Lesson 33: Review of the Assumptions

Student Outcomes

- Students examine the basic geometric assumptions from which all other facts can be derived.
- Students review the principles addressed in Module 1.

Classwork

Review Exercises (40 minutes)

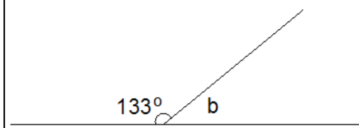
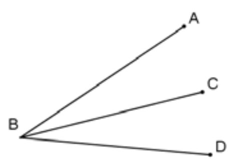
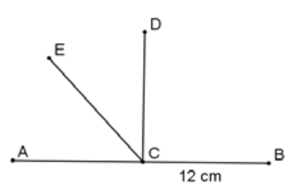
We have covered a great deal of material in Module 1. Our study has included definitions, geometric assumptions, geometric facts, constructions, unknown angle problems and proofs, transformations, and proofs that establish properties we previously took for granted.

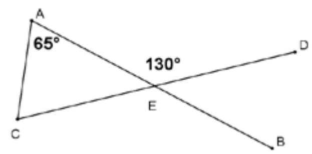
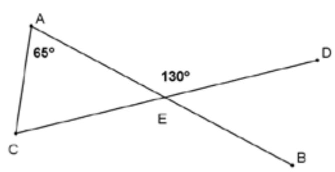
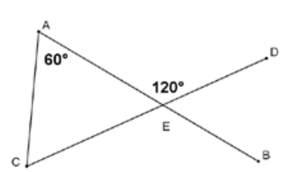
In the first list below, we compile all of the geometric assumptions we took for granted as part of our reasoning and proof-writing process. Though these assumptions were only highlights in lessons, these assumptions form the basis from which all other facts can be derived (e.g., the other facts presented in the table). College-level geometry courses often do an in-depth study of the assumptions.

The latter tables review the facts associated with problems covered in Module 1. Abbreviations for the facts are within brackets.

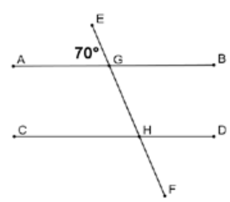
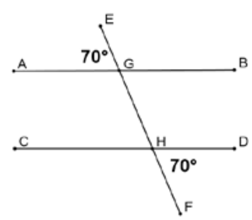
Geometric Assumptions (Mathematicians call these "Axioms.")

1. (Line) Given any two distinct points, there is exactly one line that contains them.
2. (Plane Separation) Given a line contained in the plane, the points of the plane that do not lie on the line form two sets, called half-planes, such that
   a. Each of the sets is convex,
   b. If P is a point in one of the sets and Q is a point in the other, then $\overline{PQ}$ intersects the line.
3. (Distance) To every pair of points A and B there corresponds a real number $\text{dist}\,(A,B) \geq 0$, called the distance from A to B, so that
   a. $\text{dist}(A,B) = \text{dist}(B,A)$.
   b. $\text{dist}(A,B) \geq 0$, and $\text{dist}(A,B) = 0 \Leftrightarrow A$ and B coincide.
4. (Ruler) Every line has a coordinate system.
5. (Plane) Every plane contains at least three non-collinear points.
6. (Basic Rigid Motions) Basic rigid motions (e.g., rotations, reflections, and translations) have the following properties:
   a. Any basic rigid motion preserves lines, rays, and segments. That is, for any basic rigid motion of the plane, the image of a line is a line, the image of a ray is a ray, and the image of a segment is a segment.
   b. Any basic rigid motion preserves lengths of segments and angle measures of angles.
7. ($180°$ Protractor) To every $\angle AOB$, there corresponds a real number $\text{m}\angle AOB$, called the degree or measure of the angle, with the following properties:
   a. $0° < \text{m}\angle AOB < 180°$
   b. Let $\overrightarrow{OB}$ be a ray on the edge of the half-plane H. For every r such that $0° < r < 180°$, there is exactly one ray $\overrightarrow{OA}$ with A in H such that $\text{m}\angle AOB = r°$.
   c. If C is a point in the interior of $\angle AOB$, then $\text{m}\angle AOC + \text{m}\angle COB = \text{m}\angle AOB$.
   d. If two angles $\angle BAC$ and $\angle CAD$ form a linear pair, then they are supplementary, e.g., $\text{m}\angle BAC + \text{m}\angle CAD = 180°$.
8. (Parallel Postulate) Through a given external point, there is at most one line parallel to a given line.

