



Lesson 9: Applying the Properties of Operations to Add and Subtract Rational Numbers

Student Outcomes

- Students use properties of operations to add and subtract rational numbers without the use of a calculator.
- Students recognize that any problem involving addition and subtraction of rational numbers can be written as a problem using addition and subtraction of positive numbers only.
- Students use the commutative and associative properties of addition to rewrite numerical expressions in different forms. They know that the opposite of a sum is the sum of the opposites; e.g., $-(3 - 4) = -3 + 4$.

Classwork

Fluency Exercise (6 minutes): Integer Subtraction

Photocopy the attached 2-page fluency-building exercises so that each student receives a copy. Time the students, allowing one minute to complete Side A. Before students begin, inform them that they may not skip over questions and that they must move in order. After one minute, discuss the answers. Before administering Side B, elicit strategies from those students who were able to accurately complete many problems on Side A. Administer Side B in the same fashion, and review the answers. Refer to the Sprints and Sprint Delivery Script sections in the Module Overview for directions to administer a Sprint.

Exercise 1 (6 minutes)

MP.8

Students are given scrambled steps to one possible solution to the following problem¹. They work independently to arrange the expressions in an order that leads to a solution and record their solutions in the student materials.

Exercise 1

Unscramble the cards, and show the steps in the correct order to arrive at the solution to $5\frac{2}{9} - (8.1 + 5\frac{2}{9})$.

$$0 + (-8.1)$$

$$\left(5\frac{2}{9} + \left(-5\frac{2}{9}\right)\right) + (-8.1)$$

$$-8.1$$

$$5\frac{2}{9} + \left(-8.1 + \left(-5\frac{2}{9}\right)\right)$$

$$5\frac{2}{9} + \left(-5\frac{2}{9} + (-8.1)\right)$$

¹ The scrambled steps may also be displayed on an interactive whiteboard, and students can come up one at a time to slide a step into the correct position.

$$5\frac{2}{9} + (-8.1 + (-5\frac{2}{9})) \quad \text{The opposite of a sum is the sum of its opposites.}$$

$$5\frac{2}{9} + (-5\frac{2}{9} + (-8.1)) \quad \text{Apply the commutative property of addition.}$$

$$(5\frac{2}{9} + (-5\frac{2}{9})) + (-8.1) \quad \text{Apply the associative property of addition.}$$

$$0 + (-8.1) \quad \text{A number plus its opposite equals zero.}$$

$$-8.1 \quad \text{Apply the additive identity property.}$$

Scaffolding:

- Adapt for struggling learners by having fewer steps to rearrange or by including only integers.
- Adapt for proficient students by requiring that they state the property of operations to justify each step.

After 2 minutes, students share the correct sequence of steps with the class.

- What allows us to represent operations in another form and rearrange the order of terms?
 - The properties of operations.
- Specifically which properties of operations were used in this example?
 - Students recall the additive inverse property and commutative property of addition. (Students are reminded to focus on all of the properties that justify their steps today.)
- Why did we use the properties of operations?
 - Students recognize that using the properties allows us to efficiently (more easily) calculate the answer to the problem.

Examples 1–2 (7 minutes)

Students record the following examples. Students assist in volunteering verbal explanations for each step during the whole-group discussion. Today, students' focus is *not* on memorizing the names of each property but rather knowing that each representation is justifiable through the properties of operations.

Examples 1–2

Represent each of the following expressions as one rational number. Show and explain your steps.

$$\begin{aligned}
 1. \quad & 4\frac{4}{7} - (4\frac{4}{7} - 10) \\
 &= 4\frac{4}{7} - (4\frac{4}{7} + (-10)) \quad \text{Subtracting a number is the same as adding its inverse.} \\
 &= 4\frac{4}{7} + (-4\frac{4}{7} + 10) \quad \text{The opposite of a sum is the sum of its opposites.} \\
 &= (4\frac{4}{7} + (-4\frac{4}{7})) + 10 \quad \text{The associative property of addition.} \\
 &= 0 + 10 \quad \text{A number plus its opposite equals zero.} \\
 &= 10
 \end{aligned}$$

Scaffolding:

- Provide a list of the properties of operations as a reference sheet for students.

- First, predict the answer. Explain your prediction.
 - Answers may vary.

