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Lesson 12: Ratios of Fractions and Their Unit Rates

Student Outcomes

* Students use ratio tables and ratio reasoning to compute unit rates associated with ratios of fractions in the context of measured quantities, e.g., recipes, lengths, areas, and speed.
* Students use unit rates to solve problems and analyze unit rates in the context of the problem.

Classwork

During this lesson, you are remodeling a room at your house and need to figure out if you have enough money. You will work individually and with a partner to make a plan of what is needed to solve the problem. After your plan is complete, then you will solve the problem by determining if you have enough money.

*Scaffolding:*

* Review that $12$ inches =$ 1$ foot and how to represent feet and inches as mixed fractions.
* Review the concept of area and the formula for finding area of a rectangle.
* Review how to multiply mixed numbers.
* How can estimation be used to answer this problem?

Example 1 (25 minutes): Time to Remodel

Students are given the task of determining the cost of tiling a rectangular room. The students are given the dimensions of the room, the area in square feet of one tile, and the cost of one tile.

If students are unfamiliar with completing a chart like this one, guide them in completing the first row.

Example 1: Time to Remodel

You have decided to remodel your bathroom and install a tile floor. The bathroom is in the shape of a rectangle and the floor measures $14$ feet, $8$ inches long by $5 $feet, $6$ inches wide. The tiles you want to use cost $\$5$ each, and each tile covers $4$ $\frac{2}{3}$ square feet. If you have $\$100$ to spend, do you have enough money to complete the project?

MP.2

Make a Plan: Complete the chart to identify the necessary steps in the plan and find a solution.

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| What I Know | What I Want to Find | How to Find it |
| dimensions of the floor | area | Convert inches to feet as a fraction with a denominator $12$. Area = $lw$ |
| square foot of $1$ tile | number of tiles needed | total area divided by the area of $1$ tile |
| cost of $1$ tile | total cost of all tiles | If the total money needed is more than $\$100$, then I won’t have enough money to do the remodel.  |

Compare your plan with a partner. Using your plans, work together to determine how much money you will need to complete the project and if you have enough money.

Dimensions: $5$ ft.,$ 6$ in. = $5\frac{1}{2}$ ft.

 $14$ ft., $8$ in. = $14\frac{2}{3}$ ft.

Area (square feet):

 $A = lw$

 $A =$ $\left(5\frac{1}{2}ft.\right)\left(14\frac{2}{3}ft.\right)$

 $A =$ $\left(\frac{11}{2}ft.\right)\left(\frac{44}{3}ft.\right)$

 $A =$ $\frac{242}{3}$ $=80\frac{2}{3} ft^{2}$

Number of Tiles:

$$\frac{80\frac{2}{3}}{4\frac{2}{3}}=\frac{\frac{242}{3}}{\frac{14}{3}}=\frac{242}{14}=17\frac{2}{7}$$

I cannot buy part of a tile, so I will need to purchase $18$ tiles.

Total Cost: $18\left(5\right)=\$90$

Do I have enough money?

Yes. Since the total is less than $\$100$, I have enough money.

Generate discussion about completing the plan and finding the solution. If needed, pose the following questions:

* Why was the mathematical concept of area, and not perimeter or volume, used?
	+ *Area was used because we were “covering” the rectangular floor. Area is* $2$*-dimensional, and we were given two dimensions, length and width of the room, to calculate the area of the floor. If we were just looking to put trim around the outside, then we would use perimeter. If we were looking to fill the room from floor to ceiling, then we would use volume.*
* Why would $5.6$ inches and $14.8$ inches be incorrect representations for $5$ feet, $6$ inches and $14$ feet, $8$ inches?
	+ *The relationship between feet and inches is* $12$ *inches for every* $1$ *foot. To convert to feet, you need to figure out what fractional part*$ 6$ *inches is of a foot, or* $12$ *inches. If you just wrote* $5.6$*, then you would be basing the inches out of* $10$ *inches, not* $12$ *inches. The same holds true for* $14$ *feet,* $8$ *inches.*
* How is the unit rate useful?
	+ *The unit rate for a tile is given as* $4\frac{2}{3}$*. We can find the total number of tiles needed by dividing the area (total square footage) by the unit rate.*
* Can I buy $17\frac{2}{7}$ tiles?
	+ *No, you have to buy whole tiles and cut what you may need.*
* How would rounding to $17$ tiles compare to $18 $tiles affect the job?
	+ *Even though the rules of rounding would say round down to* $17$ *tiles, we would not in this problem. If we round down, then the entire floor would not be covered, and we would be short. If we round up to* $18 $*tiles, the entire floor would be covered with a little extra.*

