Lesson 13: Finding Equivalent Ratios Given the Total Quantity

Student Outcomes

- Students use tables to find an equivalent ratio of two partial quantities given a part-to-part ratio and the total of those quantities, in the third column, including problems with ratios of fractions.

Classwork

Example 1 (12 minutes)

Have students work in pairs to complete the chart below. Teacher may allow students to utilize a calculator to assist in the multiplication step of converting mixed numbers to improper fractions.

Scaffolding:

Review 16 oz. $= 1$ lb.

Example 1

A group of 6 hikers are preparing for a one-week trip. All of the group's supplies will be carried by the hikers in backpacks. The leader decides that each hiker will carry a backpack that is the same fraction of weight to all of the other hikers' weights. This means that the heaviest hiker would carry the heaviest load. The table below shows the weight of each hiker and the weight of the backpack.

Complete the table. Find the missing amounts of weight by applying the same value of the ratio as the first two rows.

Hiker's Weight	Backpack Weight	Total Weight (lb.)
152 lb. 4 oz. $152\frac{1}{4}$	14 lb. 8 oz. $14\frac{1}{2}$	$166\frac{3}{4}$
107 lb. 10 oz. $107\frac{5}{8}$	10 lb. 4 oz. $10\frac{1}{4}$	$117\frac{7}{8}$
129 lb. 15 oz. $129\frac{15}{16}$	$12\frac{3}{8}$	$142\frac{5}{16}$
68 lb. 4 oz. $68\frac{1}{4}$	$6\frac{1}{2}$	$74\frac{3}{4}$
$91\frac{7}{8}$	8 lb. 12 oz. $8\frac{3}{4}$	$100\frac{5}{8}$
105	10 lb.	115

Scaffolding:

You may need to review the abbreviations for pound (lb.) and ounce (oz.).

Value of the ratio of backpack weight to hiker weight:	***Equations:***
$\dfrac{14\frac{1}{2}}{152\frac{1}{4}} = \dfrac{\frac{29}{2}\times 4}{\frac{609}{4}\times 4} = \dfrac{58}{609} = \dfrac{2}{21}$	B = *Backpack weight* H = *Hiker's weight* $B = \frac{2}{21}H$ $B = \frac{2}{21}\left(129\frac{15}{16}\right)$ $B = 12\frac{3}{8}$

- What challenges did you encounter when calculating the missing values?
  - *Remembering the conversions of ounces to pounds and dividing fractions.*
- How is the third column representing the total quantity found and how is it useful?
  - *To find the third column, you need to add the total weight of both the hiker and the backpack. The third column giving the total allows one to compare the overall quantities. Also, if the total and ratio are known, then you can find the weight of the backpack and the hiker.*
- How can you calculate the values that are placed in the third column?
  - *In order to find the third column, you need the first two columns or the ratio of the first two columns. Because the third column is the total, add the values in the first two columns.*
- If a value is missing from the first or second column, how can you calculate the value?
  - *If a value is missing from one of the first two columns, you need to look at the rest of the table to determine the constant rate or ratio. You can write an equation of the relationship and then substitute in or write an equivalent ratio of the unknown to the constant of proportionality.*
- Based on the given values and found values, is the backpack weight proportional to the hiker's weight? How do you know?
  - *The table shows the backpack weight is proportional to the hiker's weight because there exists a constant, $\frac{2}{21}$, that when each measure of the hiker's weight is multiplied by the constant, it results in the corresponding weight of the backpack.*
- Would these two quantities always be proportional to each other?
  - *Not necessarily. The relationship between the backpack weight and the hiker's weight will not always be in the ratio $\frac{2}{21}$, but for these 6 hikers it was proportional.*
- Describe how to use different approaches to finding the missing values of either quantity.
  - *Writing equations or writing equivalent ratios can be used.*
- Describe the process of writing and using equations to find the missing values of a quantity.
  - *First, find the constant of proportionality or unit rate of $\frac{y}{x}$.*
  - *Once that is found, write an equation in the form $y = kx$, replacing k with the constant of proportionality.*
  - *Substitute the known value in for the variable, and solve for the unknown.*

