Lesson 27: Triangle Congruency Proofs

Student Outcomes

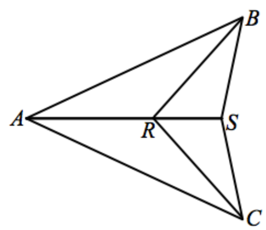
- Students complete proofs requiring a synthesis of the skills learned in the last four lessons.

Classwork

Exercises 1–6 (40 minutes)

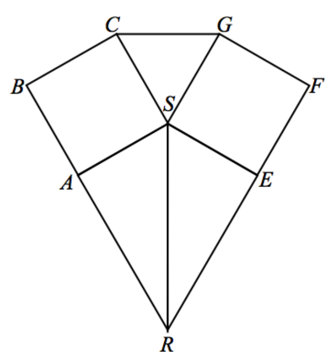
Exercises 1–6

1. Given: $AB = AC, RB = RC.$
   Prove: $SB = SC.$



$AB = AC, RB = RC$	*Given*
$AR = AR$	*Reflexive property*
$\triangle ARC \cong \triangle ARB$	*SSS*
$\text{m}\angle ARC = \text{m}\angle ARB$	*Corresponding angles of congruent triangles are equal in measure*
$\text{m}\angle ARC + \text{m}\angle SRC = 180$, $\text{m}\angle ARB + \text{m}\angle SRB = 180$	*Linear pairs form supplementary angles*
$\text{m}\angle SRC = \text{m}\angle SRB$	*Angles supplementary to either the same angle or to congruent angles are equal in measure*
$SR = SR$	*Reflexive property*
$\triangle SRB \cong \triangle SRC$	*SAS*
$SB = SC$	*Corresponding sides of congruent angles are equal in length*

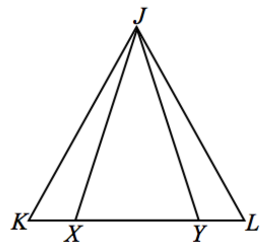
2. Given: Square $ABCS \cong$ Square $EFGS$,
   $\overleftrightarrow{RAB}, \overleftrightarrow{REF}.$
   Prove: $\triangle ASR \cong ESR.$



Square $ABCS \cong$ Square $EFGS$	*Given*
$AS = ES$	*Corresponding sides of congruent squares are equal in length*
$SR = SR$	*Reflexive property*
$\angle BAS$ and $\angle FES$ are right angles	*Definition of square*
$\angle BAS$ and $\angle SAR$ form a linear pair	*Definition of linear pair*
$\angle FES$ and $\angle SER$ form a linear pair	*Definition of linear pair*
$\angle SAR$ and $\angle SER$ are right angles	*Two angles that are supplementary and congruent each measure $90°$ and are, therefore, right angles*
$\triangle ASR$ and $\triangle ESR$ are right triangles	*Definition of right triangle*
$\triangle ASR \cong \triangle ESR$	*HL*

3. **Given:** $JK = JL, JX = JY$.

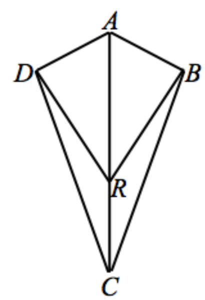
Prove: $KX = LY$.



$JX = JY$	*Given*
$m\angle JXY = m\angle JYX$	*Base angles of an isosceles triangle are equal in measure*
$m\angle JXK + m\angle JXY = 180$,	
$m\angle JYL + m\angle JYX = 180$	*Linear pairs form supplementary angles.*
$m\angle JXK + m\angle JXY = m\angle JYL + m\angle JYX$	*Substitution property of equality*
$m\angle JXK + m\angle JXY = m\angle JYL + m\angle JXY$	*Substitution property of equality*
$m\angle JXK = m\angle JYL$	*Angles supplementary to either the same angle or congruent angles are equal in measure*
$JK = JL$	*Given*
$m\angle K = m\angle L$	*Base angles of an isosceles triangle are equal in measure*
$\triangle JXK \cong \triangle JYL$	*AAS*
$KX = LY$	*Corresponding sides of congruent triangles are equal in length*

