Lesson 21: Correspondence and Transformations

Student Outcomes

- Students practice applying a sequence of rigid motions from one figure onto another figure in order to demonstrate that the figures are congruent.

Lesson Notes

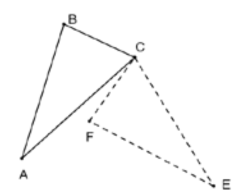
In Lesson 21, we will consolidate our understanding of congruence in terms of rigid motions with our knowledge of corresponding vertices and sides of triangles. We will identify specific sides and angles in the pre-image of a triangle that map onto specific angles and sides of the image. If a rigid motion results in every side and every angle of the pre-image mapping onto every corresponding side and angle of the image, we will assert that the triangles are congruent.

Classwork

Opening Exercise (7 minutes)

Opening Exercise

The figure to the right represents a rotation of $\triangle ABC$ $80°$ around vertex C. Name the triangle formed by the image of $\triangle ABC$. Write the rotation in function notation, and name all corresponding angles and sides.



$\triangle EFC$

$R_{D,80°}(\Delta EFC)$

Corresponding angles	*Corresponding sides*
$\angle A \rightarrow \angle E$	$\overline{AB} \rightarrow \overline{EF}$
$\angle B \rightarrow \angle F$	$\overline{BC} \rightarrow \overline{FC}$
$\angle C \rightarrow \angle C$	$\overline{AC} \rightarrow \overline{EC}$

Discussion (5 minutes)

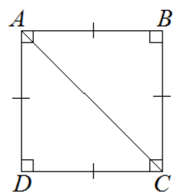
Discussion

In the Opening Exercise, we explicitly showed a single rigid motion, which mapped every side and every angle of $\triangle ABC$ onto $\triangle EFC$. Each corresponding pair of sides and each corresponding pair of angles was congruent. When each side and each angle on the pre-image maps onto its corresponding side or angle on the image, the two triangles are congruent. Conversely, if two triangles are congruent, then each side and angle on the pre-image is congruent to its corresponding side or angle on the image.

Example 1 (8 minutes)

Example 1

$ABCD$ is a square, and $\overline{AC}$ is one diagonal of the square. $\triangle ABC$ is a reflection of $\triangle ADC$ across segment AC. Complete the table below identifying the missing corresponding angles and sides.



Corresponding angles	Corresponding sides
$\angle BAC \rightarrow \angle DAC$ $\angle ABC \rightarrow \angle ADC$ $\angle BCA \rightarrow \angle DCA$	$\overline{AB} \rightarrow \overline{AD}$ $\overline{BC} \rightarrow \overline{DC}$ $\overline{AC} \rightarrow \overline{AC}$

a. **Are the corresponding sides and angles congruent? Justify your response.**

Since the triangle is a rigid transformation, all angles and sides maintain their size.

b. **Is $\triangle ABC \cong \triangle ADC$? Justify your response.**

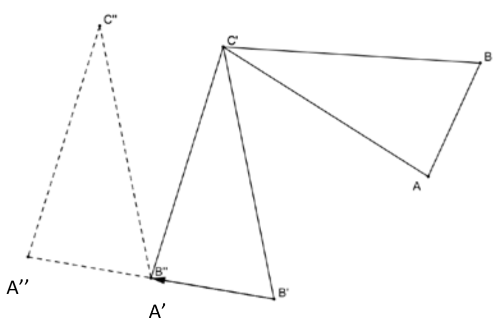
Yes. Since $\triangle ADC$ is a reflection of $\triangle ABC$, they must be congruent.

Exercises 1–3 (20 minutes)

Exercises 1–3

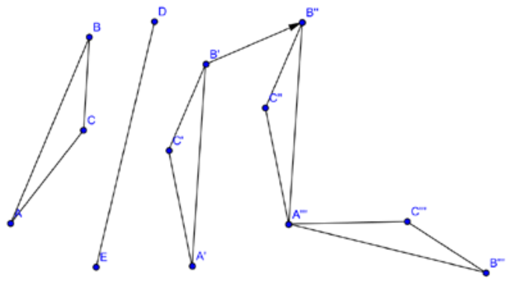
Each exercise below shows a sequence of rigid motions that map a pre-image onto a final image. Identify each rigid motion in the sequence, writing the composition using function notation. Trace the congruence of each set of corresponding sides and angles through all steps in the sequence, proving that the pre-image is congruent to the final image by showing that every side and every angle in the pre-image maps onto its corresponding side and angle in the image. Finally, make a statement about the congruence of the pre-image and final image.

1.



