Lesson 1: Opposite Quantities Combine to Make Zero

Student Outcomes

- Students add positive integers by counting up and negative integers by counting down (using curved arrows on the number line).
- Students play the Integer Game to combine integers, justifying that an integer plus its opposite add to zero.
- Students know the opposite of a number is called the additive inverse because the sum of the two numbers is zero.

Lesson Notes

There is some required preparation from teachers before the lesson begins. It is suggested that number lines are provided for students. However, it is best if students can reuse these number lines by having them laminated and using white board markers. Also, the Integer Game is used during this lesson, so the teacher should prepare the necessary cards for the game.

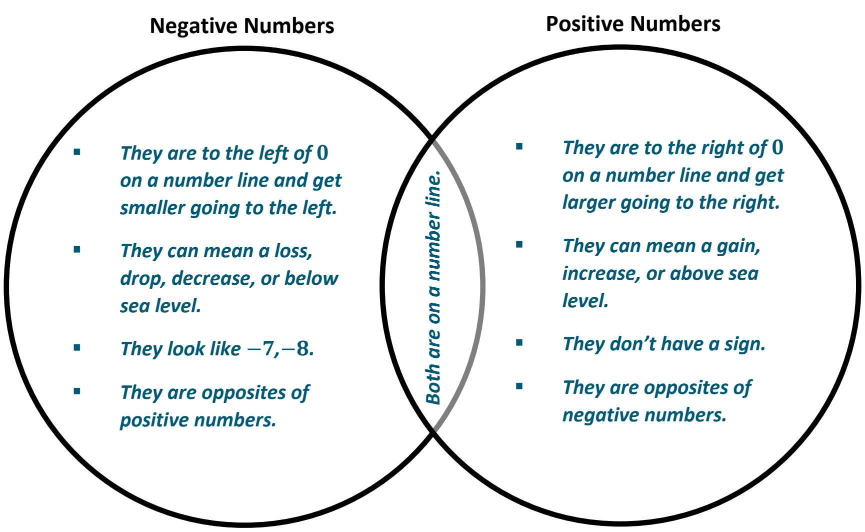
Classwork

Exercise 1 (3 minutes): Positive and Negative Numbers Review

In pairs, students will discuss "What I Know" about positive and negative integers to access prior knowledge. Have them record and organize their ideas in the graphic organizer in the student materials. At the end of discussion, the teacher will choose a few pairs to share out with the class.

Exercise 1: Positive and Negative Numbers Review

With your partner, use the graphic organizer below to record what you know about positive and negative numbers. Add or remove statements during the whole class discussion.



Scaffolding:

- Laminate (or place in sheet protectors) 1-page of number lines (vary blank and numbered) for individual use with white board markers.
- Create a number line on the floor using painters tape to model the "counting on" principle.
- Provide a wall model of the number line at the front of the room for visual reinforcement.

Example 1 (5 minutes): Introduction to the Integer Game

Read the Integer Game outline before the lesson. The teacher selects a group of 3 or 4 students to demonstrate to the whole class how to play the Integer Game.[1] The game will be played later in the lesson. The teacher should stress that the object of the game is to get a score of zero.

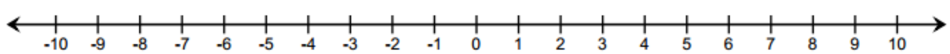
Example 2 (5 minutes): Counting Up and Counting Down on the Number Line

Model a few examples of counting with small curved arrows to locate numbers on the number line, where *counting* up corresponds to positive numbers and *counting down* corresponds to negative numbers.

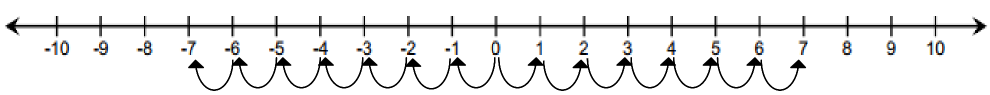
Example 2: Counting Up and Counting Down on the Number Line

Use the number line below to practice counting up and counting down.

