



Lesson 5: Understanding Subtraction of Integers and Other Rational Numbers

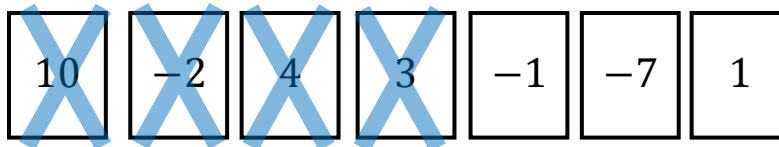
Student Outcomes

- Students justify the rule for subtraction: Subtracting a number is the same as adding its opposite.
- Students relate the rule for subtraction to the Integer Game: Removing (subtracting) a positive card changes the score in the same way as adding a corresponding negative card. Removing (subtracting) a negative card makes the same change as adding the corresponding positive card.
- Students justify the rule for subtraction for all rational numbers from the inverse relationship between addition and subtraction; i.e., subtracting a number and adding it back gets you back to where you started: $(m - n) + n = m$.

Classwork

Example 1 (7 minutes): Exploring Subtraction with the Integer Game

Students play the Integer Game 1 in groups of 3–4, recording what happens in their student materials as they select and discard cards from their hand. Students will use their previous knowledge of adding integers of same and opposite signs to help look for patterns when subtracting integers. In this example, students start with the cards 10, -2 , and 4. The “X” indicates the cards that are removed from the hand.



Example 1: Exploring Subtraction with the Integer Game

Play the Integer Game in your group. Start Round 1 by selecting four cards. Follow the steps for each round of play.

- Write the value of your hand in the Total column.
- Then, record what card values you select in the Action 1 column and discard from your hand in the Action 2 column.
- After each action, calculate your new total, and record it under the appropriate Results column.
- Based on the results, describe what happens to the value of your hand under the appropriate Descriptions column. For example, “Score increased by 3.”

¹ Refer to the Integer Game outline for player rules.

Round	Total	Action 1	Result 1	Description	Action 2	Result 2	Description
1	12	Select 3	15	Score (total) increased by 3	Discard 10	5	Score (total) decreased by 10
2	5	Select -1	4	Score (total) decreased by 1	Discard 4	0	Score (total) decreased by 4
3	0	Select -7	-7	Score (total) decreased by 7	Discard -2	-5	Score (total) increased by 2
4	-5	Select 1	-4	Score (total) increased by 1	Discard -7	3	Score (total) increased by 7
5	3						

Discussion (5 minutes): Making Connections to Integer Subtraction

The teacher leads class in a discussion. The objective of the discussion is to allow students the opportunity to discuss any patterns they noticed while playing the game; in particular, what happens to the value of the hand when cards with negative values are selected or discarded. The teacher poses questions to individual groups to elicit student feedback.

Discussion: Making Connections to Integer Subtraction

- How did selecting positive value cards change the value of your hand?
It increased my score by the value of the card.
- How did selecting negative value cards change the value of your hand?
It decreased my score by the absolute value of the card.
- How did discarding positive value cards change the value of your hand?
It decreased my score by the value of the card.
- How did discarding negative value cards change the value of your hand?
It increased my score by the absolute value of the card.
- What operation reflects selecting a card?
Addition
- What operation reflects discarding or removing a card?
Subtraction
- Based on the game, can you make a prediction about what happens to the result when
 - Subtracting a positive integer?
The result of the hand will decrease by the value of the integer.
 - Subtracting a negative integer?
The result of the hand will increase by the absolute value of the negative integer.

At the end of the lesson, the class will review its predictions.

Scaffolding:

- Display questions and give students time to discuss in their groups prior to whole-class discussion.
- Allow students to use whiteboards, number lines, or tables to formulate and justify their opinions to the group.
- Record selected student responses and examples on chart paper to help identify patterns.