- When writing equations to find the missing value(s) of a quantity, are we restricted to using the variables x and y? Explain.
  - *No, any variable can be used. Often using a variable to represent the context of the problem makes it easier to know which variable to replace with the known value. For instance, if the two quantities are hours and pay, one may use the variable p to represent pay instead of y and h to represent hours instead of x.*
- Describe the process of writing equivalent ratios to find the missing value(s) of a quantity. How is this method similar and different to writing proportions?
  - *Start with the unit rate or constant of proportionality. Determine what variable is known, and determine what you must multiply by to obtain the known value. Multiply the remaining part of the unit rate by the same number to get the value of the unknown value.*
- What must be known in order to find the missing value(s) of a quantity regardless of what method is used?
  - *The ratio of the two quantities must be known.*
- If the ratio of the two quantities and the total amount are given, can you find the remaining parts of the table?
  - *Yes, once the ratio is determined or given, find an equivalent ratio to the given ratio that also represents the total amount.*

Scaffolding:

May need to review solving a one-step equation using the multiplicative inverse to solve.

Example 2 (13 minutes)

Example 2

When a business buys a fast food franchise, it is buying the recipes used at every restaurant with the same name. For example, all Pizzeria Specialty House Restaurants have different owners, but they must all use the same recipes for their pizza, sauce, bread, etc. You are now working at your local Pizzeria Specialty House Restaurant, and listed below are the amounts of meat used on one meat-lovers pizza.

$\frac{1}{4}$ **cup of sausage**

$\frac{1}{3}$ **cup of pepperoni**

$\frac{1}{6}$ **cup of bacon**

$\frac{1}{8}$ **cup of ham**

$\frac{1}{8}$ **cup of beef**

What is the total amount of toppings used on a meat-lovers pizza? ___1___ **cups**

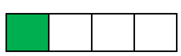
The meat must be mixed using this ratio to ensure that customers will receive the same great tasting meat-lovers pizza from every Pizzeria Specialty House Restaurant nationwide. The table below shows 3 different orders for meat-lovers pizzas on Super Bowl Sunday. Using the amounts and total for one pizza given above, fill in every row and column of the table so the mixture tastes the same.

	Order 1	Order 2	Order 3
Sausage (cups)	1	$1\frac{1}{2}$	$2\frac{1}{4}$
Pepperoni (cups)	$1\frac{1}{3}$	2	3
Bacon (cups)	$\frac{2}{3}$	1	$1\frac{1}{2}$
Ham (cups)	$\frac{1}{2}$	$\frac{3}{4}$	$1\frac{1}{8}$
Beef (cups)	$\frac{1}{2}$	$\frac{3}{4}$	$1\frac{1}{8}$
TOTAL (cups)	4	6	9

- What must you calculate or know to complete this table?
  - *You need to know the amount of each kind of meat in the original recipe and then keep each type of meat in the same ratio for each order using the given information.*
- How many pizzas were made for Order 1? Explain how you obtained and used your answer?
  - *There were 4 pizzas ordered. The amount of sausage increased from $\frac{1}{4}$ cup to 1 cup, which is 4 times as much. Knowing this, the amount of each ingredient can now be multiplied by 4 to determine how much of each ingredient is needed for Order 1.*

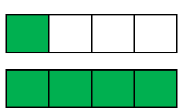
A bar model can be utilized as well:

  - *The amount of sausage is represented by the green portion in the bar model. This represents $\frac{1}{4}$ of a cup.*



If the amount of sausage becomes 1 cup, then the model should represent 1 whole (new green).

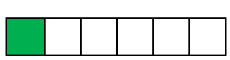
The number of $\frac{1}{4}$'s in one whole is 4.



- How many pizzas were made for Order 2? Explain how you obtained and used your answer?
  - *There were 6 pizzas ordered. The amount of bacon increased from $\frac{1}{6}$ to 1, which is 6 times as much. Each ingredient can then be multiplied by 6.*

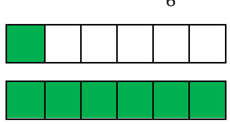
Bar Model:

The amount of bacon, $\frac{1}{6}$, is represented by the green portion in the model.



The amount of bacon became 1 cup, so the model should represent 1 whole (new green.)

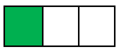
The number of $\frac{1}{6}$'s in one whole is 6.



