Lesson 2: Using the Number Line to Model the Addition of Integers

Student Outcomes

- Students model integer addition on the number line by using horizontal arrows; e.g., an arrow for -2 is a horizontal arrow of length 2 pointing in the negative direction.
- Students recognize that the length of an arrow on the number line is the absolute value of the integer.
- Students add arrows (realizing that adding arrows is the same as combining numbers in the Integer Game). Given several arrows, students indicate the number that the arrows represent (the sum).

Classwork

Exercise 1 (5 minutes): Real-World Introduction to Integer Addition

MP.4 Students answer the following question independently, as the teacher circulates around the room providing guidance and feedback as needed. Students focus on how to represent the answer using both an equation and a number line diagram. They will be able to make the connection between both representations.

Scaffolding:

- Create an anchor poster for the additive inverse to help access prior knowledge of number line features including arrow placement and direction and ordering of positive and negative numbers.

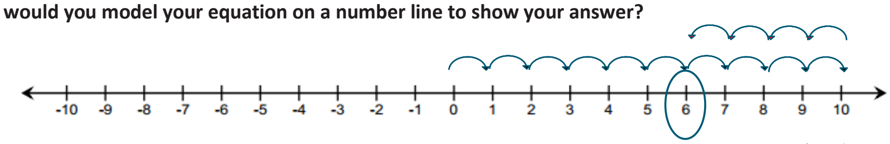
Exercise 1: Real-World Introduction to Integer Addition

Answer the questions below.

a. **Suppose you received $\$10$ from your grandmother for your birthday. You spent $\$4$ on snacks. Using addition, how would you write an equation to represent this situation?**

$10 + (-4) = 6.$

b. **How would you model your equation on a number line to show your answer?**



Real-world situations can be modeled with equations and represented on a real number line. In this exercise, positive ten represents the "$10 given as a birthday gift" because it is a gain. Negative four represents the "$4 spent on snacks" because it is a loss. Gaining $10 and then taking away $4 will leave you with $6.

Example 1 (5 minutes): Modeling Addition on the Number Line

The teacher models addition on a number line using straight arrows (vectors) to find the sum of $-2 + 3$. Elicit student responses to assist in creating the steps. Students record the steps and diagram.

- Place the tail of the arrow on 0.
- Draw the arrow 2 units to the left of 0, and stop at -2. The direction of the arrow is to the left since you are counting down from 0.
- Start the next arrow at the end of the first arrow or at -2.
- Draw the second arrow 3 units to the right since you are counting up from -2.
- Stop at 1.
- Circle the number at which the second arrow ends to indicate the ending value.

Scaffolding:

- Use counters or chips to transfer prior learning of additive inverse or zero pairs.
- Create a number line model on the floor for kinesthetic and visual learners.

Using the example, model a real-world story problem for the class.

- If the temperature outside were 2 degrees below zero and it increased by 3 degrees, the new temperature outside would be 1 degree.

Have students share a story problem involving temperature, money, or sea level that would describe the number line model. Select a few students to share their answers with the class.

- *Answers will vary. I owed my brother* $\$2$, *and my dad gave me* $\$3$. *I paid my brother, and now I have* $\$1$ *left over.*

Example 1: Modeling Addition on the Number Line

Complete the steps to find the sum of $-2+3$ by filling in the blanks. Model the equation using straight arrows called *vectors* on the number line below.

a. **Place the tail of the arrow on $\underline{\ 0\ }$.**

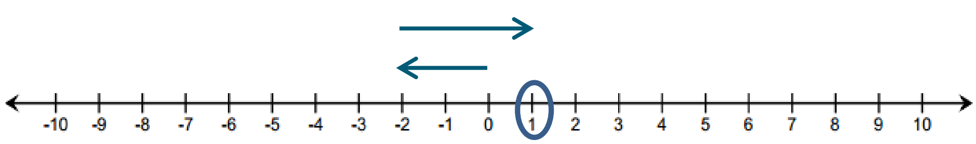
b. **Draw the arrow 2 units to the left of 0, and stop at $\underline{\ -2\ }$. The direction of the arrow is to the $\underline{\ \textit{left}\ }$ since you are counting down from 0.**

c. **Start the next arrow at the end of the first arrow or at $\underline{\ -2\ }$.**

