Lesson 3: Understanding Addition of Integers

Student Outcomes

- Students understand addition of integers as putting together or counting up. For negative numbers "counting up" is actually counting down.
- Students use arrows to show the sum of two integers, $p + q$, on a number line and to show that the sum is distance $|q|$ from p to the right if q is positive and to the left if q is negative.
- Students refer back to the Integer Game to reinforce their understanding of addition.

Classwork

Exercise 1 (15 minutes): Addition Using the Integer Game

> **Exercise 1: Addition Using the Integer Game**
>
> **Play the Integer Game with your group without using a number line.**

In pairs, students will play a modified version of the Integer Game[1] without a number line. Monitor student play and ask probing questions. When students share at the end of the game, see if anyone used the concept of additive inverse, if the opportunity occurred, when adding.

> *Scaffolding:*
>
> - Allow for the use of a number line for ELL students if needed.

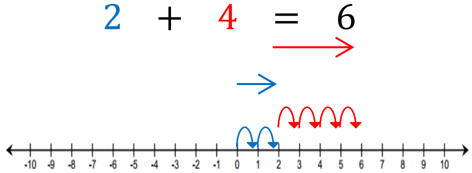
Example 1 (10 minutes): "Counting On" to Express the Sum as Absolute Value on a Number Line

The teacher leads whole class instruction using vector addition to (1) review the sum of two integers on a real number line horizontally and vertically and (2) show that the sum is the distance of the absolute value of the ***q*-value** (second addend) from the ***p*-value** (first addend).

> **Example 1: "Counting On" to Express the Sum as Absolute Value on a Number Line**

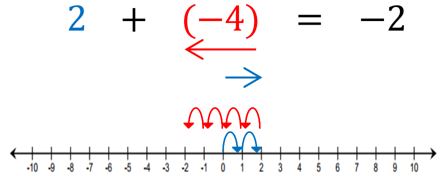
Model of Counting Up

$$2 + 4 = 6$$



Model of Counting Down

$$2 + (-4) = -2$$



[1] Refer to the Integer Game outline for complete player rules. In Exercise 1, cards are shuffled and placed face down. Players draw three cards each and calculate the sums of their hands. Once they each have the sum of their three cards, players put down their cards face up. Next, they will find the sum of all six cards that they have collectively.

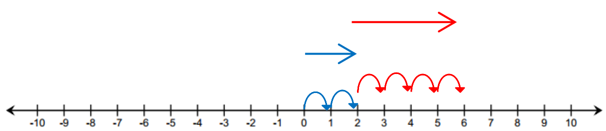
Horizontal Number Line Model of Counting Up

$$2 + 4 = 6$$

p-value
Start at 0; count up 2 units to the right.
Arrow is 2 units long and points to the right.

q-value
$|4| = 4$
Absolute value of 4 is 4.
Start at 2; count up 4 units to the right.
Arrow is 4 units long and points to the right.

Sum
6 is 4 units away from 2.



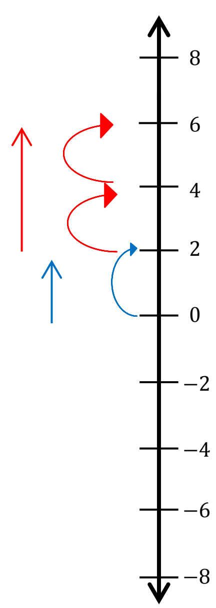
Vertical Number Line Model of Counting Up

$$2 + 4 = 6$$

p-value
Start at 0; count up 2 units above 0.
Arrow is 2 units long and points up.

q-value
$|4| = 4$
Absolute value of 4 is 4.
Start at 2; count up 4 units.
Arrow is 4 units long and points up.

Sum
6 is 4 units above 2.



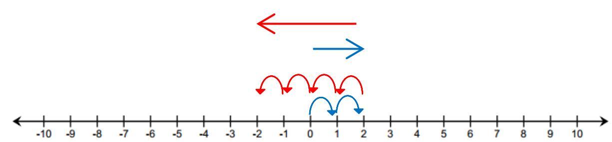
Horizontal Number Line Model of Counting Down

$$2 + (-4) = -2$$

p-value
Start at 0; count up 2 units to the right.
Arrow is 2 units long and points to the right.

q-value
$|-4| = 4$
Absolute value of -4 is 4.
Start at 2; count down 4 units to the left.
Arrow is 4 units long and points to the left.

