Lesson 34: Review of the Assumptions

Student Outcomes

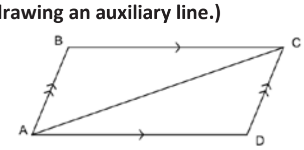
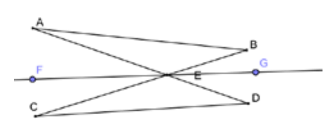
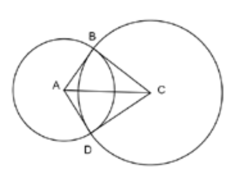
- Students review the principles addressed in Module 1.

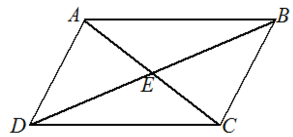
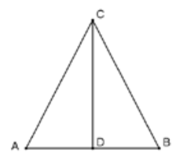
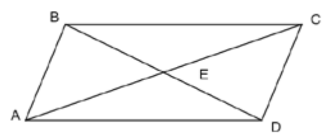
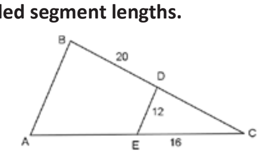
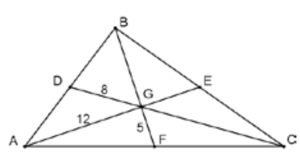
Lesson Notes

In Lesson 33, we reviewed many of the assumptions, facts, and properties used in this module to derive other facts and properties in geometry. We continue this review process with the table of facts and properties below, beginning with those related to rigid motions.

Classwork

Review Exercises (40 minutes)

Assumption/Fact/Property	Guiding Questions/Applications	Notes/Solutions
Given two triangles $\triangle ABC$ and $\triangle A'B'C'$ so that $AB = A'B'$ (Side), $\text{m}\angle A = \text{m}\angle A'$ (Angle), $AC = A'C'$(Side), then the triangles are congruent. [SAS]	The figure below is a parallelogram $ABCD$. What parts of the parallelogram satisfy the SAS triangle congruence criteria for $\triangle ABD$ and $\triangle CDB$? Describe a rigid motion(s) that will map one onto the other. (Consider drawing an auxiliary line.)  	*$AD = CB$, property of a parallelogram* *$\text{m}\angle ABD = \text{m}\angle CDB$, alternate interior angles* *$BD = BD$, reflexive property* *$\triangle ABD \cong \triangle CDB$, SAS* *$180°$ rotation about the midpoint of BD*
Given two triangles $\triangle ABC$ and $\triangle A'B'C'$, if $\text{m}\angle A = \text{m}\angle A'$ (Angle), $AB = A'B'$ (Side), and $\text{m}\angle B = \text{m}\angle B'$ (Angle), then the triangles are congruent. [ASA]	In the figure below, $\triangle CDE$ is the image of the reflection of $\triangle ABE$ across line FG. Which parts of the triangle can be used to satisfy the ASA congruence criteria?  	*$\text{m}\angle AEB = \text{m}\angle CED$, vertical angles are equal in measure.* *$BE = DE$, reflections map segments onto segments of equal length.* *$\angle B \cong \angle D$, reflections map angles onto angles of equal measure.*
Given two triangles $\triangle ABC$ and $\triangle A'B'C'$, if $AB = A'B'$ (Side), $AC = A'C'$ (Side), and $BC = B'C'$ (Side), then the triangles are congruent. [SSS]	$\triangle ABC$ and $\triangle ADC$ are formed from the intersections and center points of circles A and C. Prove $\triangle ABC \cong \triangle ADC$ by SSS.  	*AC is a common side.* *$AB = AD$, they are both radii of the same circle.* *$BC = DC$, they are both radii of the same circle.* *Thus, $\triangle ABC \cong \triangle ADC$ by SSS.*

Given two triangles, $\triangle ABC$ and $\triangle A'B'C'$, if $AB = A'B'$ (Side), $m\angle B = m\angle B'$ (Angle), and $\angle C = \angle C'$ (Angle), then the triangles are congruent. [AAS]	The AAS congruence criterion is essentially the same as the ASA criterion for proving triangles congruent. Why is this true? 	*If two angles of a triangle are congruent to two angles of a second triangle, then the third pair must also be congruent. Therefore, if one pair of corresponding sides is congruent, we treat the given corresponding sides as the included side and the triangles are congruent by ASA.*
Given two right triangles $\triangle ABC$ and $\triangle A'B'C'$ with right angles $\angle B$ and $\angle B'$, if $AB = A'B'$ (Leg) and $AC = A'C'$ (Hypotenuse), then the triangles are congruent. [HL]	In the figure below, CD is the perpendicular bisector of AB and $\triangle ABC$ is isosceles. Name the two congruent triangles appropriately, and describe the necessary steps for proving them congruent using HL. 	$\triangle ADC \cong \triangle BDC$ *Given $CD \perp AB$, both $\triangle ADC$ and $\triangle BDC$ are right triangles. CD is a common side. Given $\triangle ABC$ is isosceles, $\overline{AC} \cong \overline{CB}$.*
The opposite sides of a parallelogram are congruent. The opposite angles of a parallelogram are congruent. The diagonals of a parallelogram bisect each other.	In the figure below, $BE \cong DE$ and $\angle CBE \cong \angle ADE$. Prove $ABCD$ is a parallelogram. 	*$\angle BEC \cong \angle AED$, vertical angles are equal in measure.* *$\overline{BE} \cong \overline{DE}$, and $\angle CBE \cong \angle ADE$, given.* *$\triangle BEC \cong \triangle DEA$, ASA.* *By similar reasoning, we can show that $\triangle BEA \cong \triangle DEC$.* *Since $\overline{AB} \cong \overline{DC}$ and $\overline{BC} \cong \overline{DA}$, $ABCD$ is a parallelogram because the opposite sides are congruent (property of parallelogram).*
The midsegment of a triangle is a line segment that connects the midpoints of two sides of a triangle; the midsegment is parallel to the third side of the triangle and is half the length of the third side.	$\overline{DE}$ is the midsegment of $\triangle ABC$. Find the perimeter of $\triangle ABC$, given the labeled segment lengths. 	96
The three medians of a triangle are concurrent at the centroid; the centroid divides each median into two parts, from vertex to centroid and centroid to midpoint in a ratio of $2:1$.	If $\overline{AE}$, $\overline{BF}$, and $\overline{CD}$ are medians of $\triangle ABC$, find the lengths of segments BG, GE, and CG, given the labeled lengths. 	$BG = 10$ $GE = 6$ $CG = 16$

