Lesson 25: Congruence Criteria for Triangles—AAS and HL

Student Outcomes

- Students learn why any two triangles that satisfy the AAS or HL congruence criteria must be congruent.
- Students learn why any two triangles that meet the AAA or SSA criteria are not necessarily congruent.

Classwork

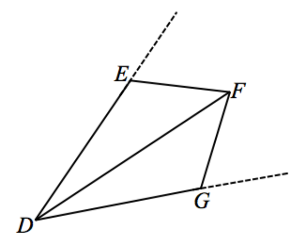
Opening Exercise (7 minutes)

Opening Exercise

Write a proof for the following question. Once done, compare your proof with a neighbor's.

Given: $DE = DG$, $EF = GF$

Prove: $\overline{DF}$ **is the angle bisector of** $\angle EDG$



$DE = DG$	*Given*
$EF = GF$	*Given*
$DF = DF$	*Reflexive property*
$\triangle DEF \cong \triangle DGF$	*SSS*
$\angle EDF \cong \angle GDF$	*Corresponding angles of congruent triangles are congruent.*
$\overline{DF}$ *is the angle bisector of* $\angle EDG$	*Definition of an angle bisector*

Exploratory Challenge (25 minutes)

The included proofs of AAS and HL are not transformational; rather, they follow from ASA and SSS, already proved.

Exploratory Challenge

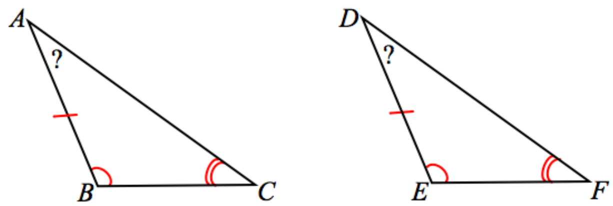
Today we are going to examine three possible triangle congruence criteria, Angle-Angle-Side (AAS), Side-Side-Angle (SSA), and Angle-Angle-Angle (AAA). Ultimately, only one of the three possible criteria will ensure congruence.

Angle-Angle-Side Triangle Congruence Criteria (AAS): Given two triangles ABC and $A'B'C'$. If $AB = A'B'$ (Side), $\mathrm{m}\angle B = \mathrm{m}\angle B'$ (Angle), and $\mathrm{m}\angle C = \mathrm{m}\angle C'$ (Angle), then the triangles are congruent.

Proof:

Consider a pair of triangles that meet the AAS criteria. If you knew that two angles of one triangle corresponded to and were equal in measure to two angles of the other triangle, what conclusions can you draw about the third angles of each triangle?

Since the first two angles are equal in measure, the third angles must also be equal in measure.



Given this conclusion, which formerly learned triangle congruence criteria can we use to determine if the pair of triangles are congruent?

ASA

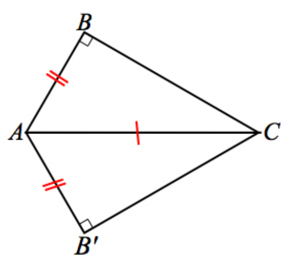
Therefore, the AAS criterion is actually an extension of the _____*ASA*_____ triangle congruence criterion.

Note that when using the Angle-Angle-Side triangle congruence criteria as a reason in a proof, you need only state the congruence and "AAS."

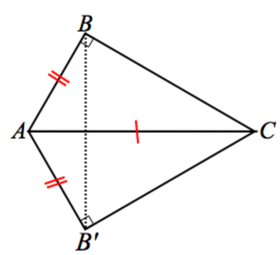
Hypotenuse-Leg Triangle Congruence Criteria (HL): Given two right triangles ABC and $A'B'C'$ with right angles B and B', if $AB = A'B'$ (Leg) and $AC = A'C'$ (Hypotenuse), then the triangles are congruent.

Proof:

As with some of our other proofs, we will not start at the very beginning, but imagine that a congruence exists so that triangles have been brought together such that $A = A'$ and $C = C'$; the hypotenuse acts as a common side to the transformed triangles.



Similar to the proof for SSS, we add a construction and draw $\overline{BB'}$.

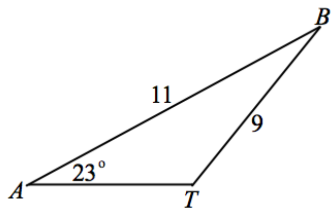


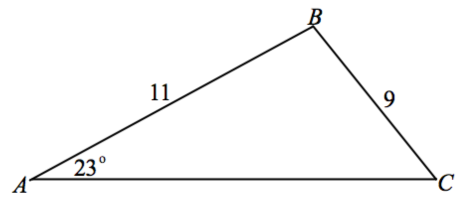
$\triangle ABB'$ is isosceles by definition, and we can conclude that base angles $\mathrm{m}\angle ABB' = \mathrm{m}\angle AB'B$. Since $\angle CBB'$ and $\angle CB'B$ are both the complements of equal angle measures ($\angle ABB'$ and $\angle AB'B$), they too are equal in measure. Furthermore, since $\mathrm{m}\angle CBB' = \mathrm{m}\angle CB'B$, the sides of $\triangle CBB'$ opposite them are equal in measure: $BC = B'C'$.