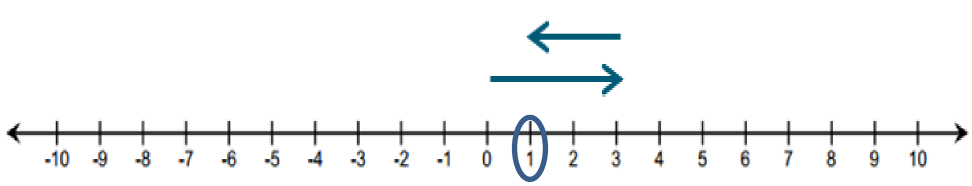
d. **Draw the second arrow $\underline{\ 3\ }$ units to the right since you are counting up from -2.**

e. **Stop at $\underline{\ 1\ }$.**

f. **Circle the number at which the second arrow ends to indicate the ending value.**



g. **Repeat the process from parts (a)–(f) for the expression $3+(-2)$.**



h. **What can you say about the sum of $-2+3$ and $3+(-2)$? Does order matter when adding numbers? Why or why not?**

$-2+3$ is the same as $3+(-2)$ because they both equal 1. The order does not matter when adding numbers because addition is commutative.

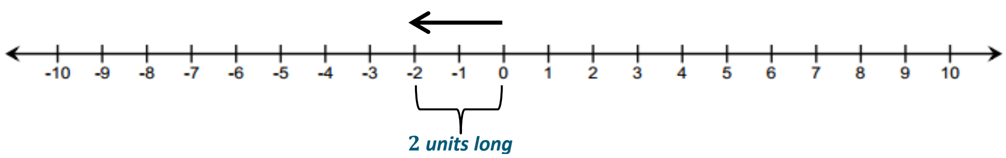
Example 2 (3 minutes): Expressing Absolute Value as the Length of an Arrow on the Real Number Line

The teacher models absolute value as the length of an arrow. Students recall that absolute value represents distance.

Example 2: Expressing Absolute Value as the Length of an Arrow on the Real Number Line

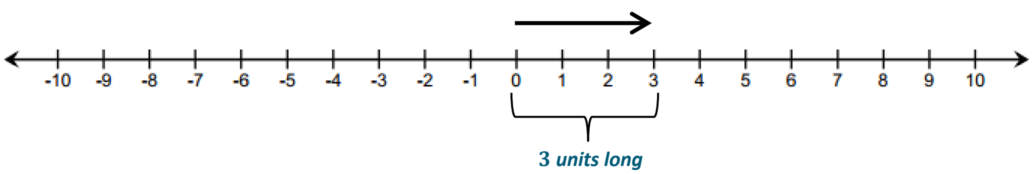
a. **How does absolute value determine the arrow length for -2?**

$|-2| = 2$, so the arrow is 2 units long. Because -2 is a negative number, the arrow points to the left.



b. **How does the absolute value determine the arrow length for 3?**

$|3| = 3$, so the arrow is 3 units long. Because 3 is positive, the arrow points to the right.



c. **How does absolute value help you to represent -10 on a number line?**

The absolute value can help me because it tells me how long my arrow should be when starting at 0 on the real number line. The $|-10| = 10$, so my arrow will be 10 units in length.

Exercise 2 (5 minutes)

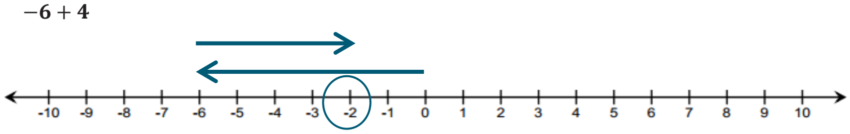
Students work independently to create a number line model to represent each of the expressions below. After 2–3 minutes, students are selected to share their responses and work with the class. Monitor student work by paying careful attention to common mistakes such as miscounting, not lining up arrows head-to-tail, and starting both arrows at 0.