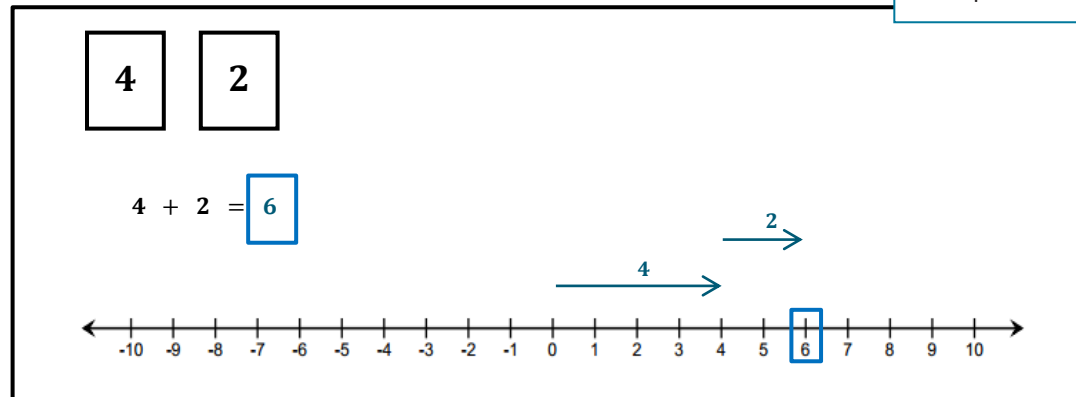
Example 2 (5 minutes): Subtracting a Positive Number

The teacher leads the whole class by modeling an Integer Game example to find the sum of $4 + 2$.

Example 2: Subtracting a Positive Number

Follow along with your teacher to complete the diagrams below.

If I had these two cards, the sum would be 6.

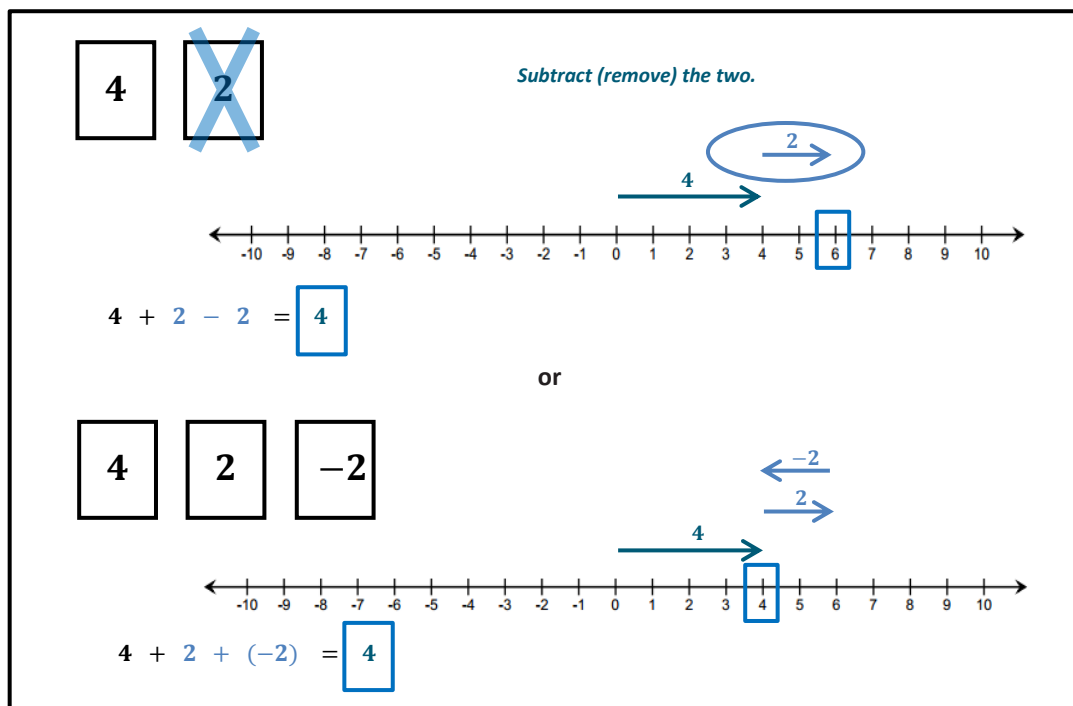


Scaffolding:

Allow students to use their Integer Cards throughout this example.

Show that discarding (subtracting) a positive card, which is the same as subtracting a positive number, decreases the value of the hand.

If I discarded or removed the 2, my score would decrease by 2 because I would still have a 4 left in my hand. $4 + 2 - 2 = 4$. Taking away, or subtracting, a 2 causes my score to decrease by 2.



Removing (subtracting) a positive card changes the score in the same way as adding a card whose value is the additive inverse (or opposite). In this case, adding the corresponding negative, such that $4 - 2 = 4 + (-2)$.