- ***Counting up*** **corresponds to** ___*positive*___ **numbers.**
- ***Counting down*** **corresponds to** ___*negative*___ **numbers.**



A negative 7 is 7 units to the left of 0. $|-7| = 7$ *A positive 7 is 7 units to the right of 0. $|7| = 7$*



a. **Where do you begin when locating a number on the number line?**

Start at 0.

b. **What do you call the distance between a number and 0 on a number line?**

The absolute value.

c. **What is the relationship between 7 and -7?**

Answers will vary. 7 and -7 both have the same absolute values. They are both the same distance from zero, 0, but in opposite directions; therefore, 7 and -7 are opposites.

Example 3 (5 minutes): Using the Integer Game and the Number Line

The teacher leads the whole class using a number line to model the concept of counting on (addition) to calculate the value of a hand when playing the Integer Game. The hand's value is the sum of the card values.

[1] Refer to the Integer Game outline for player rules.

$\boxed{5}$ $\boxed{-5}$ $\boxed{-4}$ $\boxed{8}$

First card: 5

Start at 0 and end up at positive 5. This is the first card drawn, so the value of the hand is 5.

Second Card: -5

Start at 5, the value of the hand after the first card; move 5 units to the left to end at 0.

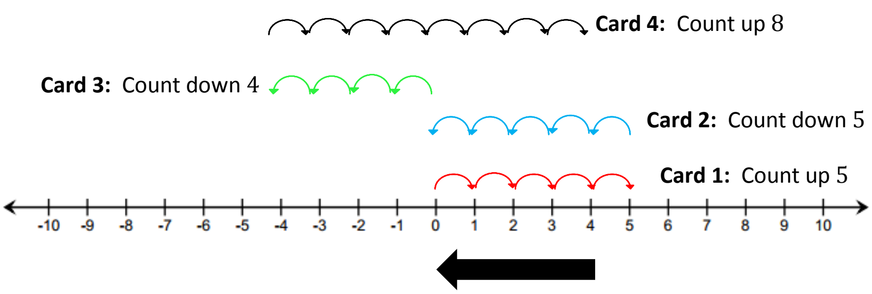
Third Card: -4

Start at 0, the value of the hand after the second card; move 4 units to the left.

Fourth Card: 8

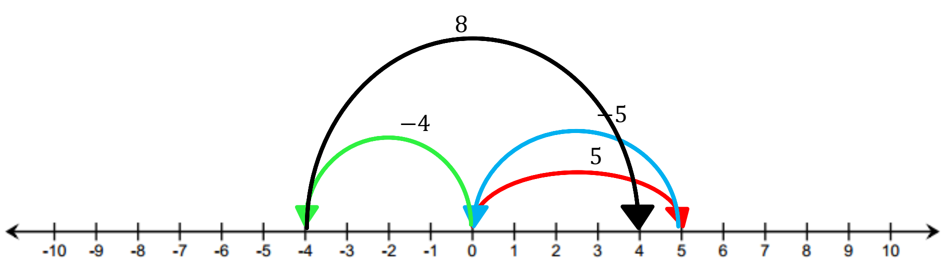
Start at -4, the value of the hand after the third card; move 8 units to the right.

- What is the final position on the number line?
  - *The final position on the number line is* 4.



Move 4 units to the left to get back to 0.

- What card or combination of cards would you need to get back to 0?
  - *In order to get a score of* 0, *I would need to count down* 4 *units. This means, I would need to draw a* -4 *card or a combination of cards whose sum is* -4, *such as* -1 *and* -3.



The final position is 4 units to the right of 0.