MP.6

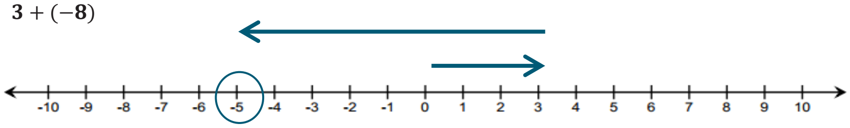
Exercise 2

Create a number line model to represent each of the expressions below.

a. $-6 + 4$



b. $3 + (-8)$



Scaffolding:

- Have early finishers explain how absolute value determined the arrow lengths for each of the addends and how they knew each arrow's direction.

Example 3 (5 minutes): Finding Sums on a Real Number Line Model

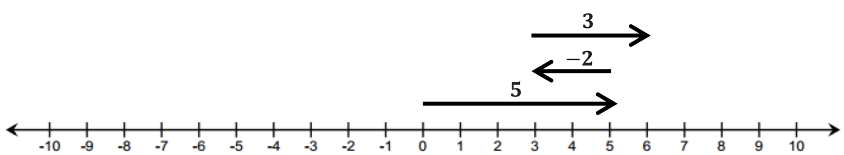
The teacher refers to the Integer Game from Lesson 1. Pose discussion questions to the class.

Scaffolding:

- Have students use their same cards to create a different addition number sentence and a new number line representation.
- Have students examine how the diagram changes when the order of addition changes to reinforce the commutative property.

Example 3: Finding Sums on a Real Number Line Model

Find the sum of the integers represented in the diagram below.



a. **Write an equation to express the sum.**

$$5+(-2)+3=6$$

b. **What three cards are represented in this model? How did you know?**

The cards are 5, -2, and 3 because the arrows show their lengths.

c. **In what ways does this model differ from the ones we used in Lesson 1?**

In Lesson 1, a movement of 5 units was shown with 5 separate hops. In this lesson, 5 units are shown as one total movement with a straight arrow. Both represent the same total movement.

d. **Can you make a connection between the sum of 6 and where the third arrow ends on the number line?**

The final position of the third arrow is 6. This means that the sum is 6.

e. **Would the sum change if we changed the order in which we add the numbers, for example, $(-2)+3+5$?**

No because addition is commutative. Order does not matter.

f. **Would the diagram change? If so, how?**

Yes, the first arrow would start at 0 and point left 2 units. The second arrow would start at -2 and point right 3 units. The third arrow would start at 1 and point 5 units right but still end on 6.

Exercise 3 (14 minutes)

In groups of 3–4 students play the Integer Game[1]. The objective of the game for Lesson 2 is to get as close to 0 as possible. During play, students work independently to create an equation and number line diagram to model integer addition. Monitor the classroom and ask probing questions.

Exercise 3

Play the Integer Game with your group. Use a number line to practice "counting on."

[1] Refer to the Integer Game outline for player rules.

Closing (3 minutes)

The teacher initiates whole-group discussion prompting students to verbally state the answers to the following questions:

- How can we use a number line to model and find the sum of $-8+5$?
  - *We would start at 0 and then draw an arrow eight units to the left to represent -8. From the end of this arrow you would draw an arrow five units to the right to represent 5. The number the final arrow ends on is the sum of $-8+5$.*
- What does the absolute value of a number tell you?
  - *The absolute value of a number tells us the length of the arrow.*

Lesson Summary

- **On a number line, arrows are used to represent integers; they show length and direction.**
- **The length of an arrow on the number line is the absolute value of the integer.**
- **Adding several arrows is the same as combining integers in the Integer Game.**
- **The sum of several arrows is the final position of the last arrow.**

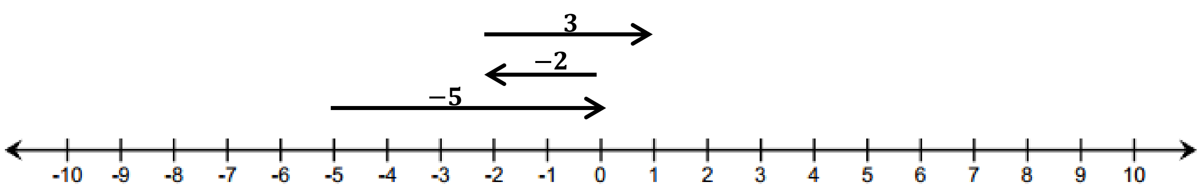
Exit Ticket (5 minutes)