Fact/Property	Guiding Questions/Applications	Notes/Solutions
Two angles that form a linear pair are supplementary.		$m\angle b = 47°$
The sum of the measures of all adjacent angles formed by three or more rays with the same vertex is $360°$.		$m\angle g = 80°$
Vertical angles have equal measure.	**Use the fact that linear pairs form supplementary angles to prove that vertical angles are equal in measure.**	$m\angle w + m\angle x = 180°$ $m\angle y + m\angle x = 180°$ $m\angle w + m\angle x = m\angle y + m\angle x$ $\therefore\ m\angle w = m\angle y$
The bisector of an angle is a ray in the interior of the angle such that the two adjacent angles formed by it have equal measure.	**In the diagram below, $\overline{BC}$ is the bisector of $\angle ABD$, which measures $64°$. What is the measure of $\angle ABC$?** 	$32°$
The perpendicular bisector of a segment is the line that passes through the midpoint of a line segment and is perpendicular to the line segment.	**In the diagram below, $\overline{DC}$ is the $\perp$ bisector of $\overline{AB}$, and $\overline{CE}$ is the angle bisector of $\angle ACD$. Find the measures of $\overline{AC}$ and $\angle ECD$.** 	$AC = 12$, $m\angle ECD = 45°$

The sum of the 3 angle measures of any triangle is $180°$.	**Given the labeled figure below, find the measures of $\angle DEB$ and $\angle ACE$. Explain your solutions.** 	$\mathrm{m}\angle DEB = 50°$, $\mathrm{m}\angle ACE = 65°$ $\mathrm{m}\angle\, DEB + \mathrm{m}\angle\, AED = 180°$ *and angle sum of a triangle*
When one angle of a triangle is a right angle, the sum of the measures of the other two angles is $90°$.	**This fact follows directly from the preceding one. How is simple arithmetic used to extend the angle sum of a triangle property to justify this property?**	*Since a right angle is $90°$ and angles of a triangle sum to $180°$, by arithmetic the other two angles must add up to $90°$.*
An exterior angle of a triangle is equal to the sum of its two opposite interior angles.	**In the diagram below, how is the exterior angle of a triangle property proved?** 	*The sum of two interior opposite angles and the third angle of a triangle is $180°$, which is equal to the angle sum of the third angle and the exterior angle. Thus, the exterior angle of a triangle is equal to the sum of the interior opposite angles.*
Base angles of an isosceles triangle are congruent.	**The triangle in the figure above is isosceles. How do we know this?**	*The base angles are equal.*
All angles in an equilateral triangle have equal measure.	**If the figure above is changed slightly, it can be used to demonstrate the equilateral triangle property. Explain how this can be demonstrated.** 	$\mathrm{m}\angle AEC$ *is* $60°$*; angles on a line.* $\mathrm{m}\angle C$ *is also* $60°$ *by the angle sum of a triangle property. Thus, each interior angle is* $60°$*.*

The facts and properties in the immediately preceding table relate to angles and triangles. In the table below, we will review facts and properties related to parallel lines and transversals.

Fact/Property	Guiding Questions/Applications	Notes/Solutions
If a transversal intersects two parallel lines, then the measures of the corresponding angles are equal.	Why does the property specify *parallel* lines?	*If the lines are not parallel, then the corresponding angles are not congruent.*
If a transversal intersects two lines such that the measures of the corresponding angles are equal, then the lines are parallel.	The converse of a statement turns the relevant property into an *if and only if* relationship. Explain how this is related to the guiding question about corresponding angles.	*The "if and only if" specifies the only case in which corresponding angles are congruent (when two lines are parallel).*
If a transversal intersects two parallel lines, then the interior angles on the same side of the transversal are supplementary.	This property is proved using (in part) the corresponding angles property. Use the diagram below ($\overline{AB} \parallel \overline{CD}$) to prove that $\angle AGH$ and $\angle CHG$ are supplementary.  	*$m\angle AGH$ is $110°$ because they form a linear pair and $\angle CHG$ is $70°$ because of corresponding angles. Thus, interior angles on the same side are supplementary.*
If a transversal intersects two lines such that the same side interior angles are supplementary, then the lines are parallel.	Given the labeled diagram below, prove that $\overline{AB} \parallel \overline{CD}$.  	*$m\angle AGH = 110°$ due to a linear pair, and $\angle GHC = 70°$ due to vertical angles. Then, $\overline{AB} \parallel \overline{CD}$ because the corresponding angles are congruent.*
If a transversal intersects two parallel lines, then the measures of alternate interior angles are equal.	1. Name both pairs of alternate interior angles in the diagram above. 2. How many different angle measures are in the diagram?	*1. $\angle GHC$, $\angle HGB$ $\angle AGH$, $\angle DHG$ 2. 2*
If a transversal intersects two lines such that measures of the alternate interior angles are equal, then the lines are parallel.	Although not specifically stated here, the property also applies to *alternate exterior angles*. Why is this true?	*The alternate exterior angles are vertical angles to the alternate interior angles.*