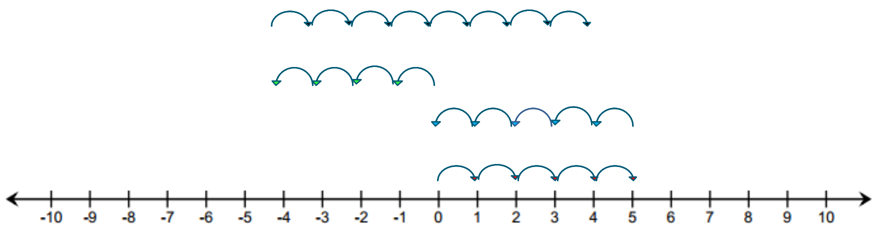
We can use smaller, curved arrows to show the number of "hops" or "jumps" that correspond to each integer. Or, we can use larger, curved arrows to show the length of the "hop" or "jump" that corresponds to the distance between the tail and the tip of the arrow along the number line. Either way, the final position is 4 units to the right of zero. Playing the Integer Game will prepare students for integer addition using arrows (vectors) in Lesson 2.

Example 3: Using the Integer Game and the Number Line

What is the sum of the card values shown? Use the counting on method on the provided number line to justify your answer.

4

5	-5	-4	8



a. **What is the final position on the number line?** $\underline{\quad 4 \quad}$

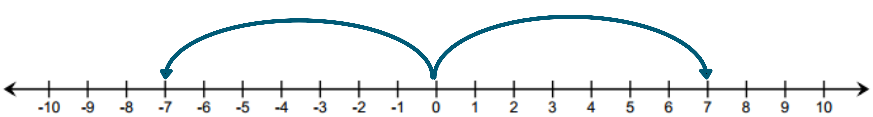
b. **What card or combination of cards would you need to get back to 0?** *-4 or -1 and -3*

Exercise 2 (5 minutes): The Additive Inverse

Before students begin, the teacher highlights part of the previous example where starting at zero and counting up five units and then back down five units brings us back to zero. This is because 5 and -5 are opposites. Students work independently to answer the questions. At the end of the exercise questions, formalize the definition of **additive inverse**.

Exercise 2: The Additive Inverse

Use the number line to answer each of the following questions.



a. **How far is 7 from 0 and in which direction?** *7 units to the right*

b. **What is the opposite of 7?** -7

c. **How far is -7 from 0 and in which direction?** *7 units to the left*

d. **Thinking back to our previous work, how would you use the counting on method to represent the following: While playing the Integer Game, the first card selected is 7, and the second card selected is -7.**

I would start at 0 and count up 7 by moving to the right. Then, I would start counting back down from 7 to 0.

e. **What does this tell us about the sum of 7 and its opposite, -7?**

The sum of 7 and -7 equals $0. 7 + (-7) = 0.$

f. **Look at the curved arrows you drew for 7 and -7. What relationship exists between these two arrows that would support your claim about the sum of 7 and -7?**

The arrows are both the same distance from 0. They are just pointing in opposite directions.

g. **Do you think this will hold true for the sum of any number and its opposite? Why?**

I think this will be true for the sum of any number and its opposite because when you start at 0 on the number line and move in one direction, moving in the opposite direction the same number of times will always take you back to zero.

For all numbers a there is a number $-a$, such that $a + (-a) = 0$.

The additive inverse of a real number is the opposite of that number on the real number line. For example, the opposite of -3 is 3. A number and its additive inverse have a sum of 0. The sum of any number and its opposite is equal to zero.

Example 4 (5 minutes): Modeling with Real-World Examples

The purpose of this example is to introduce real-world applications of opposite quantities to make zero. The teacher holds up an Integer Game card, for example -10, to the class and models how to write a story problem.

- How would the value of this card represent a temperature?
  - -10 *could mean* 10 *degrees below zero.*
- How would the temperature need to change in order to get back to 0 degrees?
  - *Temperature needs to rise* 10 *degrees.*
- With a partner, write a story problem using money that represents the expression $200 + (-200)$.
  - *Answers will vary. Timothy earned* $\$200$ *last week. He spent it on a new video game console. How much money does he have left over?*

Students share their responses to the last question with the class.

Exercise 3 (10 minutes): Playing the Integer Game

Exercise 3: Playing the Integer Game

Play the Integer Game with your group. Use a number line to practice counting on.