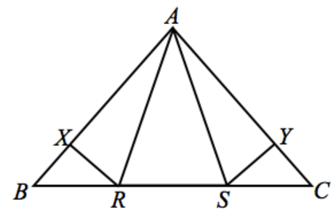
4. **Given:** $\overline{AD} \perp \overline{DR}, \overline{AB} \perp \overline{BR}$,
$\overline{AD} \cong \overline{AB}$.

Prove: $\angle DCR \cong \angle BCR$.



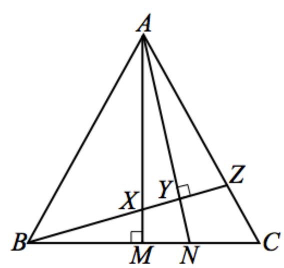
$\overline{AD} \perp \overline{DR}$, $\overline{AB} \perp \overline{BR}$	*Given*
$\triangle ADR$ and $\triangle ABR$ are right triangles	*Definition of right triangle*
$\overline{AD} \cong \overline{AB}$	*Given*
$\overline{AR} \cong \overline{AR}$	*Reflexive property*
$\triangle ADR \cong \triangle ABR$	*HL*
$\angle ARD \cong ARB$	*Corresponding angles of congruent triangles are congruent*
$m\angle ARD + m\angle DRC = 180$,	
$m\angle ARB + m\angle BRC = 180$	*Linear pairs form supplementary angles.*
$m\angle ARD + m\angle DRC = m\angle ARB + m\angle BRC$	*Transitive property*
$m\angle DRC = m\angle BRC$	*Angles supplementary to either the same angle or congruent angles are equal in measure*
$\overline{DR} \cong \overline{BR}$	*Corresponding sides of congruent triangles are congruent*
$\overline{RC} \cong \overline{RC}$	*Reflexive property*
$\triangle DRC \cong \triangle BRC$	*SAS*
$\angle DRC \cong \angle BRC$	*Corresponding angles of congruent triangles are congruent*

5. **Given:** $AR = AS$, $BR = CS$,
$\overline{RX} \perp \overline{AB}$, $\overline{SY} \perp \overline{AC}$.
Prove: $BX = CY$.



$AR = AS$	*Given*
$\text{m}\angle ARS = m\angle ASR$	*Base angles of an isosceles triangle are equal in measure*
$\text{m}\angle ARS + \text{m}\angle ARB = 180$, $\text{m}\angle ASR + \text{m}\angle ASC = 180$	*Linear pairs form supplementary angles*
$\text{m}\angle ARS + \text{m}\angle ARB = \text{m}\angle ASR + \text{m}\angle ASC$	*Transitive property*
$\text{m}\angle ARB = \text{m}\angle ASC$	*Subtraction*
$BR = CS$	*Given*
$\triangle ARB \cong \triangle ASC$	*SAS*
$\angle ABR \cong \angle ACS$	*Corresponding angles of congruent triangles are congruent*
$\overline{RX} \perp \overline{AB}, \overline{SY} \perp \overline{AC}$	*Given*
$\text{m}\angle RXB = 90° = \text{m}\angle SYC$	*Definition of perpendicular line segments.*
$\triangle BRX \cong \triangle SYC$	*AAS*
$BX = CY$	*Corresponding sides of congruent triangles are equal in length*

6. **Given:** $AX = BX$, $\text{m}\angle AMB = \text{m}\angle AYZ = 90°$.
Prove: $NY = NM$.