- The answer will be between 0 and $\frac{1}{2}$ because $5 + (-5) = 0$ and $-4\frac{4}{7}$ is close to -5 , but 5 has a larger absolute value than $-4\frac{4}{7}$. To add $5 + (-4\frac{4}{7})$, we subtract their absolute values. Since $-4\frac{4}{7}$ is close to $-4\frac{1}{2}$, the answer will be about $5 - 4\frac{1}{2} = \frac{1}{2}$.

$$\begin{aligned}
 2. \quad & 5 + \left(-4\frac{4}{7}\right) \\
 &= 5 + \left[-\left(4 + \frac{4}{7}\right)\right] && \text{The mixed number } 4\frac{4}{7} \text{ is equivalent to } 4 + \frac{4}{7}. \\
 &= 5 + \left(-4 + \left(-\frac{4}{7}\right)\right) && \text{The opposite of a sum is the sum of its opposites.} \\
 &= (5 + (-4)) + \left(-\frac{4}{7}\right) && \text{Associative property of addition.} \\
 &= 1 + \left(-\frac{4}{7}\right) && 5 + (-4) = 1 \\
 &= \frac{7}{7} + \left(-\frac{4}{7}\right) && \frac{7}{7} = 1 \\
 &= \frac{3}{7}
 \end{aligned}$$

- Does our answer match our prediction?
 - Yes, we predicted a positive number close to zero.

Exercise 2 (10 minutes): Team Work!

Students work in groups of three. Each student has a different colored pencil. Each problem has at least three steps. Students take turns writing a step to each problem, passing the paper to the next person, and rotating the student who starts first with each new problem.

MP.2
&
MP.3

After 8 minutes, students partner up with another group of students to discuss or debate their answers. Students should also explain their steps and the properties/rules that justify each step.

Exercise 2: Team Work!

$$\begin{aligned}
 \text{a.} \quad & -5.2 - (-3.1) + 5.2 \\
 &= -5.2 + 3.1 + 5.2 \\
 &= -5.2 + 5.2 + 3.1 \\
 &= 0 + 3.1 \\
 &= 3.1
 \end{aligned}$$

$$\begin{aligned}
 \text{b.} \quad & 32 + \left(-12\frac{7}{8}\right) \\
 &= 32 + \left(-12 + \left(-\frac{7}{8}\right)\right) \\
 &= (32 + (-12)) + \left(-\frac{7}{8}\right) \\
 &= 20 + \left(-\frac{7}{8}\right) \\
 &= 19\frac{1}{8}
 \end{aligned}$$

$$\text{c. } 3\frac{1}{6} + 20.3 - \left(-5\frac{5}{6}\right)$$

$$\begin{aligned} & 3\frac{1}{6} + 20.3 + 5\frac{5}{6} \\ &= 3\frac{1}{6} + 5\frac{5}{6} + 20.3 \\ &= 8\frac{6}{6} + 20.3 \\ &= 9 + 20.3 \\ &= 29.3 \end{aligned}$$

$$\text{d. } \frac{16}{20} - (-1.8) - \frac{4}{5}$$

$$\begin{aligned} &= \frac{16}{20} + 1.8 - \frac{4}{5} \\ &= \frac{16}{20} + 1.8 + \left(-\frac{4}{5}\right) \\ &= \frac{16}{20} + \left(-\frac{4}{5}\right) + 1.8 \\ &= \frac{16}{20} + \left(-\frac{16}{20}\right) + 1.8 \\ &= 0 + 1.8 \\ &= 1.8 \end{aligned}$$

Exercise 3 (5 minutes)

Students work independently to answer the following question, then after 3 minutes, group members share their responses with one another and come to a consensus.

Exercise 3

Explain step by step, how to arrive at a single rational number to represent the following expression. Show both a written explanation and the related math work for each step.

$$-24 - \left(-\frac{1}{2}\right) - 12.5$$

Subtracting $\left(-\frac{1}{2}\right)$ is the same as adding its inverse $\frac{1}{2}$:

$$= -24 + \frac{1}{2} + (-12.5)$$

Next, I used the commutative property of addition to rewrite the expression:

$$= -24 + (-12.5) + \frac{1}{2}$$

Next, I added both negative numbers:

$$= -36.5 + \frac{1}{2}$$

Next, I wrote $\frac{1}{2}$ in its decimal form:

$$= -36.5 + 0.5$$

Lastly, I added $-36.5 + 0.5$:

$$= -36$$

Closing (3 minutes)