MP.6 & MP.7

Students will play the Integer Game in groups. Students will practice counting using their number lines. Let students explore how they will model addition on the number line. Monitor student understanding by ensuring that the direction of the arrows appropriately represents positive or negative integers.

Closing (2 minutes)

Students will discuss the following questions in their groups to summarize the lesson.

- How do you model addition using a number line?
  - *When adding a positive number on a number line, you count up by moving to the right. When adding a negative number on a number line, you count down by moving to the left.*
- Using a number line, how could you find the sum of $(-5) + 6$?
  - *Start at zero, then count down or move to the left five. From this point, count up or move to the right six.*
- Peter says he found the sum by thinking of it as $(-5) + 5 + 1$. Is this an appropriate strategy? Why do you think Peter did this?
  - *Peter did use an appropriate strategy to determine the sum of $(-5) + 6$. Peter did this because 5 and -5 are additive inverses, so they have a sum of zero. This made it easier to determine the sum to be one.*
- Why is the opposite of a number also called the additive inverse? What is the sum of a number and its opposite?
  - *The opposite of a number is called the additive inverse because the two numbers' sum is zero.*

Lesson Summary

- **Add positive integers by counting up, and add negative integers by counting down.**
- **An integer plus its opposite sum to zero.**
- **The opposite of a number is called the additive inverse because the two numbers' sum is zero.**

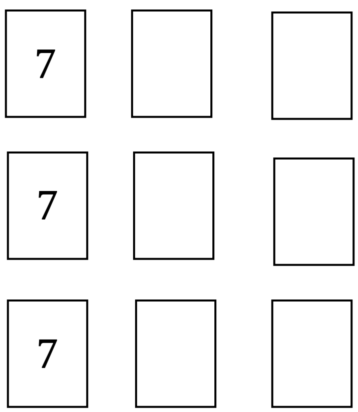
Exit Ticket (5 minutes)

Name ____ Date ____________________

Lesson 1: Opposite Quantities Combine to Make Zero

Exit Ticket

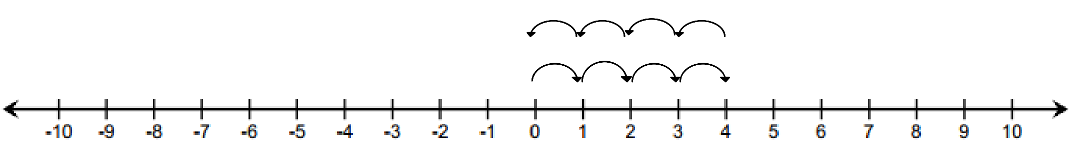
1. Your hand starts with the 7 card. Find three different pairs that would complete your hand and result in a value of zero.

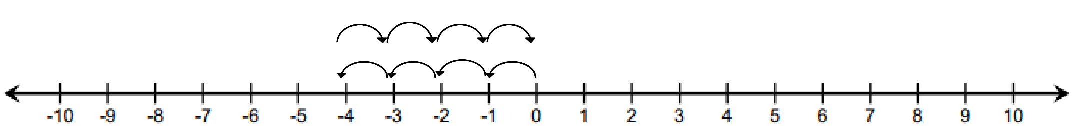


2. Write an equation to model the sum of the situation below.

   A hydrogen atom has a zero charge because it has one negatively charged electron and one positively charged proton.

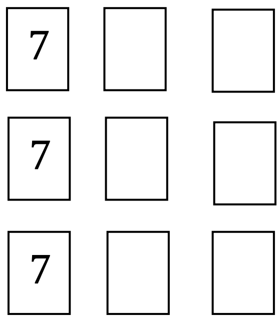
3. Write an equation for each diagram below. How are these equations alike? How are they different? What is it about the diagrams that lead to these similarities and differences?





Exit Ticket Sample Solutions

1. Your hand starts with the 7 cards. Find three different pairs that would complete your hand and result in a value of zero.



