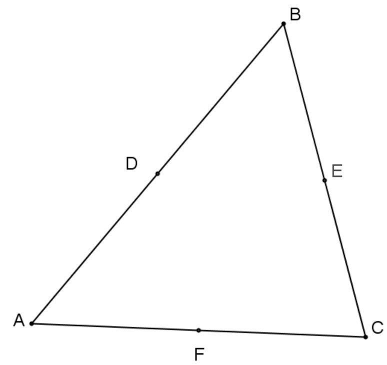
Lesson 30: Special Lines in Triangles

Classwork

Opening Exercise

In $\triangle ABC$ at the right, D is the midpoint of $\overline{AB}$; E is the midpoint of $\overline{BC}$, and F is the midpoint of $\overline{AC}$. Complete each statement below.



$\overline{DE}$ is parallel to __________ and measures __________ the length of __________.

$\overline{DF}$ is parallel to __________ and measures __________ the length of __________.

$\overline{EF}$ is parallel to __________ and measures __________ the length of __________.

Discussion

In the previous two lessons, we proved that (a) the midsegment of a triangle is parallel to the third side and half the length of the third side and (b) diagonals of a parallelogram bisect each other. We use both of these facts to prove the following assertion:

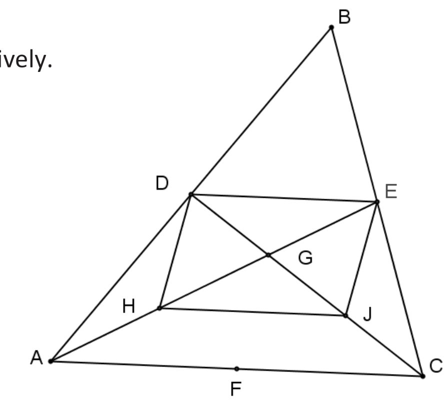
All medians of a triangle are ____________________. That is, the three medians of a triangle (the segments connecting each vertex to the midpoint of the opposite side) meet at a single point. This point of concurrency is called the ______________________, or the center of gravity, of the triangle. The proof will also show a length relationship for each median: The length from the vertex to the centroid is _________________ the length from the centroid to the midpoint of the side.

Example 1

Provide a valid reason for each step in the proof below.

Given: $\triangle ABC$ with D, E, and F the midpoints of sides AB, BC, and AC, respectively.

Prove: The three medians of $\triangle ABC$ meet at a single point.



(1) Draw AE and DC; label their intersection as point G.

(2) Construct and label the midpoint of AG as point H and the midpoint of GC as point J.

(3) $DE \parallel AC$, ____________________________________

(4) $HJ \parallel AC$, ____________________________________

(5) $DE \parallel HJ$, ____________________________________

(6) $DE = \frac{1}{2}AC$ and $HJ = \frac{1}{2}AC$, ____________________________________

(7) $DEJH$ is a parallelogram, ____________________________________

(8) $HG = EG$ and $JG = DG$, ____________________________________

(9) $AH = HG$ and $CJ = JG$, ____________________________________

(10) $AH = HG = GE$ and $CJ = JG = GD$, ____________________________________

(11) $AG = 2GE$ and $CG = 2GD$,

____

(12) We can complete steps (1)–(11) to include the median from B; the third median, $\overline{BF}$, passes through point G, which divides it into two segments such that the longer part is twice the shorter.

(13) The intersection point of the medians divides each median into two parts with lengths in a ratio of 2:1; therefore, all medians are concurrent at that point.

The three medians of a triangle are concurrent at the ______________________, or the center of gravity. This point of concurrency divides the length of each median in a ratio of _________________; the length from the vertex to the centroid is ____________________ the length from the centroid to the midpoint of the side.

Example 2

In the figure to the right, $DF = 4$, $BF = 16$, $AG = 30$. Find each of the following measures.

a. $FC =$ ________________________

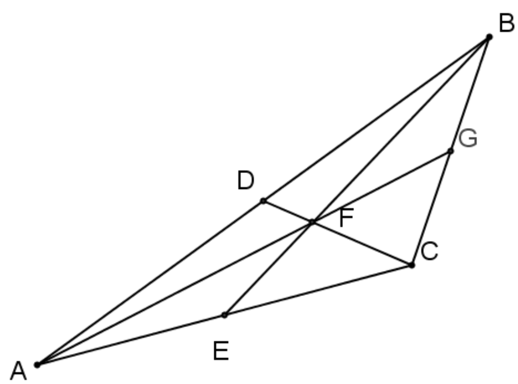
b. $DC =$ ________________________

c. $AF =$ ________________________

d. $BE =$ ________________________

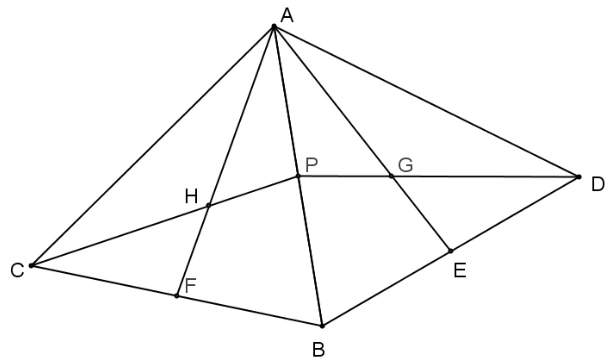
e. $FG =$ ________________________

f. $EF =$ ________________________



Example 3

In the figure to the right, $\triangle ABC$ is reflected over $\overline{AB}$ to create $\triangle ABD$. Points P, E, and F are midpoints of $\overline{AB}$, $\overline{BD}$, and $\overline{BC}$, respectively. If $AH = AG$, prove that $PH = GP$.



Problem Set

Ty is building a model of a hang glider using the template below. To place his supports accurately, Ty needs to locate the center of gravity on his model.

1. Use your compass and straightedge to locate the center of gravity on Ty's model.

2. Explain what the center of gravity represents on Ty's model.

3. Describe the relationship between the longer and shorter sections of the line segments you drew as you located the center of gravity.