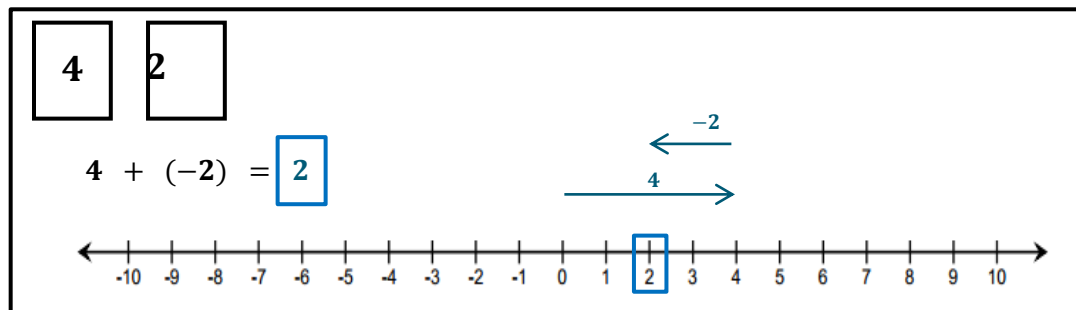
Subtracting a positive q -value is represented on the number line as moving to the left on a number line.

Example 3 (7 minutes): Subtracting a Negative Number

The teacher leads the whole class by modeling an Integer Game example to find the sum of $4 + (-2)$.

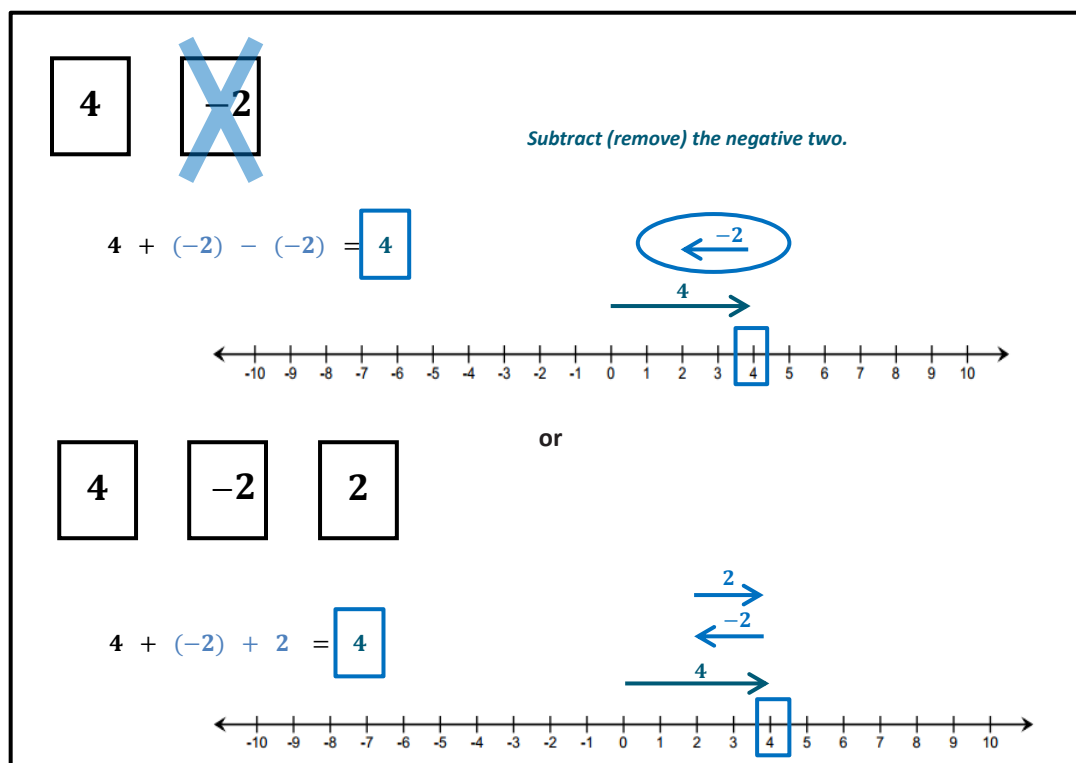
Example 3: Subtracting a Negative Number

Follow along with your teacher to complete the diagrams below.



How does removing a negative card change the score, or value, of the hand?

If I discarded or removed the -2 , my score would increase by 2 because I would still have a 4 left in my hand. $4 + (-2) - (-2) = 4$. Taking away, or subtracting, a -2 causes my score to increase by 2.



Removing (subtracting) a negative card changes the score in the same way as adding a card whose value is the additive inverse (or opposite). In this case, adding the corresponding positive such that $4 - (-2) = 4 + 2$.