Exercise (10 minutes)

*Scaffolding:*

Since the students are at a young age, they may not be familiar with cars, distance, and miles per gallon relationships. Students may select the car with a lower unit rate because they may be confused with the better buy and lower unit prices. Further clarification may be needed to explain how a higher miles per gallon value is more favorable.

Exercise

Which car can travel further on $1$ gallon of gas?

Blue Car: travels $18$ $\frac{2}{5}$ miles using $0.8$ gallons of gas

Red Car: travels $17$ $\frac{2}{5}$ miles using $0.75$ gallons of gas

Finding the Unit Rate:

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| --- | --- |
| Blue Car: | Red Car: |
| $$\frac{18\frac{2}{5}}{\frac{4}{5}} = \frac{\frac{92}{5}}{\frac{4}{5}} =23$$ | $$\frac{17\frac{2}{5}}{\frac{3}{4}}=\frac{\frac{87}{5}}{\frac{3}{4}}=23\frac{1}{5}$$ |

Rate:

$23$ miles/gallon $23\frac{1}{5}$ miles/gallon

The red car traveled $\frac{1}{5}$ mile further on one gallon of gas.

Closing (5 minutes)

The red car traveled 1/5 mile further on one gallon of gas.

* How can unit rates with fractions be applied in the real-world?

Exit Ticket (5 minutes)

Name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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Exit Ticket

If $3$ $\frac{3}{4}$ lb. of candy cost $\$20.25$, how much would $1$ lb. of candy cost?

Exit Ticket Sample Solutions

If $3$ $\frac{3}{4} $lb. of candy cost $\$20.25$, how much would $1$ lb. of candy cost?

$$5\frac{2}{5}=5.4$$

Students may find the unit rate by first converting $\$20.25$ to $\frac{81}{4}$ and then dividing by $\frac{15}{4}$.

Problem Set Sample Solutions

1. You are getting ready for a family vacation. You decide to download as many movies as possible before leaving for the road trip. If each movie takes $1\frac{2}{5}$ hours to download and you downloaded for $5\frac{1}{4}$ hours, how many movies did you download?

$3\frac{3}{4}$ movies; however since you cannot download $\frac{3}{4}$ of a movie then you downloaded $3$ movies.

1. The area of a blackboard is $1\frac{1}{3}$ square yards. A poster’s area is $\frac{8}{9}$ square yards. Find the unit rate and explain, in words, what the unit rate means in the context of this problem. Is there more than one unit rate that can be calculated? How do you know?

$1\frac{1}{2}$. The area of the blackboard is $1\frac{1}{2}$ time the area of the poster.

Yes. There is another possible unit rate: $\frac{2}{3}$ the area of the poster is $\frac{2}{3}$ the area of the blackboard.

1. A toy jeep is $12\frac{1}{2}$ inches long, while an actual jeep measures $18\frac{3}{4}$ feet long. What is the value of the ratio of the length of the toy jeep to length of the actual jeep? What does the ratio mean in this situation?

$$\frac{ 12\frac{1}{2} }{18\frac{3}{4}}=\frac{ \frac{25}{2} }{\frac{75}{4}}=\frac{2}{3}$$

Every $2$ inches in length on the toy jeep corresponds to $3$ feet in length on the actual jeep.

1. $\frac{1}{3}$ cup of flour is used to make $5$ dinner rolls.
	1. How much flour is needed to make one dinner roll?

$\frac{1}{15}$ cup

* 1. How many cups of flour are needed to make$ 3$ dozen dinner rolls?

$2\frac{2}{5}$ cups

* 1. How many rolls can you make with $5\frac{2}{3}$ cups of flour?

$85$ rolls