Lesson 26: Triangle Congruency Proofs

Student Outcomes

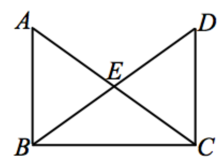
- Students complete proofs requiring a synthesis of the skills learned in the last four lessons.

Classwork

Exercises 1–6 (40 minutes)

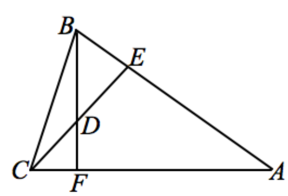
Exercises 1–6

1. Given: $\overline{AB} \perp \overline{BC}$, $\overline{BC} \perp \overline{DC}$.
   $\overline{DB}$ bisects $\angle ABC$, $\overline{AC}$ bisects $\angle DCB$.
   $EB = EC$.
   Prove: $\triangle BEA \cong \triangle CED$.



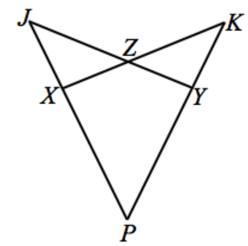
$\overline{AB} \perp \overline{BC}, \overline{BC} \perp \overline{DC}$	*Given*
$m\angle ABC = 90°$, $m\angle DCB = 90°$	*Definition of perpendicular lines*
$m\angle ABC = m\angle DCB$	*Transitive property*
$\overline{DB}$ *bisects* $\angle ABC$, $\overline{AC}$ *bisects* $\angle DCB$	*Given*
$m\angle ABE = 45°$, $m\angle DCE = 45°$	*Definition of an angle bisector*
$EB = EC$	*Given*
$m\angle AEB = m\angle DEC$	*Vertical angles are equal in measure*
$\triangle BEA \cong \triangle CED$	*ASA*

2. Given: $\overline{BF} \perp \overline{AC}$, $\overline{CE} \perp \overline{AB}$.
   $AE = AF$.
   Prove: $\triangle ACE \cong \triangle ABF$.



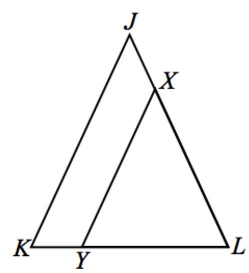
$\overline{BF} \perp \overline{AC}, \overline{CE} \perp \overline{AB}$	*Given*
$m\angle BFA = 90°$, $m\angle CEA = 90°$	*Definition of perpendicular*
$AE = AF$	*Given*
$m\angle A = m\angle A$	*Reflexive property*
$\triangle ACE \cong \triangle ABF$	*ASA*

3. Given: $XJ = YK$, $PX = PY$, $\angle ZXJ \cong \angle ZYK$.
   Prove: $JY = KX$.



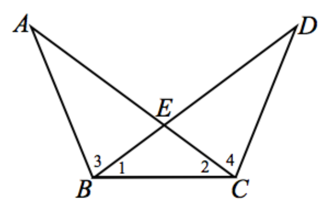
$XJ = YK$, $PX = PY$, $\angle ZXJ \cong \angle ZYK$	*Given*
$\overline{JP} \cong \overline{KP}$	*Segment addition*
$\mathrm{m}\angle JZX = \mathrm{m}\angle KZY$	*Vertical angles are equal in measure.*
$\triangle JZX \cong \triangle KZY$	*AAS*
$\angle J \cong \angle K$	*Corresponding angles of congruent triangles are congruent*
$\angle P \cong \angle P$	*Reflexive property*
$\triangle PJY \cong \triangle PKX$	*AAS*
$\overline{JY} \cong \overline{KX}$	*Corresponding sides of congruent triangles are congruent*
$JY = KX$	*Definition of congruent segments*

4. Given: $JK = JL$, $\overline{JK} \parallel \overline{XY}$.
   Prove: $XY = XL$.



$JK = JL$	*Given*
$\mathrm{m}\angle K = \mathrm{m}\angle L$	*Base angles of an isosceles triangle are equal in measure*
$\overline{JK} \parallel \overline{XY}$	*Given*
$\mathrm{m}\angle K = \mathrm{m}\angle XYL$	*When two parallel lines are cut by a transversal, corresponding angles are equal in measure*
$\mathrm{m}\angle XYL = \mathrm{m}\angle L$	*Transitive property*
$XY = XL$	*If two angles of a triangle are congruent, then the sides opposite the angles are equal in length*