Sum
-2 is 4 units away from 2.



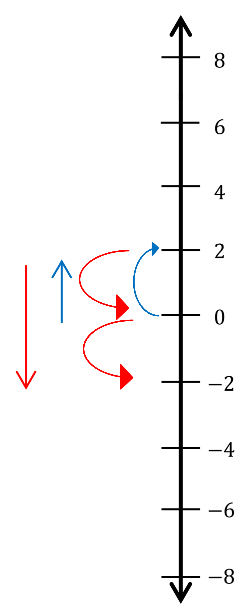
Vertical Number Line Model of Counting Down

$$2 + (-4) = -2$$

p-value
Start at 0; count up 2 units above 0.
Arrow is 2 units long and points up.

q-value
$|-4| = 4$
Absolute value of -4 is 4.
Start at 2; count down 4 units.
Arrow is 4 units long and points down.

Sum
-2 is 4 units below 2.



The teacher poses the following questions to the class for open discussion. Students record their responses in the space provided.

Counting up -4 is the same as "the opposite of counting up 4" and also means counting down 4.

a. **For each example above, what is the distance between 2 and the sum?**

4 units

b. **Does the sum lie to the right or left of 2 on a horizontal number line? Above or below on a vertical number line?**

Horizontal: On the first model, the sum lies to the right of 2. On the second model, it lies to the left of 2.

Vertical: On the first model, the sum lies above 2. On the second model, it lies below 2.

c. **Given the expression $54 + 81$, determine, without finding the sum, the distance between 54 and the sum. Explain.**

The distance will be 81 units. When the q-value is positive, the sum will be to the right of (or above) the p-value the same number of units as the q-value.

d. **Is the sum to the right or left of 54 on the horizontal number line? Above or below on a vertical number line?**

The sum is to the right of 54 on a horizontal number line and above 54 on a vertical number line.

e. **Given the expression $14 + (-3)$, determine, without finding the sum, the distance between 14 and the sum? Explain.**

The distance will be 3 units. When the q-value is negative, the sum will be to the left of (or below) the p-value the same number of units as the q-value.

f. **Is the sum to the right or left of 14 on the number line? Above or below on a vertical number line?**

The sum is to the left of 14 on a horizontal number line and below 14 on a vertical number line.

Scaffolding:

- Review the concept of "sum" with the whole class for ELL students.
- Provide written stems for ELL students. For example, "The sum is ____ units to the ____ of ____."

Exercise 2 (5 minutes)

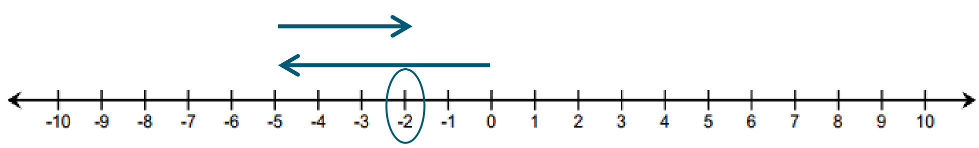
Students work in pairs to create a number line model to represent each of the following expressions. After 2 or 3 minutes, students are selected to share their responses and work with the class. Ask students to describe the sum using distance from the first addend along the number line.

Exercise 2

Work with a partner to create a horizontal number line model to represent each of the following expressions. Describe the sum using distance from the p-value along the number line.

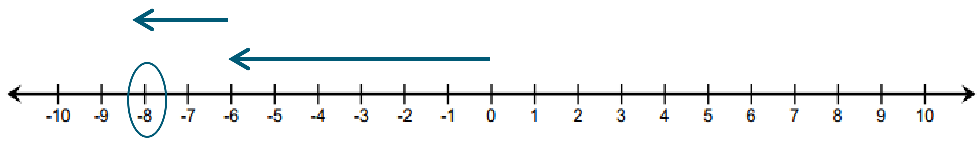
a. $-5 + 3$

$-5 + 3 = -2$. *The sum is 3 units to the right of -5.*



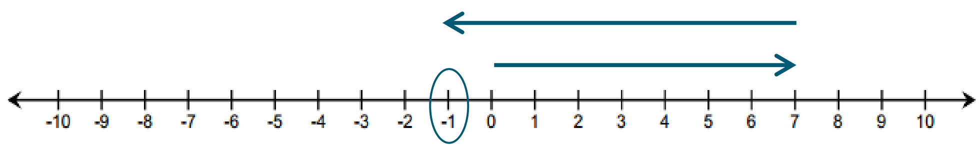
b. $-6 + (-2)$

$-6 + (-2) = -8$. The sum is 2 units to the left of -6.



c. $7 + (-8)$

$7 + (-8) = -1$. The sum is 8 units to the left of 7.