- How are the properties of operations helpful when finding the sums and differences of rational numbers?
 - *The properties of operations allow us to add and subtract rational numbers more efficiently.*
- Do you think the properties of operations could be used in a similar way to aid in the multiplication and division of rational numbers?
 - *Answers will vary.*

Lesson Summary

- Use the properties of operations to add and subtract rational numbers more efficiently. For instance,

$$-5\frac{2}{9} + 3.7 + 5\frac{2}{9} = \left(-5\frac{2}{9} + 5\frac{2}{9}\right) + 3.7 = 0 + 3.7 = 3.7.$$

- The opposite of a sum is the sum of its opposites as shown in the examples that follow:

$$-4\frac{4}{7} = -4 + \left(-\frac{4}{7}\right).$$

$$-(5 + 3) = -5 + (-3).$$

Exit Ticket (8 minutes)

Name _____

Date _____

Lesson 9: Applying the Properties of Operations to Add and Subtract Rational Numbers

Exit Ticket

1. Jamie was working on his math homework with his friend, Kent. Jamie looked at the following problem.

$$-9.5 - (-8) - 6.5$$

He told Kent that he did not know how to subtract negative numbers. Kent said that he knew how to solve the problem using only addition. What did Kent mean by that? Explain. Then, show your work and represent the answer as a single rational number.

Work Space:

Answer: _____

2. Use one rational number to represent the following expression. Show your work.

$$3 + (-0.2) - 15\frac{1}{4}$$

Exit Ticket Sample Solutions

1. Jamie was working on his math homework with his friend, Kent. Jamie looked at the following problem

$$-9.5 - (-8) - 6.5$$

He told Kent that he did not know how to subtract negative numbers. Kent said that he knew how to solve the problem using only addition. What did Kent mean by that? Explain. Then, show your work and represent the answer as a single rational number.

Kent meant that since any subtraction problem can be written as an addition problem by adding the opposite of the number you are subtracting, Jamie can solve the problem by using only addition.

Work Space:

$$\begin{aligned} & -9.5 - (-8) - 6.5 \\ &= -9.5 + 8 + (-6.5) \\ &= -9.5 + (-6.5) + 8 \\ &= -16 + 8 \\ &= -8 \end{aligned}$$

Answer: -8

2. Use one rational number to represent the following expression. Show your work.

$$\begin{aligned} & 3 + (-0.2) - 15\frac{1}{4} \\ &= 3 + (-0.2) + \left(-15 + \left(-\frac{1}{4}\right)\right) \\ &= 3 + (-0.2 + (-15) + (-0.25)) \\ &= 3 + (-15.45) \\ &= -12.45 \end{aligned}$$

Problem Set Sample Solutions

Show all steps taken to rewrite each of the following as a single rational number.

1. $80 + \left(-22\frac{4}{15}\right)$

$$\begin{aligned} &= 80 + \left(-22 + \left(-\frac{4}{15}\right)\right) \\ &= (80 + (-22)) + \left(-\frac{4}{15}\right) \\ &= 58 + \left(-\frac{4}{15}\right) \\ &= 57\frac{11}{15} \end{aligned}$$

2. $10 + \left(-3\frac{3}{8}\right)$

$$\begin{aligned} &= 10 + \left(-3 + \left(-\frac{3}{8}\right)\right) \\ &= (10 + (-3)) + \left(-\frac{3}{8}\right) \\ &= 7 + \left(-\frac{3}{8}\right) \\ &= 6\frac{5}{8} \end{aligned}$$

$$\begin{aligned}
 3. \quad & \frac{1}{5} + 20.3 - \left(-5\frac{3}{5}\right) \\
 &= \frac{1}{5} + 20.3 + 5\frac{3}{5} \\
 &= \frac{1}{5} + 5\frac{3}{5} + 20.3 \\
 &= 5\frac{4}{5} + 20.3 \\
 &= 5\frac{4}{5} + 20\frac{3}{10} \\
 &= 5\frac{8}{10} + 20\frac{3}{10} \\
 &= 25\frac{11}{10} \\
 &= 26\frac{1}{10}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & \frac{11}{12} - (-10) - \frac{5}{6} \\
 &= \frac{11}{12} + 10 + \left(-\frac{5}{6}\right) \\
 &= \frac{11}{12} + \left(-\frac{5}{6}\right) + 10 \\
 &= \frac{11}{12} + \left(-\frac{10}{12}\right) + 10 \\
 &= \frac{1}{12} + 10 \\
 &= 10\frac{1}{12}
 \end{aligned}$$

