Lesson 18: Computing Actual Lengths from a Scale Drawing

Student Outcomes

- Given a scale drawing, students compute the lengths in the actual picture using the scale. Students identify the scale factor in order to make intuitive comparisons of size, and then devise a strategy for efficiently finding actual lengths using the scale.

Classwork

Example 1 (14 minutes): Basketball at Recess?

The first example has students building upon the previous lesson by applying the scale factor to find missing dimensions. This leads into a discussion of whether this method is the most efficient and whether they could find another approach that would be simpler, as demonstrated in Example 2. Guide students to record responses and additional work in their student materials.

Scaffolding:

A reduction has a scale factor less than 1, and an enlargement has a scale factor greater than 1.

- How can we use the scale factor to determine the actual measurements?
  - *Divide each drawing length by the scale factor to find the actual measurement.*

See table below.

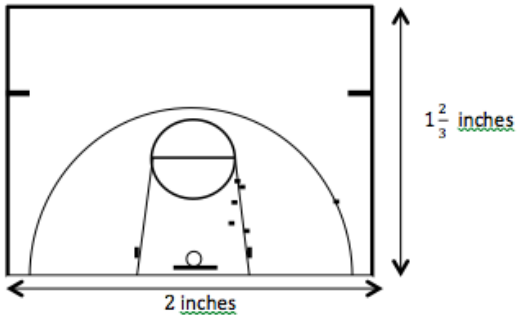
MP.2

- How can we use the scale factor to write an equation relating the scale drawing lengths to the actual lengths?
  - *The scale factor is the constant of proportionality, or the k in the equation $y = kx$ or $x = \frac{y}{k}$ or even $k = \frac{y}{x}$. It is the ratio of drawing length to actual length.*

Example 1: Basketball at Recess?

Vincent proposes an idea to the Student Government to install a basketball hoop along with a court marked with all the shooting lines and boundary lines at his school for students to use at recess. He presents a plan to install a half-court design as shown below. After checking with school administration, he is told it will be approved if it will fit on the empty lot that measures 25 feet by 75 feet on the school property. Will the lot be big enough for the court he planned? Explain.

Scale Drawing: 1 inch on the drawing corresponds to 15 feet of actual length.



Scale Drawing Lengths	1 *in.*	2 *in.*	$1\frac{2}{3}$ *in.*
Actual Court Lengths	15 *ft.*	30 *ft.*	25 *ft.*

Scale Factor: 1 inch corresponds to $(15 \cdot 12)$ inches, or 180 inches, so the scale factor is 180. Let $k = 180$, x represent the scale drawing lengths in inches, and y represent the actual court lengths in inches. The y-values must be converted from feet to inches.

To find actual length: $y = 180x$

$y = 180(2)$

$y = 360$ *inches, or* 30 *feet*

To find actual width: $y = 180x$

$y = 180\left(1\frac{2}{3}\right)$

$y = \frac{180}{1} \cdot \frac{5}{3}$

$y = 300$ *inches, or* 25 *feet*

The actual court measures 25 feet by 30 feet. Yes, the lot will be big enough for the court Vincent planned. The court will take up the entire width of the lot.

Example 2 (5 minutes)

Guide the whole class through the completion of the examples below while encouraging student participation through questioning. Students should record the information in their student materials.

Hold a discussion with students regarding the use of the word *scale*.

- Where have you seen this term used?
  - *Bottom of a map, blueprint, etc.*
- The word scale refers to a type of ratio. 1 cm represents 20 m is an example of a ratio relationship, and the ratio $1:20$ is sometimes called a scale ratio or a scale. Why isn't this called the scale factor?
  - *The scale factor in a scaled drawing is always a scalar between distances measured in the same units.*
- Do we always need to use the scale factor in order to find actual measurements from a scale drawing, or could we just use the given scale ratio (or scale)? (See below.)
- Take a few minutes to try to find the actual length of the garden. Give your answer in meters. Be prepared to explain you how you found your answer.

Allow for students to share approaches with the class. Students could calculate the scale factor and follow the steps from Example 1, or they may realize that it is not necessary to find the scale factor. They may apply the scale ratio and work the problem using the ratio $1:20$, perhaps setting up the proportional relationship $y = 20x$, where x represents the drawing measurement and y represents the actual length.

