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Lesson 23: Base Angles of Isosceles Triangles

Student Outcomes

* Students examine two different proof techniques via a familiar theorem.
* Students complete proofs involving properties of an isosceles triangle.

Lesson Notes

In Lesson 23, students study two proofs to demonstrate why the base angles of isosceles triangles are congruent. The first proof uses transformations, while the second uses the recently acquired understanding of the SAS triangle congruency. The demonstration of both proofs will highlight the utility of the SAS criteria. Encourage students to articulate why the SAS criteria is so useful.

The goal of this lesson is to compare two different proof techniques by investigating a familiar theorem. Be careful not to suggest that proving an established fact about isosceles triangles is somehow the significant aspect of this lesson. However, if you need to, you can help motivate the lesson by appealing to history. Point out that Euclid used SAS and that the first application he made of it was in proving that base angles of an isosceles triangle are congruent. This is a famous part of the Elements. Today, proofs using SAS and proofs using rigid motions are valued equally.

Classwork

Opening Exercise (7 minutes)

Opening Exercise

Describe the additional piece of information needed for each pair of triangles to satisfy the SAS triangle congruence criteria.

1. Given:

 , , or

Prove:



1. Given:

 or

Prove:

Exploratory Challenge (25 minutes)


Exploratory Challenge

Today we examine a geometry fact that we already accept to be true. We are going to prove this known fact in two ways: (1) by using transformations and (2) by using SAS triangle congruence criteria.

Here is isosceles triangle . We accept that an isosceles triangle, which has (at least) two congruent sides, also has congruent base angles.

Label the congruent angles in the figure.

Now we will prove that the base angles of an isosceles triangle are always congruent.

**Prove Base Angles of an Isosceles are Congruent: Transformations**

While simpler proofs do exist, this version is included to reinforce the idea of congruence as defined by rigid motions. Allow 15 minutes for the first proof.


Prove Base Angles of an Isosceles are Congruent: Transformations

Given: Isosceles , with

Prove:

*Construction*: Draw the angle bisector of , whereis the intersection of the bisector and . We need to show that rigid motions will map point to point and pointto point .

Let be the reflection through . Through the reflection, we want to demonstrate two pieces of information that map to point and vice versa: (1) maps to , and (2) .

Since is on the line of reflection, , . Reflections preserve angle measures, so the measure of the reflected angle equals the measure of ; therefore, . Reflections also preserve lengths of segments; therefore, the reflection of will still have the same length as . By hypothesis, , so the length of the reflection will also be equal to . Then *.* Using similar reasoning, we can show that .

Reflections map rays to rays, so and . Again, since reflections preserve angle measures, the measure of is equal to the measure of .

We conclude that . Equivalently, we can state that . In proofs, we can state that “base angles of an isosceles triangle are equal in measure” or that “base angles of an isosceles triangle are congruent.”

**Prove Base Angles of an Isosceles are Congruent: SAS**

Allow 10 minutes for the second proof.


Prove Base Angles of an Isosceles are Congruent: SAS

Given: Isosceles , with

Prove:

*Construction:* Draw the angle bisector of , where is the intersection of the bisector and . We are going to use this auxiliary line towards our SAS criteria.

Allow students five minutes to attempt the proof on their own.

 Given

 Reflexive property

 *Definition of an angle bisector*

 Side Angle Side criteria

 Corresponding angles of congruent triangles are congruent

Exercises (10 minutes)

Note that in Exercise 4, students are asked to use transformations in the proof. This is intentional to reinforce the notion that congruence is defined in terms of rigid motions.

Exercises

1. Given: ; bisects

Prove:

 Given

 Base angles of an isosceles triangle are congruent.

 Definition of a segment bisector

 SAS

 Corresponding angles of congruent triangles are congruent.