Answers will vary. $(-3$ *and* $-4)$, $(-5$ *and* $-2)$, $(-10$ *and* $3)$

2. Write an equation to model the sum of the situation below.

A hydrogen atom has a zero charge because it has one negatively charged electron and one positively charged proton.

$(-1)+1=0$ *or* $1+(-1)=0$

3. Write an equation for each diagram below. How are these equations alike? How are they different? What is it about the diagrams that lead to these similarities and differences?

Diagram A:

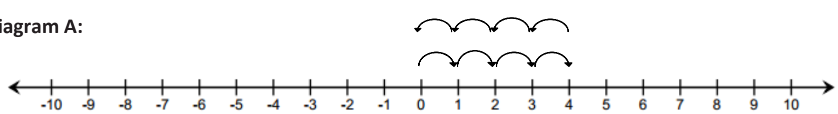
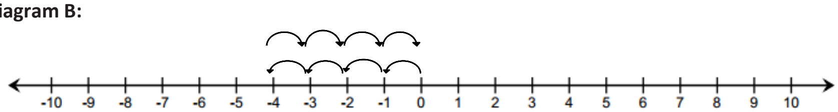


Diagram B:



A: $4+(-4)=0$

B: $-4+4=0$

Answers will vary. Both equations are adding 4 *and* -4*. The order of the numbers is different. The direction of A shows counting up* 4*, then counting down* 4*. The direction of* B *shows counting down* 4*, then counting up* 4*.*

Students may also mention that both diagrams demonstrate a sum of zero, adding opposites, or adding additive inverses.

Problem Set Sample Solutions

The Problem Set will provide practice with real-world situations involving the additive inverse such as temperature and money. Students will also explore more scenarios from the Integer Game to provide a solid foundation for Lesson 2.

For Problems 1 and 2, refer to the Integer Game.

1. **You have two cards with a sum of (-12) in your hand.**

   a. **What two cards could you have?**

   Answers will vary. $(-6$ *and* $-6)$

   b. **You add two more cards to your hand, but the total sum of the cards remains the same, (-12). Give some different examples of two cards you could choose.**

   Answers will vary, but numbers must be opposites. $(-2$ *and* $2)$ *and* $(4$ *and* $-4)$

2. **Choose one card value and its additive inverse. Choose from the list below to write a real-world story problem that would model their sum.**

   a. **Elevation: above and below sea level**

   Answers will vary. (A scuba diver is 20 feet below sea level. He had to rise 20 feet in order to get back on the boat.)

   b. **Money: credits and debits, deposits and withdrawals**

   Answers will vary. (The bank charges a fee of $\$5$ for replacing a lost debit card. If you make a deposit of $\$5$, what would be the sum of the fee and the deposit?)

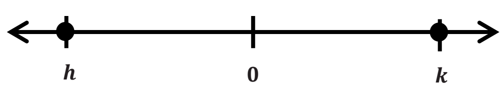
   c. **Temperature: above and below 0 degrees**

   Answers will vary. (The temperature of one room is 5 degrees above 0. The temperature of another room is 5 degrees below zero. What is the sum of both temperatures?)

   d. **Football: loss and gain of yards**

   Answers will vary. (A football player gained 25 yards on the first play. On the second play, he lost 25 yards. What is his net yardage after both plays?)

3. **On the number line below, the numbers h and k are the same distance from 0. Write an equation to express the value of $h + k$. Explain.**



$h + k = 0$ because their absolute values are equal, but their directions are opposite. k is the additive inverse of h, and h is the additive inverse of k because they have a sum of zero.

4. During a football game, Kevin gained five yards on the first play. Then he lost seven yards on the second play. How many yards does Kevin need on the next play to get the team back to where they were when they started? Show your work.

He has to gain 2 yards. $5 + (-7) + 2 = 0$, $5 + (-7) = -2$, *and* $-2 + 2 = 0$.

5. Write an addition number sentence that corresponds to the arrows below.

$10 + (-5) + (-5) = 0$.