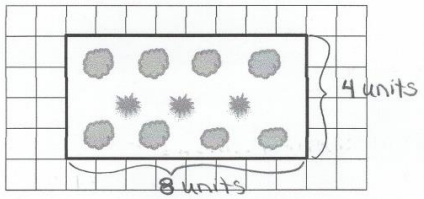
- So then, what two quantities does the constant of proportionality, k, relate?
  - *The constant of proportionality relates the drawing length to the actual length, when converted to the same units if a scale factor is being used. If just the scale ratio is used, then the quantities do not need to be converted to the same units.*
- What method was more efficient? Explain why.
  - *Allow for students to respond. If we apply the scale ratio, it requires fewer steps.*

- Then why would we consider the scale factor?
  - *The scale factor gives us a sense of the comparison. In this example, the scale factor is* $2{,}000$*, so the scale drawing lengths are* $\frac{1}{2{,}000}$ *of the actual lengths. It is not always easy to see that comparison when you are basing your calculations on the scale. The scale factor helps us reason through the problem and make sense of our results.*
- Now, go back and find the actual width of the garden using the scale ratio.

Elicit responses from students, including an explanation of how they arrived at their answers. Record results on the board for students to see, and be sure students have recorded correct responses in their student materials.

Example 2

The diagram shown represents a garden. The scale is 1 centimeter for every 20 meters. Each square in the drawing measures 1 cm by 1 cm. Find the actual length and width of the garden based upon the given drawing.



	Scale	*Length*	*Width*
Drawing, x	1 *cm*	8 *cm*	4 *cm*
Actual, y	20 *m (or* $2{,}000$ *cm)*	160 *m (or* $16{,}000$ *cm)*	80 *m (or* $8{,}000$ *cm)*

Method 1:

Using the given scale: 1 *cm of scale drawing length corresponds to* 20 *m of actual length.*

$k = 20$ *drawing length to actual length*

To find the actual length: $y = 20x$ *where x represents the the scale drawing measurements in centimeters and y represents the actual measurement in meters*

$y = 20(8)$ *substitute scale drawing length in place of x*

$y = 160$

The actual length is 160 *m.*

To find actual width: *Divide the actual length by* 2 *since its drawing width is half the length.*

The actual width is 80 *m.*

Method 2:

Use the scale factor: 1 *cm of scale drawing length corresponds to* $2{,}000$ *cm of actual length.*

$k = 2{,}000$ *drawing length to actual length (in same units)*

To find actual length: $y = 2{,}000x$ *where x represents the drawing measurement in centimeters and y represents the actual measurement in centimeters.*

$y = 2{,}000(8)$ *substitute the scale drawing length in place of x*

$y = 16{,}000$

The actual length is $16{,}000$ *cm, or* 160 *m.*

To find actual width: $y = 2{,}000x$

$y = 2{,}000(4)$ *substitute the scale drawing width in place of x*

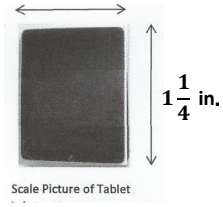
$y = 8{,}000$

The actual width is $8{,}000$ *cm, or* 80 *m.*

Example 3 (10 minutes)

Example 3

A graphic designer is creating an advertisement for a tablet. She needs to enlarge the picture given here so that 0.25 inches on the scale picture will correspond to 1 inch on the actual advertisement. What will be the length and width of the tablet on the advertisement?



Using a Table:

	Scale	*Length*	*Width*
Picture, x	0.25 *in.*	$1\frac{1}{4}$ *in.*	$1\frac{1}{8}$ *in.*
Actual Advertisement, y	1 *in.*	5 *in.*	$4\frac{1}{2}$ *in.*

Using an Equation:

Find the constant of proportionality, k: $k = 4$

$k = 4$ *(scale factor since units of measure are the same; it is an enlargement)*

To find Actual Length: $y = 4x$ *where x represents the picture measurement and y represents the actual advertisement measurement*

$y = 4\left(1\frac{1}{4}\right)$ *Substitute the picture length in place of x.*

$y = 5$ *in.*

To find Actual Width: $y = 4x$

$y = 4\left(1\frac{1}{8}\right)$ *Substitute the picture width in place of y.*

$y = 4\frac{1}{2}$

The tablet will be 5 inches by $4\frac{1}{2}$ inches on the actual advertisement.

- Is it always necessary to write and solve an equation $y = kx$ to find actual measurements?
  - *Guide students to conclude that the actual measurement can be found by applying any of the three relationships: $y = kx$, $x = \frac{y}{k}$, or even $k = \frac{y}{x}$. Encourage students to try any of these approaches in the next exercise.*

Exercises (10 minutes)

Hold a brief discussion of the problem as a class, and identify how to find the answer. Guide students to identify the following big ideas to address as they solve the problem:

- We need to find the relationship between the lengths in the scale drawing and the corresponding actual lengths.
- Use this relationship to calculate the width of the actual mall entrance.
- Compare this with the width of the panels.

Allow time for students to measure and complete the problem (see the measurement on the diagram below). Encourage students to check with a partner to ensure that their measurements match each other's.