Then, by SSS, we can conclude $\triangle ABC \cong \triangle A'B'C'$. Note that when using the Hypotenuse-Leg triangle congruence criteria as a reason in a proof, you need only state the congruence and "HL."

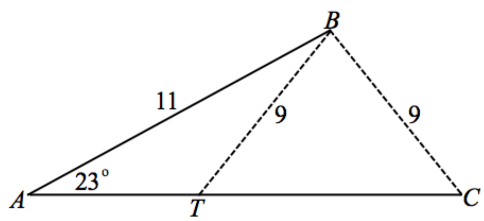
Criteria that *do not* determine two triangles as congruent: SSA and AAA

<u>Side-Side-Angle (SSA)</u>: Observe the diagrams below. Each triangle has a set of adjacent sides of measures 11 and 9, as well as the non-included angle of $23°$. Yet, the triangles are not congruent.





Examine the composite made of both triangles. The sides of lengths 9 each have been dashed to show their possible locations.



The triangles that satisfy the conditions of SSA cannot *guarantee* congruence criteria. In other words, two triangles under SSA criteria may or may not be congruent; therefore, we cannot categorize SSA as congruence criterion.

<u>Angle-Angle-Angle (AAA)</u>: A correspondence exists between $\triangle ABC$ and $\triangle DEF$. Trace $\triangle ABC$ onto patty paper, and line up corresponding vertices.

Based on your observations, why isn't AAA categorized as congruence criteria? Is there any situation in which AAA does guarantee congruence?

Even though the angle measures may be the same, the sides can be proportionally larger; you can have similar triangles in addition to a congruent triangle.

List all the triangle congruence criteria here:

SSS, SAS, ASA, AAS, HL

List the criteria that do not determine congruence here:

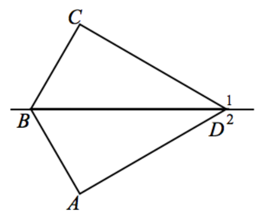
SSA, AAA

Examples (8 minutes)

Examples

1. **Given:** $\overline{BC} \perp \overline{CD}$, $\overline{AB} \perp \overline{AD}$, $\mathrm{m}\angle 1 = \mathrm{m}\angle 2$

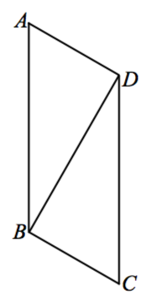
   Prove: $\triangle BCD \cong \triangle BAD$



$\mathrm{m}\angle 1 = \mathrm{m}\angle 2$	*Given*
$\overline{AB} \perp \overline{AD}$	*Given*
$\overline{BC} \perp \overline{CD}$	*Given*
$BD = BD$	*Reflexive property*
$\mathrm{m}\angle 1 + \mathrm{m}\angle CDB = 180°$	*Linear pairs form supplementary angles*
$\mathrm{m}\angle 2 + \mathrm{m}\angle ADB = 180°$	*Linear pairs form supplementary angles*
$\mathrm{m}\angle CDB = \mathrm{m}\angle ADB$	*If two angles are equal in measure, then their supplements are equal in measure*
$\mathrm{m}\angle BCD = \mathrm{m}\angle BAD = 90°$	*Definition of perpendicular line segments*
$\triangle BCD \cong \triangle BAD$	*AAS*

2. **Given:** $\overline{AD} \perp \overline{BD}$, $\overline{BD} \perp \overline{BC}$, $AB = CD$

   Prove: $\triangle ABD \cong \triangle CDB$



$\overline{AD} \perp \overline{BD}$	*Given*
$\overline{BD} \perp \overline{BC}$	*Given*
$\triangle ABD$ *is a right triangle*	*Definition of perpendicular line segments*
$\triangle CDB$ *is a right triangle*	*Definition of perpendicular line segments*
$AB = CD$	*Given*
$BD = BD$	*Reflexive property*
$\triangle ABD \cong \triangle CDB$	*HL*

Exit Ticket (5 minutes)

Name ____ Date____________________

Lesson 25: Congruence Criteria for Triangles—AAS and HL

Exit Ticket

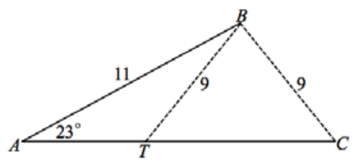
1. Sketch an example of two triangles that meet the AAA criteria but are not congruent.

2. Sketch an example of two triangles that meet the SSA criteria that are not congruent.

Exit Ticket Sample Solutions

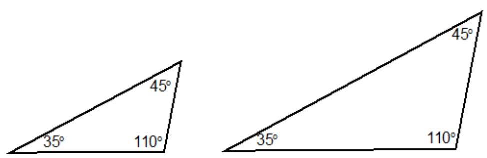
1. **Sketch an example of two triangles that meet the AAA criteria but are not congruent.**

   Responses should look something like the example below.