5. Explain, step by step, how to arrive at a single rational number to represent the following expression. Show both a written explanation and the related math work for each step.

$$1 - \frac{3}{4} + \left(-12\frac{1}{4}\right)$$

First, I rewrote the subtraction of $\frac{3}{4}$ as the addition of its inverse $-\frac{3}{4}$:

$$= 1 + \left(-\frac{3}{4}\right) + \left(-12\frac{1}{4}\right)$$

Next, I used the associative property of addition to regroup addend:

$$= 1 + \left(\left(-\frac{3}{4}\right) + \left(-12\frac{1}{4}\right)\right)$$

Next, I separated $-12\frac{1}{4}$ into the sum of -12 and $-\frac{1}{4}$:

$$= 1 + \left(\left(-\frac{3}{4}\right) + (-12) + \left(-\frac{1}{4}\right)\right)$$

Next, I used the commutative property of addition:

$$= 1 + \left(\left(-\frac{3}{4}\right) + \left(-\frac{1}{4}\right) + (-12)\right)$$

Next, I found the sum of $-\frac{3}{4}$ and $-\frac{1}{4}$:

$$= 1 + ((-1) + (-12))$$

Next, I found the sum of -1 and -12 :

$$= 1 + (-13)$$

Lastly, since the absolute value of 13 is greater than the absolute value of 1, and it is a negative 13, the answer will be a negative number. The absolute value of 13 minus the absolute value of 1 equals 12, so the answer is -12 .

$$= -12$$

Number Correct: _____

Integer Subtraction – Round 1**Directions:** Determine the difference of the integers, and write it in the column to the right.

1.	$4 - 2$	
2.	$4 - 3$	
3.	$4 - 4$	
4.	$4 - 5$	
5.	$4 - 6$	
6.	$4 - 9$	
7.	$4 - 10$	
8.	$4 - 20$	
9.	$4 - 80$	
10.	$4 - 100$	
11.	$4 - (-1)$	
12.	$4 - (-2)$	
13.	$4 - (-3)$	
14.	$4 - (-7)$	
15.	$4 - (-17)$	
16.	$4 - (-27)$	
17.	$4 - (-127)$	
18.	$14 - (-6)$	
19.	$23 - (-8)$	
20.	$8 - (-23)$	
21.	$51 - (-3)$	
22.	$48 - (-5)$	

23.	$(-6) - 5$	
24.	$(-6) - 7$	
25.	$(-6) - 9$	
26.	$(-14) - 9$	
27.	$(-25) - 9$	
28.	$(-12) - 12$	
29.	$(-26) - 26$	
30.	$(-13) - 21$	
31.	$(-25) - 75$	
32.	$(-411) - 811$	
33.	$(-234) - 543$	
34.	$(-3) - (-1)$	
35.	$(-3) - (-2)$	
36.	$(-3) - (-3)$	
37.	$(-3) - (-4)$	
38.	$(-3) - (-8)$	
39.	$(-30) - (-45)$	
40.	$(-27) - (-13)$	
41.	$(-13) - (-27)$	
42.	$(-4) - (-3)$	
43.	$(-3) - (-4)$	
44.	$(-1,066) - (-34)$	

Integer Subtraction – Round 1 [KEY]**Directions:** Determine the difference of the integers, and write it in the column to the right.

1.	$4 - 2$	2
2.	$4 - 3$	1
3.	$4 - 4$	0
4.	$4 - 5$	-1
5.	$4 - 6$	-2
6.	$4 - 9$	-5
7.	$4 - 10$	-6
8.	$4 - 20$	-16
9.	$4 - 80$	-76
10.	$4 - 100$	-96
11.	$4 - (-1)$	5
12.	$4 - (-2)$	6
13.	$4 - (-3)$	7
14.	$4 - (-7)$	11
15.	$4 - (-17)$	21
16.	$4 - (-27)$	31
17.	$4 - (-127)$	131
18.	$14 - (-6)$	20
19.	$23 - (-8)$	31
20.	$8 - (-23)$	31
21.	$51 - (-3)$	54
22.	$48 - (-5)$	53