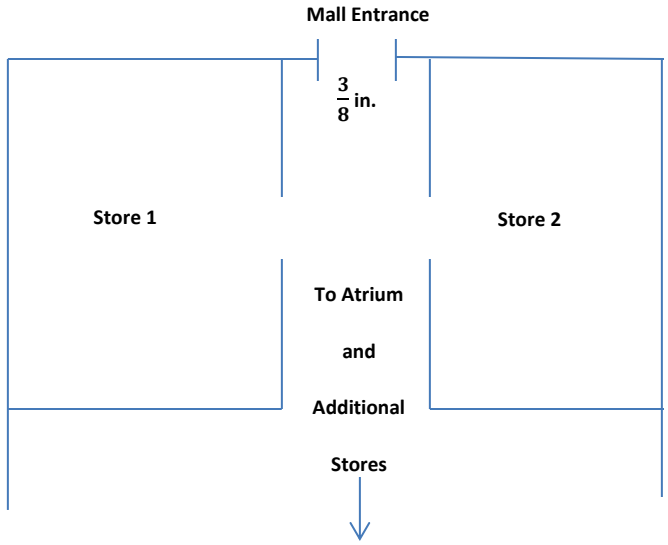
Scaffolding:

- The map distance of the mall entrance could be noted so that students would not need to measure.
- When determining what unit to use when measuring, look at the given scale.

Sample responses shown below include work for two different approaches. Students do not need to apply both and will receive credit for using either method.

Exercises

1. Students from the high school are going to perform one of the acts from their upcoming musical at the atrium in the mall. The students want to bring some of the set with them so that the audience can get a better feel for the whole production. The backdrop that they want to bring has panels that measure 10 feet by 10 feet. The students are not sure if they will be able to fit these panels through the entrance of the mall since the panels need to be transported flat (horizontal). They obtain a copy of the mall floor plan, shown below, from the city planning office. Use this diagram to decide if the panels will fit through the entrance. Use a ruler to measure.



Scale:

$\frac{1}{8}$ inch on the drawing represents $4\frac{1}{2}$ feet of actual length

Answer the following questions.

a. Find the actual distance of the mall entrance, and determine whether the set panels will fit.

Step 1: Relationship between lengths in drawing and lengths in actual

Scale: $\frac{4\frac{1}{2} \text{ ft.}}{\frac{1}{8} \text{ in.}}$, *or the value of the ratio* $\frac{36}{1}$ *feet to inches*

Scale factor calculations: $\frac{54}{\frac{1}{8}}$ *inches to inches*

$$= \frac{(54)\cdot 8}{(\frac{1}{8})\cdot 8}$$

$$= 432, \text{ an enlargement}$$

Step 2: Find the actual distance of entrance

Use the given scale: $\frac{3}{8} \cdot \frac{36}{1}$

$$= 13\frac{1}{2} \text{ feet wide}$$

OR

Using scale factor: $\frac{3}{8} \cdot \frac{432}{1}$

$$= 162 \text{ inches, or } 13\frac{1}{2} \text{ feet wide}$$

Yes, the set panels which are 10 ft. $\times$ 10 ft. will fit (lying flat) through the mall entrance.

b. What is the scale factor? What does it tell us?

The scale factor is 432. Each length on the scale drawing is $\frac{1}{432}$ of the actual length. The actual lengths are 432 times larger than the lengths in the scale drawing.

Closing (1 minute)

- What does the scale factor tell us about the relationship between the actual picture and the scale drawing?
  - *It gives us an understanding of how much larger or smaller the scale drawing is compared to the actual picture.*
- How does a scale drawing differ from other drawings?
  - *In a scale drawing, there exists a constant ratio of scale drawing length to actual length, whereas other drawings may not have a constant scale ratio between all corresponding lengths of the drawing and the actual picture or object.*

Exit Ticket (5 minutes)

Name _____ Date_____

Lesson 18: Computing Actual Lengths from a Scale Drawing

Exit Ticket

A drawing of a surfboard in a catalog shows its length as $8\frac{4}{9}$ inches. Find the actual length of the surfboard if $\frac{1}{2}$ inch length on the drawing corresponds to $\frac{3}{8}$ foot of actual length.

Exit Ticket Sample Solutions

A drawing of a surfboard in a catalog shows its length as $8\frac{4}{9}$ inches. Find the actual length of the surfboard if $\frac{1}{2}$ inch length on the drawing corresponds to $\frac{3}{8}$ foot of actual length.