Exercise 3 (5 minutes): Writing an Equation Using Verbal Descriptions

Students continue to work in pairs to complete the following task.

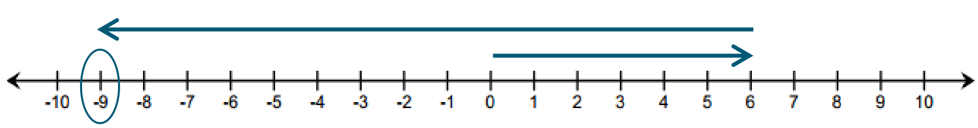
Exercise 3: Writing an Equation Using Verbal Descriptions

Write an equation, and using the number line, create an "arrow diagram" given the following information:

"The p-value is 6, and the sum lies 15 units to the left of the p-value."

Equation:

$6 + (-15) = -9$



Closing (3 minutes)

The teacher uses whole-group discussion with students verbally stating the answers to the following questions.

- What role does the $|-16| = 16$ play in modeling the expression $2 + (-16)$?
  - *The absolute value of the second value (q) represents the distance between the first addend (p) and the sum.*
- What is one important fact to remember when modeling addition on a horizontal number line? On a vertical number line?
  - *One important fact to remember when adding integers on a number line is that counting up a negative number of times is the same as counting down.*

- What is the difference between counting up and counting down?
  - *Counting up represents a positive addend and counting down represents a negative addend.*

> **Lesson Summary**
>
> - **Addition of integers is represented on a number line as "counting up," where counting up a negative number of times is the same as "counting down."**
> - **Arrows show the sum of two integers on a number line.**
> - **The sum is the distance $|q|$ from the p*-value (the first addend)* to the right if q is positive and to the left if q is negative.**

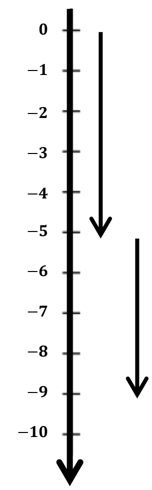
Exit Ticket (7 minutes)

Name ____ Date ____________________

Lesson 3: Understanding Addition of Integers

Exit Ticket

1. Refer to the diagram to the right.



   a. Write an equation for the diagram to the right. ______________________

   b. Find the sum. ______________________

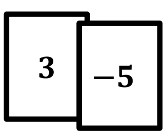
   c. Describe the sum in terms of the distance from the p-value. Explain.

   d. What integers do the arrows represent? ________________________

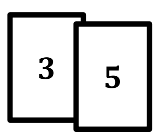
2. Jenna and Jay are playing the Integer Game. Below are the two cards they selected.

   a. How do the models for these two addition problems differ on a number line? How are they the same?

Jenna's Hand

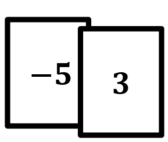


Jay's Hand

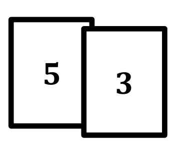


   b. If the order of the cards changed, how do the models for these two addition problems differ on a number line? How are they the same?

Jenna's Hand

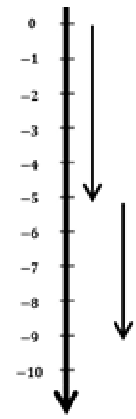


Jay's Hand



Exit Ticket Sample Solutions

1. Refer to the diagram to the right.



   a. Write an equation for the diagram below.

      $-5 + (-4) = -9$

   b. Find the sum.

      -9

   c. Describe the sum in terms of the distance from the p-value. Explain.

      The sum is 4 units below -5 because $|-4| = 4$. I counted down from -5 four times and stopped at -9.

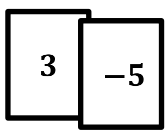
   d. What integers do the arrows represent?

      The arrows represent the integers -4 and -5.