2. **Sketch an example of two triangles that meet the SSA criteria that are not congruent.**

   Responses should look something like the example below.

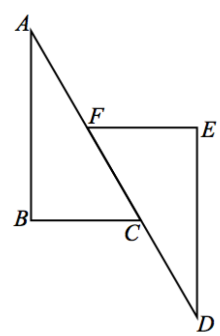


Problem Set Sample Solutions

Use your knowledge of triangle congruence criteria to write proofs for each of the following problems.

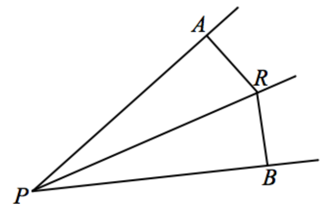
1. **Given:** $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$, $\overline{BC} \parallel \overline{EF}$, $AF = DC$

   Prove: $\triangle ABC \cong \triangle DEF$



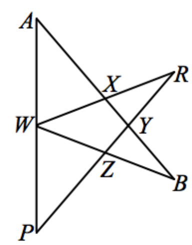
$\overline{AB} \perp \overline{BC}$	*Given*
$\overline{DE} \perp \overline{EF}$	*Given*
$\overline{BC} \parallel \overline{EF}$	*Given*
$AF = DC$	*Given*
$m\angle B = m\angle E = 90°$	*Definition of perpendicular lines*
$m\angle C = m\angle F$	*When two parallel lines are cut by a transversal, the alternate interior angles are equal in measure*
$FC = FC$	*Reflexive property*
$AF + FC = FC + CD$	*Addition property of equality*
$AC = DF$	*Segment addition*
$\triangle ABC \cong \triangle DEF$	*AAS*

2. In the figure, $\overline{PA} \perp \overleftrightarrow{AR}$ and $\overline{PB} \perp \overleftrightarrow{RB}$ and R is equidistant from $\overleftrightarrow{PA}$ and $\overleftrightarrow{PB}$. Prove that $\overline{PR}$ bisects $\angle APB$.



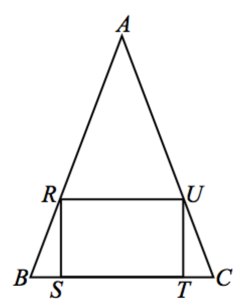
$\overline{PA} \perp \overline{AR}$	*Given*
$\overline{PB} \perp \overline{BR}$	*Given*
$RA = RB$	*Given*
$\text{m}\angle A = \text{m}\angle R = 90°$	*Definition of perpendicular lines*
$\triangle PAR$, $\triangle PBR$ *are right triangles*	*Definition of right triangle*
$PR = PR$	*Reflexive property*
$\triangle PAR \cong \triangle PBR$	*HL*
$\angle APR \cong \angle RPB$	*Corresponding angles of congruent triangles are congruent.*
$\overline{PR}$ *bisects* $\angle APB$	*Definition of an angle bisector*

3. Given: $\angle A \cong \angle P$, $\angle B \cong \angle R$, W is the midpoint of $\overline{AP}$
Prove: $\overline{RW} \cong \overline{BW}$



$\angle A \cong \angle P$	*Given*
$\angle B \cong \angle R$	*Given*
W *is the midpoint of* $\overline{AP}$	*Given*
$AW = PW$	*Definition of midpoint*
$\triangle AWB \cong \triangle PWR$	*AAS*
$\overline{RW} \cong \overline{BW}$	*Corresponding sides of congruent triangles are congruent.*

4. Given: $BR = CU$, rectangle $RSTU$
Prove: $\triangle ARU$ is isosceles



$BR = CU$	*Given*
Rectangle $RSTU$	*Given*
$\overline{BC} \parallel \overline{RU}$	*Definition of a rectangle*
$\text{m}\angle RBS = \text{m}\angle ARU$	*When two para. lines are cut by a trans., the corr. angles are equal in measure*
$\text{m}\angle UCT = \text{m}\angle AUR$	*When two para. lines are cut by a trans., the corr. angles are equal in measure*
$\text{m}\angle RST = 90°$, $\text{m}\angle UTS = 90°$	*Definition of a rectangle*
$\text{m}\angle RSB + \text{m}\angle RST = 180$	*Linear pairs form supplementary angles.*
$\text{m}\angle UTC + \text{m}\angle UTS = 180$	*Linear pairs form supplementary angles.*
$\text{m}\angle RSB = 90°$, $\text{m}\angle UTC = 90°$	*Subtraction property of equality*
$\triangle BRS$ *and* $\triangle TUC$ *are right triangles*	*Definition of right triangle*
$RS = UT$	*Definition of a rectangle*
$\triangle BRS \cong \triangle TUC$	*HL*
$\text{m}\angle RBS = \text{m}\angle UCT$	*Corresponding angles of congruent triangles are equal in measure*
$\text{m}\angle ARU = \text{m}\angle AUR$	*Substitution property of equality*
$\triangle ARU$ *is isosceles*	*If two angles in a triangle are equal in measure, then it is isosceles.*