Exit Ticket (5 minutes)

Name ____ Date____________________

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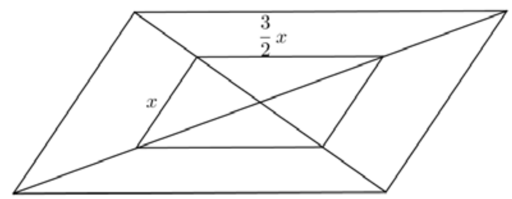
Exit Ticket

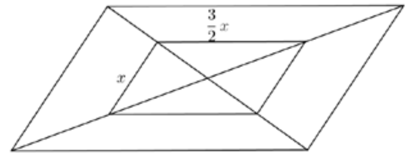
The inner parallelogram in the figure is formed from the midsegments of the four triangles created by the outer parallelogram's diagonals. The lengths of the smaller and larger midsegments are as indicated. If the perimeter of the outer parallelogram is 40, find the value of x.



Exit Ticket Sample Solutions

The inner parallelogram in the figure is formed from the mid-segments of the four triangles created by the outer parallelogram's diagonals. The lengths of the smaller and larger mid-segments are as indicated. If the perimeter of the outer parallelogram is 40, find the value of x.

$x = 4$

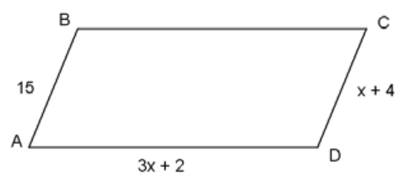


Problem Set Sample Solutions

Use any of the assumptions, facts, and/or properties presented in the tables above to find x and/or y in each figure below. Justify your solutions.

1. **Find the perimeter of parallelogram $ABCD$. Justify your solution.**

   $100, 15 = x + 4, x = 11$



2. $AC = 34$

   $AB = 26$

   $BD = 28$

   Given parallelogram $ABCD$, find the perimeter of $\triangle CED$. Justify your solution.

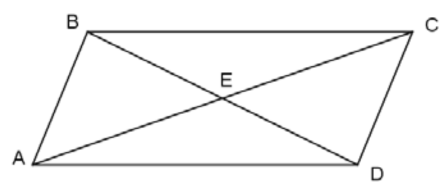
   57

   $CE = \frac{1}{2}AC; CE = 17$

   $CD = AB; CE = 26$

   $ED = \frac{1}{2}BD; ED = 14$

   Perimeter $= 17 + 26 + 14 = 57$



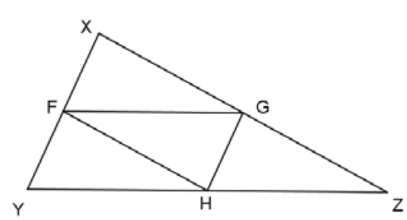
3. $XY = 12$

   $XZ = 20$

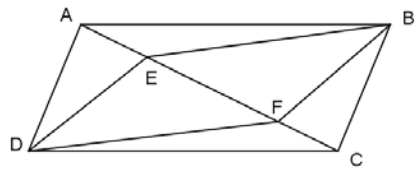
   $ZY = 24$

   F, G, and H are midpoints of the sides on which they are located. Find the perimeter of $\triangle FGH$. Justify your solution.

   28. *The midsegment is half the length of the side of the triangle it is parallel to.*

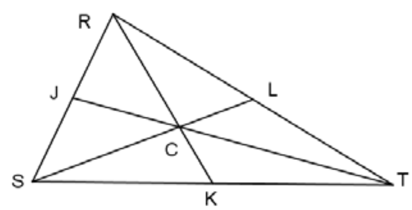


4. $ABCD$ is a parallelogram with $AE = CF$.
Prove that $DEBF$ is a parallelogram.



$AE = CF$	*Given*
$AD = BC$	*Property of a parallelogram*
$\mathrm{m}\angle DAE = \mathrm{m}\angle BCF$	*If parallel lines are cut by a transversal, then alternate interior angles are equal in measure*
$\triangle ADE \cong \triangle CBF$	*SAS*
$DE = BF$	*Corresponding sides of congruent triangles are congruent*
$AB = DC$	*Property of a parallelogram*
$\mathrm{m}\angle BAE = \mathrm{m}\angle DCF$	*If parallel lines are cut by a transversal, then alternate interior angles are equal in measure*
$\triangle BAE \cong \triangle DCF$	*SAS*
$BE = DF$	*Corresponding sides of congruent triangles are congruent*
$\therefore$ *ABCD is a parallelogram*	*If both sets of opp. sides of a quad. are equal in length, then the quad. is a parallelogram.*

5. C is the centroid of $\triangle RST$.
$RC = 16, CL = 10, TJ = 21$



$SC =$ ___20___

$TC =$ ___14___

$KC =$ ___8___