Explain.

   The row outlined shows the same pattern as the outlined column. The problems have the same answers, but the signs of the integer factors are switched. For example, $3 \times (-1) = -3 \times 1$. This shows that the product of two integers with opposite signs is equal to the product of their opposites.

2. **Explain why $(-4) \times (-5) = 20$. Use patterns, an example from the Integer Game, or the properties of operations to support your reasoning.**

   Answers may vary. Losing four -5 cards will increase a score in the Integer Game by 20 because a negative value decreases a score, but the score increases when it is removed.

3. **Each time that Samantha rides the commuter train, she spends $\$4$ for her fare. Write an integer that represents the change in Samantha's money from riding the commuter train to and from work for 13 days. Explain your reasoning.**

   If Samantha rides to and from work for 13 days, then she rides the train a total of 26 times. The cost of each ride can be represented by -4. So, the change to Samantha's money is represented by $-4 \times 26 = -104$. The change to Samantha's money after 13 days of riding the train to and from work is $-\$104$.

4. **Write a real-world problem that can be modeled by $4 \times (-7)$.**

   Answers will vary. Every day, Alec loses 7 pounds of air pressure in a tire on his car. At that rate, what is the change in air pressure in his tire after 4 days?

Challenge:

5. **Use properties to explain why for each integer a, $-a = -1 \times a$. (Hint: What does $(1 + (-1)) \times a$ equal? What is the additive inverse of a?)**

   $0 \times a = 0$ *Zero product property*

   $(1 + (-1)) \times a = 0$ *Substitution*

   $a + (-1 \times a) = 0$ *Distributive property*

   Since a and $(-1 \times a)$ have a sum of zero, they must be additive inverses. By definition, the additive inverse of a is $-a$, so $(-1 \times a) = -a$.