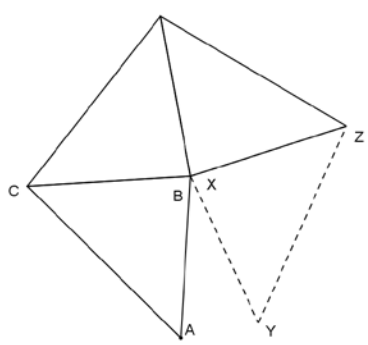
Sequence of rigid motions (2)	*rotation, translation*
Composition in function notation	$T_{\overline{B'B''}}\left(R_{c,90^\circ}(\triangle ABC)\right)$
Sequence of corresponding sides	$\overline{AB} \rightarrow \overline{A''B''}$ $\overline{BC} \rightarrow \overline{B''C''}$ $\overline{AC} \rightarrow \overline{A''C''}$
Sequence of corresponding angles	$A \rightarrow A''$ $B \rightarrow B''$ $C \rightarrow C''$
Triangle congruence statement	$\triangle ABC \cong \triangle A''B''C''$

2.



Sequence of rigid motions (3)	*reflection, translation, rotation*
Composition in function notation	$R_{A'',100^\circ}\left(T_{\overline{B'B''}}\left(r_{\overline{DE}}(\triangle ABC)\right)\right)$
Sequence of corresponding sides	$\overline{AB} \rightarrow \overline{A'''B'''}$ $\overline{BC} \rightarrow \overline{B'''C'''}$ $\overline{AC} \rightarrow \overline{A'''C'''}$
Sequence of corresponding angles	$A \rightarrow A'''$ $B \rightarrow B'''$ $C \rightarrow C'''$
Triangle congruence statement	$\triangle ABC \cong \triangle A'''B'''C'''$

3.



Sequence of rigid motions (3)	*reflections*
Composition in function notation	$r_{\overline{XZ}}\left(r_{\overline{BA'}}\left(r_{\overline{BC}}(\triangle ABC)\right)\right)$
Sequence of corresponding sides	$\overline{AB} \rightarrow \overline{YX}$ $\overline{AC} \rightarrow \overline{YZ}$ $\overline{BC} \rightarrow \overline{XZ}$
Sequence of corresponding angles	$A \rightarrow Y$ $B \rightarrow X$ $C \rightarrow Z$
Triangle congruence statement	$\triangle ABC \cong \triangle YXZ$

Exit Ticket (5 minutes)

Name ____ Date____________________

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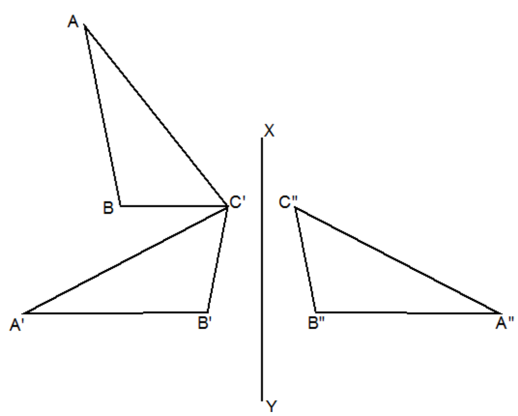
Exit Ticket

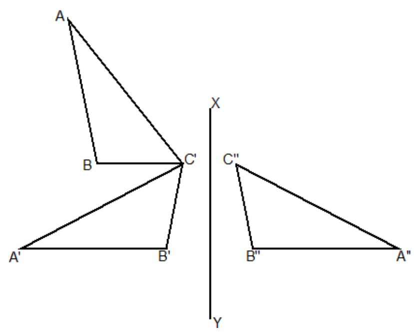
Complete the table based on the series of rigid motions performed on $\triangle ABC$ below.



Sequence of rigid motions (2)	
Composition in function notation	
Sequence of corresponding sides	
Sequence of corresponding angles	
Triangle congruence statement	

Exit Ticket Sample Solutions

Complete the table based on the series of rigid motions performed on $\triangle ABC$ below.



Sequence of rigid motions (2)	*rotation, reflection*
Composition in function notation	$\left(r_{\overline{XY}}\left(R_{C,90^\circ}\right)\right)$
Sequence of corresponding sides	$\overline{AB} \rightarrow \overline{A''B''}$ $\overline{BC} \rightarrow \overline{B''C''}$ $\overline{AC} \rightarrow \overline{A''C''}$
Sequence of corresponding angles	$A \rightarrow A'$ $B \rightarrow B'$ $C \rightarrow C'$
Triangle congruence statement	$\triangle ABC \cong \triangle A''B''C''$

Problem Set Sample Solutions

1. Exercise 3 above mapped $\triangle ABC$ onto $\triangle YXZ$ in three *steps*. Construct a fourth step that would map $\triangle YXZ$ back onto $\triangle ABC$.

   Construct an angle bisector for the $\angle ABY$, and reflect $\triangle YXZ$ over that line.

2. Explain triangle congruence in terms of rigid motions. Use the terms corresponding sides and corresponding angles in your explanation.

   Triangle congruence can be found using a series of rigid motions in which you map an original or pre-image of a figure onto itself. By doing so, all the corresponding sides and angles of the figure will map onto their matching corresponding sides or angles, which proves the figures are congruent.