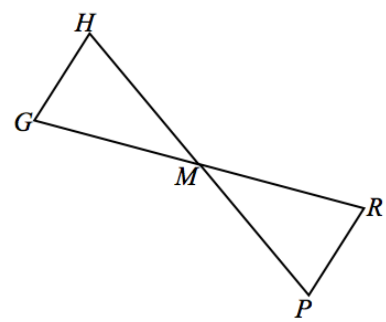
$AX = BX$	*Given*
$\text{m}\angle AMB = \text{m}\angle AYZ = 90°$	*Given*
$\text{m}\angle BXM = \text{m}\angle AXY$	*Vertical angles are equal in measure*
$\triangle BXM \cong \triangle AXY$	*AAS*
$BX + XY = BY$, $AX + XM = AM$	*Segments add*
$XM = XY$	*Corresponding sides of congruent triangles are equal in length*
$BY = AM$	*Substitution property of equality*
$\text{m}\angle BYN = 90°$	*Vertical angles are equal in measure*
$\text{m}\angle AMB + \text{m}\angle AMN = 180°$	*Linear pairs form supplementary angles*
$\text{m}\angle AMN = 90°$	*Subtraction property of equality*
$\text{m}\angle MNY = \text{m}\angle MNY$	*Reflexive property*
$\triangle BYN \cong \triangle AMN$	*AAS*
$NY = NM$	*Corresponding sides of congruent triangles are equal in length*

Exit Ticket (5 minutes)

Name ____ Date____________________

Lesson 27: Triangle Congruency Proofs

Exit Ticket



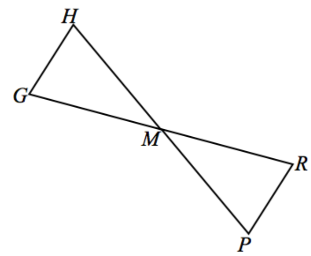
Given: M is the midpoint of GR, $\angle G \cong \angle R$.

Prove: $\triangle GHM \cong \triangle RPM$.

Exit Ticket Sample Solutions

Given: M is the midpoint of GR, $\angle G = \angle R$.

Prove: $\triangle GHM \cong \triangle RPM$.

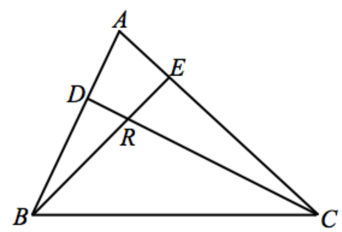


M is the midpoint of GR	*Given*
$\angle G \cong \angle R$	*Given*
$GM = RM$	*Definition of midpoint*
$\angle HMG \cong \angle PMR$	*Vertical angles are congruent.*
$\triangle GHM \cong \triangle RPM$	*ASA*

Problem Set Sample Solutions

Use your knowledge of triangle congruence criteria to write a proof for the following:

In the figure $\overline{BE} \cong \overline{CE}$, $\overline{DC} \perp \overline{AB}$, $\overline{BE} \perp \overline{AC}$, prove $\overline{AE} \cong \overline{RE}$.



$\mathrm{m}\angle ERC = \mathrm{m}\angle BRD$	*Vertical angles are equal in measure*
$\overline{DC} \perp \overline{AB}, \overline{BE} \perp \overline{AC}$	*Given*
$\mathrm{m}\angle BDR = 90°$, $\mathrm{m}\angle REC = 90°$	*Definition of perpendicular lines*
$\mathrm{m}\angle ABE = \mathrm{m}\angle RCE$	*Sum of the angle measures in a triangle is $180°$*
$\mathrm{m}\angle BAE = \mathrm{m}\angle BRD$	*Sum of the angle measures in a triangle is $180°$*
$\mathrm{m}\angle BAE = \mathrm{m}\angle ERC$	*Substitution property of equality*
$\overline{BE} \cong \overline{CE}$	*Given*
$\triangle BAE \cong \triangle CRE$	*AAS*
$\overline{AE} \cong \overline{RE}$	*Corresponding sides of congruent triangles are congruent*