23.	$(-6) - 5$	-11
24.	$(-6) - 7$	-13
25.	$(-6) - 9$	-15
26.	$(-14) - 9$	-23
27.	$(-25) - 9$	-34
28.	$(-12) - 12$	-24
29.	$(-26) - 26$	-52
30.	$(-13) - 21$	-34
31.	$(-25) - 75$	-100
32.	$(-411) - 811$	-1,222
33.	$(-234) - 543$	-777
34.	$(-3) - (-1)$	-2
35.	$(-3) - (-2)$	-1
36.	$(-3) - (-3)$	0
37.	$(-3) - (-4)$	1
38.	$(-3) - (-8)$	5
39.	$(-30) - (-45)$	15
40.	$(-27) - (-13)$	-14
41.	$(-13) - (-27)$	14
42.	$(-4) - (-3)$	-1
43.	$(-3) - (-4)$	1
44.	$(-1,066) - (-34)$	-1,032

Number Correct: _____

Improvement: _____

Integer Subtraction – Round 2**Directions:** Determine the difference of the integers, and write it in the column to the right.

1.	$3 - 2$	
2.	$3 - 3$	
3.	$3 - 4$	
4.	$3 - 5$	
5.	$3 - 6$	
6.	$3 - 9$	
7.	$3 - 10$	
8.	$3 - 20$	
9.	$3 - 80$	
10.	$3 - 100$	
11.	$3 - (-1)$	
12.	$3 - (-2)$	
13.	$3 - (-3)$	
14.	$3 - (-7)$	
15.	$3 - (-17)$	
16.	$3 - (-27)$	
17.	$3 - (-127)$	
18.	$13 - (-6)$	
19.	$24 - (-8)$	
20.	$5 - (-23)$	
21.	$61 - (-3)$	
22.	$58 - (-5)$	

23.	$(-8) - 5$	
24.	$(-8) - 7$	
25.	$(-8) - 9$	
26.	$(-15) - 9$	
27.	$(-35) - 9$	
28.	$(-22) - 22$	
29.	$(-27) - 27$	
30.	$(-14) - 21$	
31.	$(-22) - 72$	
32.	$(-311) - 611$	
33.	$(-345) - 654$	
34.	$(-2) - (-1)$	
35.	$(-2) - (-2)$	
36.	$(-2) - (-3)$	
37.	$(-2) - (-4)$	
38.	$(-2) - (-8)$	
39.	$(-20) - (-45)$	
40.	$(-24) - (-13)$	
41.	$(-13) - (-24)$	
42.	$(-5) - (-3)$	
43.	$(-3) - (-5)$	
44.	$(-1,034) - (-31)$	



Integer Subtraction – Round 2 [KEY]

Directions: Determine the difference of the integers, and write it in the column to the right.

1.	$3 - 2$	1
2.	$3 - 3$	0
3.	$3 - 4$	-1
4.	$3 - 5$	-2
5.	$3 - 6$	-3
6.	$3 - 9$	-6
7.	$3 - 10$	-7
8.	$3 - 20$	-17
9.	$3 - 80$	-77
10.	$3 - 100$	-97
11.	$3 - (-1)$	4
12.	$3 - (-2)$	5
13.	$3 - (-3)$	6
14.	$3 - (-7)$	10
15.	$3 - (-17)$	20
16.	$3 - (-27)$	30
17.	$3 - (-127)$	130
18.	$13 - (-6)$	19
19.	$24 - (-8)$	32
20.	$5 - (-23)$	28
21.	$61 - (-3)$	64
22.	$58 - (-5)$	63

23.	$(-8) - 5$	-13
24.	$(-8) - 7$	-15
25.	$(-8) - 9$	-17
26.	$(-15) - 9$	-24
27.	$(-35) - 9$	-44
28.	$(-22) - 22$	-44
29.	$(-27) - 27$	-54
30.	$(-14) - 21$	-35
31.	$(-22) - 72$	-94
32.	$(-311) - 611$	-922
33.	$(-345) - 654$	-999
34.	$(-2) - (-1)$	-1
35.	$(-2) - (-2)$	0
36.	$(-2) - (-3)$	1
37.	$(-2) - (-4)$	2
38.	$(-2) - (-8)$	6
39.	$(-20) - (-45)$	25
40.	$(-24) - (-13)$	-11
41.	$(-13) - (-24)$	11
42.	$(-5) - (-3)$	-2
43.	$(-3) - (-5)$	2
44.	$(-1,034) - (-31)$	-1,003