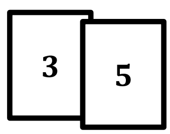
2. Jenna and Jay are playing the Integer Game. Below are the two cards they selected.

   a. How do the models for these two addition problems differ on a number line? How are they the same?

      Jenna's Hand



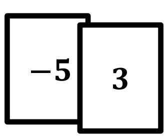
      Jay's Hand



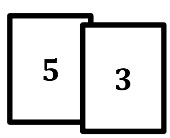
      The p-values are the same. They are both 3, so the heads of the first arrows will be at the same point on the number line. The sums will both be five units from this point but in opposite directions.

   b. If the order of the cards changed, how do the models for these two addition problems differ on a number line? How are they the same?

      Jenna's Hand



      Jay's Hand



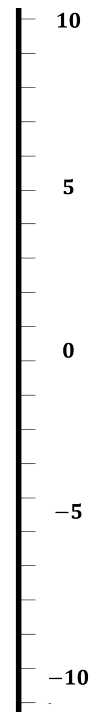
      The p-values are different, so the head of the first arrow in each model will be at different points on the number line. The sums are both three units to the right of the p-values.

Problem Set Sample Solutions

Practice problems will help students build fluency and improve accuracy when adding integers with and without the use of a number line. Students need to be comfortable with using vectors to represent integers on the number line, including the application of absolute value to represent the length of a vector.

1. Below is a table showing the change in temperature from morning to afternoon for one week.

a. Use the vertical number line to help you complete the table. As an example, the first row is completed for you.



Change in Temperatures from Morning to Afternoon

Morning Temperature	Change	Afternoon Temperature	Equation
$1°\text{C}$	Rise of $3°\text{C}$	$4°\text{C}$	$1+3=4$
$2°\text{C}$	Rise of $8°\text{C}$	$10°\text{C}$	$2+8=10$
$-2°\text{C}$	Fall of $6°\text{C}$	$-8°\text{C}$	$-2+(-6)=-8$
$-4°\text{C}$	Rise of $7°\text{C}$	$3°\text{C}$	$-4+7=3$
$6°\text{C}$	Fall of $9°\text{C}$	$-3°\text{C}$	$6+(-9)=-3$
$-5°\text{C}$	Fall of $5°\text{C}$	$-10°\text{C}$	$-5+(-5)=-10$
$7°\text{C}$	Fall of $7°\text{C}$	$0°\text{C}$	$7+(-7)=0$

b. Do you agree or disagree with the following statement: "A rise of $-7°C$" means "a fall of $7°C$?" Explain. (Note: No one would ever say, "A rise of -7 degrees;" however, mathematically speaking, it is an equivalent phrase.)

Sample response: I agree with this statement because a rise of -7 is the opposite of a rise of 7. The opposite of a rise of 7 is a fall of 7.

For Questions 2–3, refer to the Integer Game.

2. Terry selected two cards. The sum of her cards is -10.

a. Can both cards be positive? Explain why or why not.

No. In order for the sum to be -10, one of the addends would have to be negative. If both cards are positive, then Terry would count up twice going to the right. Negative integers are to the left of 0.

b. Can one of the cards be positive and the other be negative? Explain why or why not.

Yes. Since both cards cannot be positive, this means that one can be positive and the other negative. She could have -11 and 1 or -12 and 2. The card with the greatest absolute value would have to be negative.

c. Can both cards be negative? Explain why or why not.

Yes, both cards could be negative. She could have -8 and -2. On a number line, the sum of two negative integers will be to the left of 0.

3. When playing the Integer Game, the first two cards you selected were -8 and -10.

a. What is the value of your hand? Write an equation to justify your answer.

$-8+(-10)=-18$

b. For part (a), what is the distance of the sum from -8? Does the sum lie to the right or left of -8 on the number line?

The distance is 10 units from -8, and it lies to the left of -8 on the number line.

c. If you discarded the -10 and then selected a 10, what would be the value of your hand? Write an equation to justify your answer.

The value of the hand would be 2. $-8+10=2$.

4. Given the expression $67+(-35)$, can you determine, without finding the sum, the distance between 67 and the sum? Is the sum to the right or left of 67 on the number line?

The distance would be 35 units from 67. The sum is to the left of 67 on the number line.

5. Use the information given below to write an equation. Then create an "arrow diagram" of this equation on the number line provided below.

"The p-value is -4, and the sum lies 12 units to the right of the p-value."

$-4+12=8$