Subtracting a negative q -value is represented on the number line as moving to the right on a number line because it is the opposite of subtracting a positive q -value (move to the left).

The Rule of Subtraction: Subtracting a number is the same as adding its additive inverse (or opposite).

Exercises 1–3 (8 minutes): Subtracting Positive and Negative Integers

Students will work independently to find the differences below. Students may use the number line as additional support. Before students work on the exercises, model the examples below to help students make the connection between subtraction and addition of the additive inverse.

- To solve the problem $8 - 12$

$8 + 12$ Step 1: Change the subtraction sign to addition. (Rule of subtraction)

$8 + (-12)$ Step 2: Change the positive 12 to a negative 12. (Rule of subtraction)

$$|8| = 8 \quad |-12| = 12$$

Steps 3–5: Follow the steps for adding numbers with opposite signs.

$$12 - 8 = 4 \quad \text{Subtract the absolute values.}$$

$$-4 \quad \text{Take the sign of the number with the greater absolute value.}$$

- Likewise, to solve the problem $4 - (-2)$

$4 + 2$ Step 1: Change the subtraction sign to addition and the -2 to 2. (Rule of subtraction).

$4 + 2 = 6$ Step 2: Follow the steps for adding numbers with same signs.

Scaffolding:

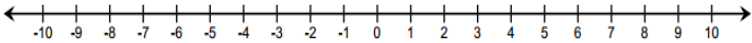
- Have students circle the integer with the greater absolute value to determine the final sign of the integer.

Exercises 1–3: Subtracting Positive and Negative Integers

- Using the rule of subtraction, rewrite the following subtraction sentences as addition sentences and solve. Use the number line below if needed.

a. $8 - 2$

$8 + (-2) = 6$



b. $4 - 9$

$4 + (-9) = -5$

c. $-3 - 7$

$-3 + (-7) = -10$

d. $-9 - (-2)$

$-9 + 2 = -7$

2. Find the differences.

a. $-2 - (-5)$

$$-2 + 5 = 3$$

b. $11 - (-8)$

$$11 + 8 = 19$$

c. $-10 - (-4)$

$$-10 + 4 = -6$$

3. Write two equivalent expressions that represent the situation. What is the difference in their elevations?

"An airplane flies at an altitude of 25,000 feet. A submarine dives to a depth of 600 feet below sea level."

$$25,000 - (-600) \text{ and } 25,000 + 600 = 25,600 \text{ feet}$$

Closing (6 minutes)

Summarize the rules for subtracting rational numbers by posing the following questions to the class.

- Review your predictions made earlier in class. Were you correct? If not, how were your predictions different from the correct responses?
 - *Answers will vary.*
- When playing the Integer Game, give two ways you can increase the value of your hand.
 - *To increase the value of your hand during the Integer Game, you can either pick up a positive card or discard a negative card.*
- Give two ways you can decrease the value of your hand.
 - *To decrease the value of your hand during the Integer Game, you can either pick up a negative card or discard a positive card.*

Lesson Summary

- **The Rule for Subtraction:** Subtracting a number is the same as adding its opposite.
- Removing (subtracting) a positive card changes the score in the same way as adding a corresponding negative card.
- Removing (subtracting) a negative card makes the same change as adding the corresponding positive card.
- For all rational numbers, subtracting a number and adding it back gets you back to where you started:
 $(m - n) + n = m$.

Exit Ticket (7 minutes)

Name _____

Date _____

Lesson 5: Understanding Subtraction of Integers and Other Rational Numbers

Exit Ticket

1. If a player had the following cards, what is the value of his hand?

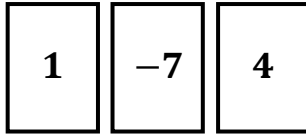
1	-7	4
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- a. Identify two different ways the player could get to a score of 5 by adding or removing only one card. Explain.
- b. Write two equations for part (a), one for each of the methods you came up with for arriving at a score of 5.
2. Using the rule of subtraction, rewrite the following subtraction expressions as addition expressions and find the sums.
- a. $5 - 9$
- b. $-14 - (-2)$

Exit Ticket Sample Solutions

1. If a player had the following cards, what is the value of his hand?

The current value of the hand is -2 . $1 + (-7) + 4 = -2$.



- a. Identify two different ways the player could get to a score of 5 by adding or removing only one card. Explain.

He could remove the -7 or add 7. If we remove the -7 , the value of the hand will be 5, which is 7 larger than -2 . I could also get a sum of 5 by adding 7 to the hand. Therefore, removing the -7 gives me the same result as adding 7.