Exit Ticket (5 minutes)

Name ____Date____________________

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Exit Ticket

1. Which assumption(s) must be used to prove that vertical angles are congruent?

2. If two lines are cut by a transversal such that corresponding angles are NOT congruent, what must be true? Justify your response.

Exit Ticket Sample Solutions

1. **Which assumption(s) must be used to prove that vertical angles are congruent?**

   The "protractor postulate" must be used. If two angles, $\angle BAC$ and $\angle CAD$, form a linear pair, then they are supplementary, e.g., $m\angle BAC + m\angle CAD = 180$.

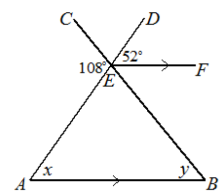
2. **If two lines are cut by a transversal such that corresponding angles are NOT congruent, what must be true? Justify your response.**

   The lines are not parallel. Corresponding angles are congruent if and only if the lines are parallel. The "and only if" part of this statement requires that, if the angles are NOT congruent, then the lines are NOT parallel.

Problem Set Sample Solutions

Use any of the assumptions, facts, and/or properties presented in the tables above to find x and y in each figure below. Justify your solutions.

1. $x = 52°$, $y = 56°$



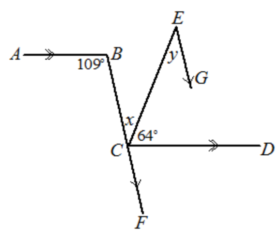
$m\angle AEB$ is $72°$	*Linear pairs form supplementary angles*
$m\angle FEB$ is $56°$	*Linear pairs form supplementary angles*
$x = 52°$	*If two parallel lines are cut by a transversal, then the corresponding angles are congruent.*
$y = 56°$	*Angles in a triangle add up to $180°$*

2. **You will need to draw an auxiliary line to solve this problem.**

   $x = 45°$, $y = 45°$

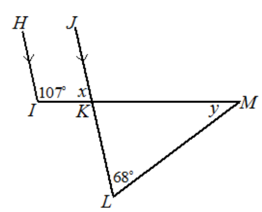
   $\angle ABC$ and $\angle DCB$ are alternate interior angles because $\overline{AB} \parallel \overline{CD}$; $x = 45°$.

   Angles x and y are also alternate interior angles because $\overline{BC} \parallel \overline{EG}$; $y = 45°$.

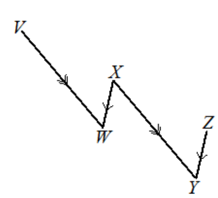


3. $x = 73°$, $y = 39°$

   $\angle HIK$ and $\angle JKI$ are supplementary because they are same side interior angles and $\overline{JK} \parallel \overline{HI}$; therefore, $x = 73°$. $\angle MKL$ and $\angle JKI$ are vertical angles. So, using the fact that the sum of angles in a triangle is $180°$, we find that $y = 39°$.



4. **Given the labeled diagram at the right, prove that $\angle VWX \cong \angle XYZ$.**



$\angle VWX \cong \angle YXW$	*When two parallel lines are cut by a transversal, the alternate interior angles are congruent*
$\angle XYZ \cong \angle YXW$	*When two parallel lines are cut by a transversal, the alternate interior angles are congruent*
$\therefore\ \angle VWX = \angle XYZ$	*Substitution property of equality*