 *Linear pairs form supplementary angles.*

 Substitution property of equality

 Division property of equality

 Definition of perpendicular lines

1. Given: ,

Prove: bisects

 Given

 *Base angles of an isosceles triangle are congruent.*

 *Given*

 *Base angles of an isosceles triangle are equal in measure*

*,*

 *Angle addition postulate*

*,*

 *Subtraction property of equality*

 *Substitution property of equality*

 SAS

 Corresponding part of congruent triangles are congruent

 bisects . Definition of an angle bisector



1. Given: ,

Prove: is isosceles

 Given

 Given

 *Base angles of an isosceles triangle are equal in measure*

 *Linear pairs form supplementary angles*

 *Linear pairs form supplementary angles*

 *Subtraction property of equality*

 *Subtraction property of equality*

 *Substitution property of equality*

 SAS

 Corresponding segments of congruent triangles are congruent.

 is isosceles Definition of an isosceles triangle

1. Given: , with

Prove:

(Converse of base angles of isosceles triangle)

Hint: Use a transformation.

Proof:

We can prove that by using rigid motions. Construct the perpendicular bisector to . Note that we do not know whether the point is on . If we did, we would know immediately that , since all points on the perpendicular bisector are equidistant from and . We need to show that is on , or equivalently, that the reflection across takes to .

Let be the transformation that reflects the across . Since is the perpendicular bisector, and . We do not know where the transformation takes ; for now, let us say that .

Since and rigid motions preserve angle measures, after applying to , we get that . Angle and angle share a ray, are of equal measure, and and are both in the same half-plane with respect to line . Hence, and are the same ray. In particular, is a point somewhere on .

Likewise, applying to gives , and for the same reasons in the previous paragraph, must also be a point somewhere on . Therefore, is common to both and .

The only point common to both and is point ; thus, and must be the same point, i.e., .

Hence, the reflection takes to , which means is on the line and , or .

1. Given: , with is the angle bisector of , and

Prove:

 Given

 *When two parallel lines are cut by a transversal, the alternate interior angles are equal*

 *When two parallel lines are cut by a transversal, the corresponding angles are equal*

 *Definition of an angles bisector*

 *Substitution property of equality*

 When the base angles of a triangle are equal in measure, the triangle is isosceles

Exit Ticket (3 minutes)

Name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Lesson 23: Base Angles of Isosceles Triangles

Exit Ticket



For each of the following, if the given congruence exists, name the isosceles triangle and name the pair of congruent angles for the triangle based on the image above.

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Exit Ticket Sample Solutions



For each of the following, if the given congruence exists, name the isosceles triangle and name the pair of congruent angles for the triangle based on the image above.

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| ,  | ,  |
| ,  | ,  |
| ,  | ,  |

Problem Set Sample Solutions



1. Given: ,

Prove: and are right triangles

 Given

 is isosceles Definition of isosceles triangle

 *Base angles of an isosceles triangle are equal in measure*

 *Given*

 *SAS*

 *Corresponding angles of congruent triangles are equal in measure*

 *Linear pairs form supplementary angles*

 *Substitution property of equality*

 *Division property of equality*

 *Transitive property*

 and are right triangles Definition of a right triangle

1. Given: and

Prove:

 Given

 is isosceles Definition of isosceles triangle

 *Base angles of an isosceles triangle are equal in measure*

 Given

 *If parallel lines are cut by a transversal, then corresponding angles are equal in measure*

 *If parallel lines are cut by a transversal, then corresponding angles are equal in measure*

 Transitive property

 is isosceles If the base angles of a triangle are equal in measure, the triangle is isosceles

 Definition of an isosceles triangle

1. In the diagram, is isosceles with. In your own words, describe how transformations and the properties of rigid motions can be used to show that .

Answers will vary. A possible response would summarize the key elements of the proof from the lesson, such as the response below.

It can be shown that using a reflection across the bisector of . The two angles formed by the bisector of would be mapped onto one another since they share a ray, and rigid motions preserve angle measure.

Since segment length is also preserved, is mapped onto and vice versa.

Finally, since rays are taken to rays and rigid motions preserve angle measure, is mapped onto , is mapped onto , and is mapped onto . From this, we see that .