- How many pizzas were made for Order 3? Explain how you obtained and used your answer?
  - *There were* 9 *pizzas ordered. The amount of pepperoni increased from* $\frac{1}{3}$ *to* 3*, which is* 9 *times as much. The other ingredients can then be multiplied by* 9.

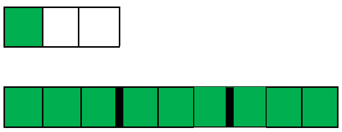
    Bar Model:

    The number of pepperoni, $\frac{1}{3}$*, is represented by the green portion in the model.*

    The amount of pepperoni becomes 3 *or* 3 *wholes, so we need to draw* 3 *whole models broken in thirds.*

    The amount of thirds in the total models is 9.

- Is it possible to order $1\frac{1}{2}$ or $2\frac{1}{2}$ pizzas? If so, describe the steps to determine the amount of each ingredient necessary.
  - *Yes, pizzas can be sold by the halves. This may not be typical, but it is possible. Most pizza places can put the ingredients on only half of a pizza. To determine the amount of each ingredient necessary, multiply the ingredient's original amount by the number of pizzas ordered.*

Exercise (12 minutes)

Exercise

The table below shows 6 different-sized pans that could be used to make macaroni and cheese. If the ratio of ingredients stays the same, how might the recipe be altered to account for the different sized pans?

Noodles (cups)	Cheese (cups)	Pan Size (number of cups)
4	1	5
3	$\frac{3}{4}$	$3\frac{3}{4}$
1	$\frac{1}{4}$	$1\frac{1}{4}$
$\frac{2}{3}$	$\frac{1}{6}$	$\frac{5}{6}$
$5\frac{1}{3}$	$1\frac{1}{3}$	$6\frac{2}{3}$
$4\frac{1}{2}$	$1\frac{1}{8}$	$5\frac{5}{8}$

MP.2

Method 1: Equations

Find the constant rate. To do this, use the row that gives both quantities, not the total. To find the unit rate:

$$\frac{\frac{3}{4}}{3} = \frac{3}{4} \cdot \frac{1}{3} = \frac{1}{4}$$

Write the equation of the relationship. $c = \frac{1}{4}n$, *where* c *represents the cups of cheese and* n *represents the cups of noodles.*

$$c = \frac{1}{4}n \qquad \frac{1}{4} = \frac{1}{4}n \qquad 1 = n$$

$$c = \frac{1}{4}n \qquad c = \frac{1}{4} \cdot \frac{2}{3} \qquad c = \frac{1}{6}$$

$$c = \frac{1}{4}n \qquad c = \frac{1}{4}\left(5\frac{1}{3}\right) \qquad c = \frac{1}{4}\left(\frac{16}{3}\right) \qquad c = \frac{4}{3} = 1\frac{1}{3}$$

Method 2: Proportions

Find the constant rate as describe in Method 1.

Set up proportions.

y *represents the number of cups of cheese and* x *represents the number of cups of noodles*

$$\frac{\frac{1}{4}}{x} = \frac{1}{4} \qquad x = 1$$

$$\frac{y}{\frac{2}{3}} = \frac{1}{4} \qquad 4y = \frac{2}{3} \qquad y = \frac{\frac{2}{3}}{4} \qquad y = \frac{2}{3} \cdot \frac{1}{4} \qquad y = \frac{2}{12} = \frac{1}{6}$$

$$\frac{y}{5\frac{1}{3}} = \frac{1}{4} \qquad 4y = 5\frac{1}{3} \qquad y = \frac{5\frac{1}{3}}{4} = \frac{16}{3} \cdot \frac{1}{4} = \frac{4}{3}$$

Method 3: Writing Equivalent Ratios

Multiply both the numerator and denominator of the original fraction by a common fraction that will give the known value. For example, what is multiplied by 1 *to get* $\frac{1}{4}$*? Multiply both the numerator and denominator by that fraction, likewise, what is multiplied by* 4 *to get* $\frac{2}{3}$*. Multiply both the numerator and denominator by that fraction.*

$$\frac{1}{4} = \frac{1 \cdot \frac{1}{4}}{4 \cdot \frac{1}{4}} = \frac{\frac{1}{4}}{x = 1} \qquad \frac{1}{4} = \frac{1 \cdot \frac{1}{6}}{4 \cdot \frac{1}{6}} = \frac{y = \frac{1}{6}}{\frac{2}{3}} \qquad \frac{1}{4} = \frac{1 \cdot \frac{4}{3}}{4 \cdot \frac{4}{3}} = \frac{\frac{4}{3}}{\frac{16}{3}} = \frac{y = \frac{4}{3}}{5\frac{1}{3}}$$

Closing (3 minutes)

- Describe how you can calculate the missing information in a table that includes the total quantity.