5. Given: $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$.
   Prove: $\overline{AC} \cong \overline{BD}$.

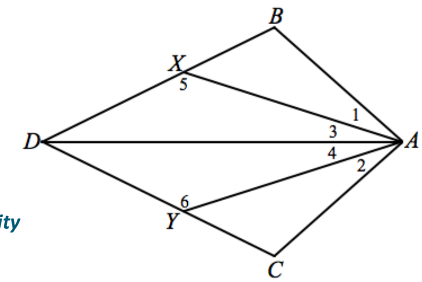


$\angle 1 \cong \angle 2$	*Given*
$\overline{BE} \cong \overline{CE}$	*When two angles of a triangle are congruent, it is an isosceles triangle*
$\angle 3 \cong \angle 4$	*Given*
$\angle AEB \cong \angle DEC$	*Vertical angles are congruent*
$\triangle ABC \cong \triangle DCB$	*ASA*
$\angle A \cong \angle D$	*Corresponding angles of congruent triangles are congruent*
$\overline{BC} \cong \overline{BC}$	*Reflexive property*
$\triangle ABC \cong \triangle DCB$	*AAS*
$\overline{AC} \cong \overline{BD}$	*Corresponding sides of congruent triangles are congruent*

6. **Given:** $\mathrm{m}\angle 1 = \mathrm{m}\angle 2,\ \mathrm{m}\angle 3 = \mathrm{m}\angle 4,\ AB = AC.$

Prove: (a) $\triangle ABD \cong \triangle ACD$.

(b) $\angle 5 \cong \angle 6$.



$\mathrm{m}\angle 1 = \mathrm{m}\angle 2,\ \mathrm{m}\angle 3 = \mathrm{m}\angle 4$	*Given*
$\mathrm{m}\angle 1 + \mathrm{m}\angle 3 = \mathrm{m}\angle DAB,$	
$\mathrm{m}\angle 2 + \mathrm{m}\angle 4 = \mathrm{m}\angle DAC$	*Angle addition postulate*
$\mathrm{m}\angle DAB = \mathrm{m}\angle DAC$	*Substitution property of equality*
$AD = AD$	*Reflexive property*
$\triangle ABD \cong \triangle ACD$	*SAS*
$\angle BDA \cong \angle VDA$	*Corresponding angles of congruent triangles are congruent*
$\triangle DXA \cong \triangle DYA$	*ASA*
$\angle 5 \cong \angle 6$	*Corresponding angles of congruent triangles are congruent*

Exit Ticket (5 minutes)

Name ____ Date____________________

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Exit Ticket

Identify the two triangle congruence criteria that do NOT guarantee congruence. Explain why they do not guarantee congruence and provide illustrations that support your reasoning.

Exit Ticket Sample Solutions

Identify the two triangle congruence criteria that do NOT guarantee congruence. Explain why they do not guarantee congruence and provide illustrations that support your reasoning.

Students should identify AAA and SSA as the two types of criteria that do not guarantee congruence. Appropriate illustrations should be included with their justifications.

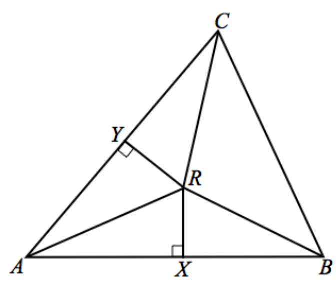
Problem Set Sample Solutions

Use your knowledge of triangle congruence criteria to write a proof for the following:

In the figure $\overline{RX}$ and $\overline{RY}$ are the perpendicular bisectors of $\overline{AB}$ and $\overline{AC}$, respectively.

Prove: (a) $\triangle RAX \cong \triangle RAY$.

(b) $\overline{RA} \cong \overline{RB} \cong \overline{RC}$.



$\overline{RX}$ *is the perpendicular bisector of* $\overline{AB}$	*Given*
$\overline{RY}$ *is the perpendicular bisector of* $\overline{AC}$	*Given*
$m\angle RYA = 90°$, $m\angle RXA = 90°$	*Definition of perpendicular bisector*
$\overline{AR} \cong \overline{AR}$	*Reflexive property*
$\triangle RAX$, $\triangle RAY$ *are right triangles*	*Definition of right triangle*
$\triangle RAX \cong \triangle RAY$	*HL*
$m\angle RYC = 90°$, $m\angle RXB = 90°$	*Definition of perpendicular bisector*
$\overline{AX} \cong \overline{XB}$, $\overline{AY} \cong \overline{YC}$	*Definition of perpendicular bisector*
$\overline{YR} \cong \overline{YR}$, $\overline{XR} \cong \overline{XR}$	*Reflexive property*
$\triangle RAY \cong \triangle RCY$, $\triangle RAX \cong \triangle RBX$	*SAS*
$\triangle RBX \cong \triangle RAX \cong \triangle RAY \cong \triangle RCY$	*Transitive property*
$\overline{RA} \cong \overline{RB} \cong \overline{RC}$	*Corresponding sides of congruent triangles are congruent*