- b. Write two equations for part (a), one for each of the methods you came up with for arriving at a score of 5.

$-2 - (-7)$ and $-2 + 7 = 5$

2. Using the rule of subtraction, rewrite the following subtraction expressions as addition expressions and find the sums.

a. $5 - 9$

$5 + (-9) = -4$

b. $-14 - (-2)$

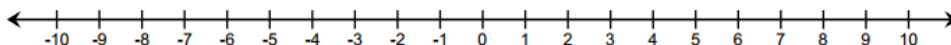
$-14 + 2 = -12$

Problem Set Sample Solutions

The Problem Set provides students with skill practice and application of the rules for integer subtraction. Students will solve problems with and without a number line.

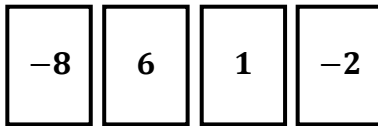
1. On a number line, find the difference of each number and 4? Complete the table to support your answers. The first example is provided.

Number	Subtraction Expression	Addition Expression	Answer
10	$10 - 4$	$10 + (-4)$	6
2	$2 - 4$	$2 + (-4)$	-2
-4	$-4 - 4$	$-4 + (-4)$	-8
-6	$-6 - 4$	$-6 + (-4)$	-10
1	$1 - 4$	$1 + (-4)$	-3

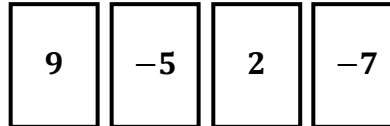


2. You and your partner were playing the Integer Game in class. Here are the cards in both hands.

Your hand



Your partner's hand



- a. Find the value of each hand. Who would win based on the current scores? (The score closest to 0 wins.)

My hand: $-8 + 6 + 1 + (-2) = -3$

Partner's hand: $9 + (-5) + 2 + (-7) = -1$

My partner would win because -1 is closer to 0. It is 1 unit to the left of 0.

- b. Find the value of each hand if you discarded the -2 and selected a 5, and your partner discarded the -5 and selected a 5. Show your work to support your answer.

My hand: Discard the -2, $-3 - (-2) = -1$; Select a 5: $-1 + 5 = 4$.

Partner's hand: Discard the -5, $-1 - (-5) = 4$; Select a 5: $4 + 5 = 9$.

- c. Use your score values from part (b) to determine who would win the game now.

I would win now because 4 is closer to zero.

3. Write the following expressions as a single integer.

a. $-2 + 16$

14

b. $-2 - (-16)$

14

c. $18 - 26$

-8

d. $-14 - 23$

-37

e. $30 - (-45)$

75

4. Explain what is meant by the following and illustrate with an example:

"For any real numbers, p and q , $p - q = p + (-q)$."

Subtracting a number is the same as adding its additive inverse. Examples will vary. A sample response is shown below.

$p = 4$, $q = 6$, $4 - 6$ is the same as $4 + (-6)$ because -6 is the opposite of 6.

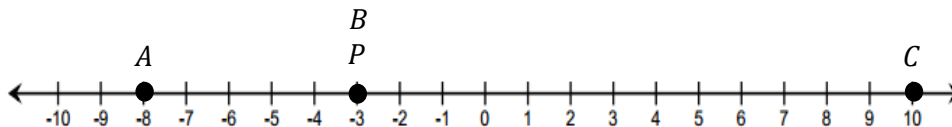
$$4 - 6 = -2$$

$$4 + (-6) = -2$$

So, $4 - 6 = 4 + (-6)$ because they both equal -2 .

5. Choose an integer between -1 and -5 on the number line, and label it point P . Locate and label the following points on the number line. Show your work.

Answers will vary. A sample response is shown below given the student chose -3 for P .



- a. Point A: $P - 5$

Point A: $-3 - 5 = -8$

- b. Point B: $(P - 4) + 4$

Point B: $(-3 - 4) + 4 = -3$ (same as P)

- c. Point C: $-P - (-7)$

Point C: $-(-3) - (-7) = 3 + 7 = 10$

Challenge Problem:

6. Write two equivalent expressions that represent the situation. What is the difference in their elevations?

"An airplane flies at an altitude of 26,000 feet. A submarine dives to a depth of 700 feet below sea level."

Two equivalent expressions are $26,000 - (-700)$ and $26,000 + 700$. The difference in their elevations is 26,700 feet.