> Lesson Summary
>
> **To find missing quantities in a ratio table where a total is given, determine the unit rate from the ratio of two given quantities and use it to find the missing quantities in each equivalent ratio.**

Exit Ticket (5 minutes)

Name ______________________________ Date____________________

Lesson 13: Finding Equivalent Ratios Given the Total Quantity

Exit Ticket

The table below shows the combination of a dry prepackaged mix and water to make concrete. The mix says for every 1 gallon of water stir 60 pounds of dry mix. We know that 1 gallon of water is equal to 8 pounds of water. Using the information provided in the table, complete the remaining parts of the table.

Dry Mix (pounds)	Water (pounds)	Total (pounds)
	8	
75	10	
		$14\frac{1}{6}$
$4\frac{1}{2}$		

Exit Ticket Sample Solutions

The table below shows the combination of a dry prepackaged mix and water to make concrete. The mix says for every 1 gallon of water stir 60 pounds of dry mix. We know that 1 gallon of water is equal to 8 pounds of water. Using the information given in the table, complete the remaining parts of the table.

Dry Mix (pounds)	Water (pounds)	Total (pounds)
60	8	68
75	10	85
$12\frac{1}{2}$	$1\frac{2}{3}$	$14\frac{1}{6}$
$4\frac{1}{2}$	$\frac{3}{5}$	$5\frac{1}{10}$

Problem Set Sample Solutions

1. Students in 6 classes, displayed below, ate the same ratio of cheese pizza slices to pepperoni pizza slices. Complete the following table, which represents the number of slices of pizza students in each class ate.

Slices of Cheese Pizza	Slices of Pepperoni Pizza	Total Pizza
2	5	7
6	15	21
8	20	28
$5\frac{1}{2}$	$13\frac{3}{4}$	$19\frac{1}{4}$
$3\frac{1}{3}$	$8\frac{1}{3}$	$11\frac{2}{3}$
$\frac{3}{5}$	$1\frac{1}{2}$	$2\frac{1}{10}$

2. To make green paint, students mixed yellow paint with blue paint. The table below shows how many yellow and blue drops from a dropper several students used to make the same shade of green paint.

   a. Complete the table.

Yellow (Y) (ml)	Blue (B) (ml)	Total
$3\frac{1}{2}$	$5\frac{1}{4}$	$8\frac{3}{4}$
2	3	5
$4\frac{1}{2}$	$6\frac{3}{4}$	$11\frac{1}{4}$
$6\frac{1}{2}$	$9\frac{3}{4}$	$16\frac{1}{4}$

   b. Write an equation to represent the relationship between the amount of yellow paint and blue paint.

   $B = 1.5Y$

3. The ratio of the number of miles run to the number of miles biked is equivalent for each row in the table.

a. Complete the table.

Distance Run (miles)	Distance Biked (miles)	Total Amount of Exercise (miles)
2	4	6
$3\frac{1}{2}$	7	$10\frac{1}{2}$
$2\frac{3}{4}$	$5\frac{1}{2}$	$8\frac{1}{4}$
$2\frac{1}{8}$	$4\frac{1}{4}$	$6\frac{3}{8}$
$1\frac{2}{3}$	$3\frac{1}{3}$	5

b. What is the relationship between distances biked and distances run?

The distances biked were twice as far as the distances run.

4. The following table shows the number of cups of milk and flour that are needed to make biscuits. Complete the table.

Milk (cups)	Flour (cups)	Total (cups)
7.5	9	16.5
$8\frac{3}{4}$	10.5	$19\frac{1}{4}$
12.5	15	27.5
5	6	11