Name ____ Date ____________________

Lesson 2: Using the Number Line to Model the Addition of Integers

Exit Ticket

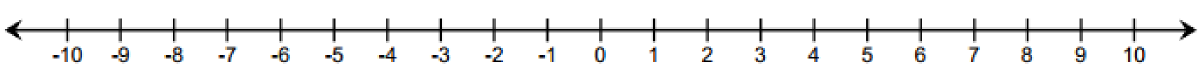
Jessica made the addition model below of the expression $(-5) + (-2) + 3$.



a. Do the arrows correctly represent the numbers that Jessica is using in her expression?

b. Jessica used the number line diagram above to conclude that the sum of the three numbers is 1. Is she correct?

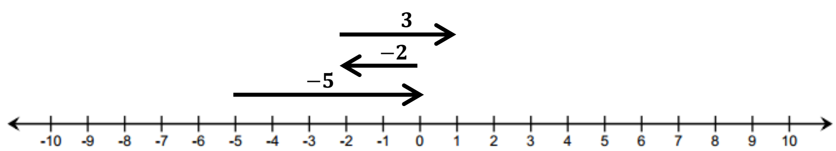
c. If she is incorrect, find the sum, and draw the correct model.



d. Write a real-world situation that would represent the sum.

Exit Ticket Sample Solutions

Jessica made the addition model below of the expression $(-5) + (-2) + 3$.



a. **Do the arrows correctly represent the numbers that Jessica is using in her expression?**

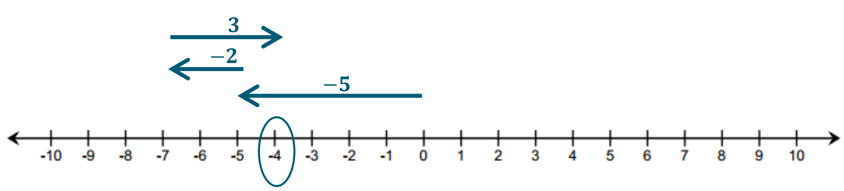
No. Jessica started her first arrow at -5 instead of 0. Negative numbers should be shown as counting down, so the arrow should have started at 0 and pointed left, ending on -5. The other arrows are drawn correctly, but they are in the wrong places because the starting arrow is in the wrong place.

b. **Jessica used the number line diagram above to conclude that the sum of the three numbers is 1. Is she correct?**

Jessica is incorrect.

c. **If she is incorrect, find the sum, and draw the correct model.**

The sum should be -4. $-5 + (-2) + 3 = -4$.



d. **Write a real-world situation that would represent the sum.**

Answers will vary. A football team lost 5 yards on the first play. On the second play, the team lost another 2 yards. Then, the team gained 3 yards. After three plays, the team had a total yardage of -4 yards.

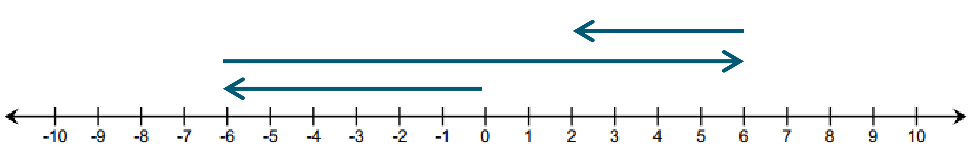
Problem Set Sample Solutions

The Problem Set provides students practice with integer addition using the Integer Game, number lines, and story problems. Students should show work with accuracy in order to demonstrate mastery.

For Questions 1–3, represent each of the following problems using both a number line diagram and an equation.