	Scale	*Equivalent Scale Ratio*	*Surfboard*
Scale Drawing Length, x	$\frac{1}{2}$ *inch*	1 *inch*	$8\frac{4}{9}$ *inches*
Actual Length, y	$\frac{3}{8}$ *foot*	$\frac{6}{8}$ *ft. or* $\frac{3}{4}$ *ft.*	?

$\cdot\frac{3}{4}$

$$y = kx$$
$$y = \frac{3}{4}x$$
$$= 8\frac{4}{9}\cdot\frac{3}{4}$$
$$= \frac{76}{9}\cdot\frac{3}{4}$$
$$= \frac{19}{3}\cdot\frac{1}{1}$$

The actual surfboard measures $6\frac{1}{3}$ feet long.

Note: Students could also use an equation where y represents the scale drawing and x represents the actual measurement, in which case, k would equal $\frac{4}{3}$.

Problem Set Sample Solutions

1. **A toy company is redesigning their packaging for model cars. The graphic design team needs to take the old image shown below and resize it so that $\frac{1}{2}$ inch on the old packaging represents $\frac{1}{3}$ inch on the new package. Find the length of the image on the new package.**

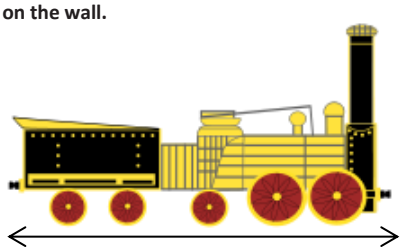
Car image length on old packaging measures 2 inches

$\frac{4}{3}$ inches; The scale $\frac{1}{2}$ to $\frac{1}{3}$ and the length of the original figure is 2, which is 4 halves, so in the scale drawing the length will be 4 thirds.

2. The city of St. Louis is creating a welcome sign on a billboard for visitors to see as they enter the city. The following picture needs to be enlarged so that $\frac{1}{2}$ inch represents 7 feet on the actual billboard. Will it fit on a billboard that measures 14 feet in height?

Yes, the drawing measures 1 inch in height, which corresponds to 14 feet on the actual billboard.

3. Your mom is repainting your younger brother's room. She is going to project the image shown below onto his wall so that she can paint an enlarged version as a mural. Use a ruler to determine the length of the image of the train. Then determine how long the mural will be if the projector uses a scale where 1 inch of the image represents $2\frac{1}{2}$ feet on the wall.



The scale drawing measures 2 inches, so the image will measure 2×2.5 or 5 feet long on the wall.

4. A model of a skyscraper is made so that 1 inch represents 75 feet. What is the height of the actual building if the height of the model is $18\frac{3}{5}$ inches?

1,395 feet

5. The portrait company that takes little league baseball team photos is offering an option where a portrait of your baseball pose can be enlarged to be used as a wall decal (sticker). Your height in the portrait measures $3\frac{1}{2}$ inches. If the company uses a scale where 1 inch on the portrait represents 20 inches on the wall decal, find the height on the wall decal. Your actual height is 55 inches. If you stand next to the wall decal, will it be larger or smaller than you?

Your height on the wall decal is 70 inches. The wall decal will be larger than your actual height (when you stand next to it).

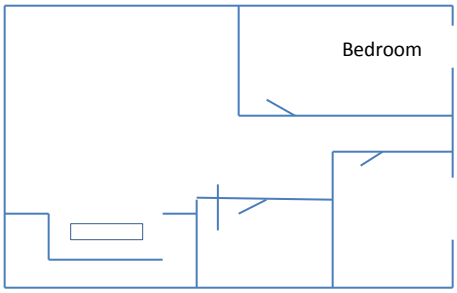
6. The sponsor of a 5K run/walk for charity wishes to create a stamp of its billboard to commemorate the event. If the sponsor uses a scale where 1 inch represents 4 feet, and the billboard is a rectangle with a width of 14 feet and a length of 48 feet, what will be the shape and size of the stamp?

The stamp will be a rectangle measuring $3\frac{1}{2}$ inches by 12 inches.

7. Danielle is creating a scale drawing of her room. The rectangular room measures $20\frac{1}{2}$ ft. by 25 ft. If her drawing uses the scale where 1 inch represents 2 feet of the actual room, will her drawing fit on an $8\frac{1}{2}$ in. by 11 in. piece of paper?

No, the drawing would be $10\frac{1}{4}$ inches by $12\frac{1}{2}$ inches, which is larger than the piece of paper.

8. A model of an apartment is shown below where $\frac{1}{4}$ inch represents 4 feet in the actual apartment. Use a ruler to measure the drawing and find the actual length and width of the bedroom.



Ruler measurements: $1\frac{1}{8}$ inches by $\frac{9}{16}$ inches.

The actual length would be 18 feet and the actual width would be 9 feet.