1. **David and Victoria are playing the Integer Card Game. David drew three cards, -6, 12, and -4. What is the sum of the cards in his hand? Model your answer on the number line below.**

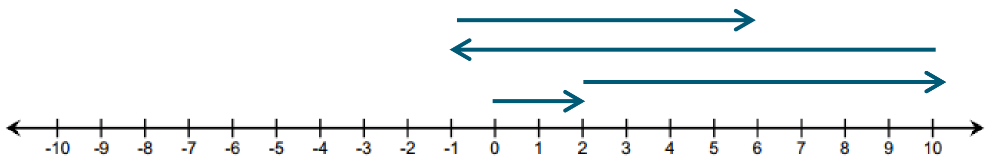
   $(-6) + 12 + (-4) = 2$



2. In the Integer Card Game, you drew the cards, 2, 8, and -11. Your partner gave you a 7 from his hand.

   a. What is your total? Model your answer on the number line below.

   $2 + 8 + (-11) + 7 = 6$



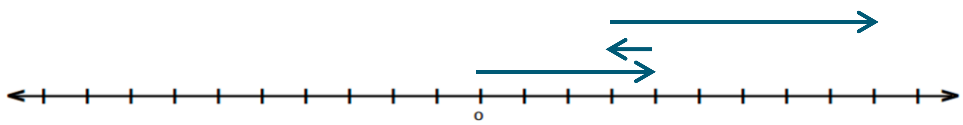
   b. What card(s) would you need to get your score back to zero? Explain. Use and explain the term "additive inverse" in your answer.

   You would need any combination of cards that sum to -6 because the additive inverse of 6 is -6.

   $6 + (-6) = 0$

3. If a football player gains 40 yards on a play, but on the next play, he loses 10 yards, what would his total yards be for the game if he ran for another 60 yards? What did you count by to label the units on your number line?

   90 yards because $40 + (-10) + 60 = 90$. Student answers may vary, but they should not choose to count by 1.



4. Find the sums.

   a. $-2 + 9$

   7

   b. $-8 + -8$

   -16

   c. $-4 + (-6) + 10$

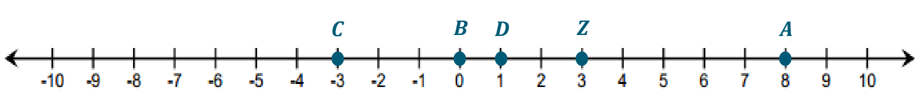
   0

   d. $5 + 7 + (-11)$

   1

5. Mark an integer between 1 and 5 on a number line, and label it point Z. Then, locate and label each of the following points by finding the sums.

   Answers will vary. Sample student response below.



   a. Point A: $Z + 5$

   Point A: $3 + 5 = 8$

b. Point B: $Z + (-3)$

Point B: $3 + (-3) = 0$

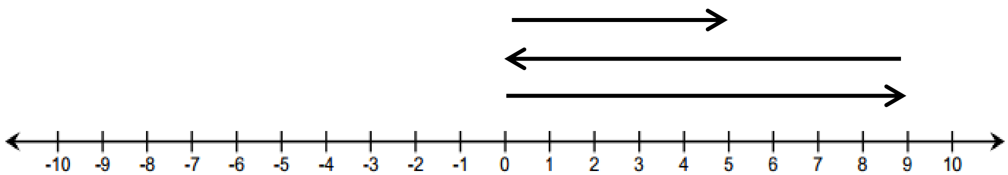
c. Point C: $(-4) + (-2) + Z$

Point C: $(-4) + (-2) + 3 = -3$

d. Point D: $-3 + Z + 1$

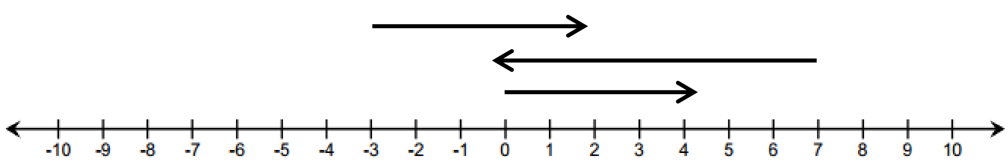
Point D: $-3 + 3 + 1 = 1$

6. Write a story problem that would model the sum of the arrows in the number diagram below.



Answers will vary. Jill got on an elevator and went to the 9th floor. She accidently pressed the down button and went back to the lobby. She pressed the button for the 5th floor and got off the elevator.

7. Do the arrows correctly represent the equation $4 + (-7) + 5 = 2$? If not, draw a correct model below.



No, the arrows are incorrect. The correct model is shown.