Lesson 16: Translations

Classwork

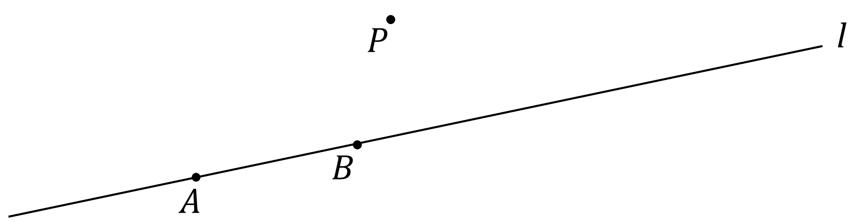
Exploratory Challenge

In Lesson 4, you completed a construction exercise that resulted in a pair of parallel lines (Problem 1 from the Problem Set). Now we examine an alternate construction.

Construct the line parallel to a given line AB through a given point P.

1. Draw circle P: Center P, radius AB.
2. Draw circle B: Center B, radius AP.
3. Label the intersection of circle P and circle B as Q.
4. Draw $\overleftrightarrow{PQ}$.

Note: Circles P and B intersect in two locations. Pick the intersection Q so that points A and Q are in opposite half-planes of line PB.



Discussion

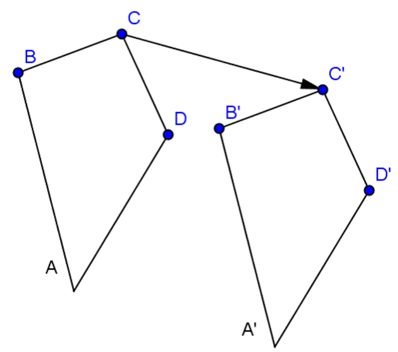
To perform a translation, we need to use the above construction. Let us investigate the definition of translation.

For vector $\overrightarrow{AB}$, the *translation along* $\overrightarrow{AB}$ is the transformation $T_{\overrightarrow{AB}}$ of the plane defined as follows:

1. For any point P on the line AB, $T_{\overrightarrow{AB}}(P)$ is the point Q on $\overleftrightarrow{AB}$ so that $\overrightarrow{PQ}$ has the same length and the same direction as $\overrightarrow{AB}$, and
2. For any point P not on $\overleftrightarrow{AB}$, $T_{\overrightarrow{AB}}(P)$ is the point Q obtained as follows. Let l be the line passing through P and parallel to $\overleftrightarrow{AB}$. Let m be the line passing through B and parallel to line AP. The point Q is the intersection of l and m.

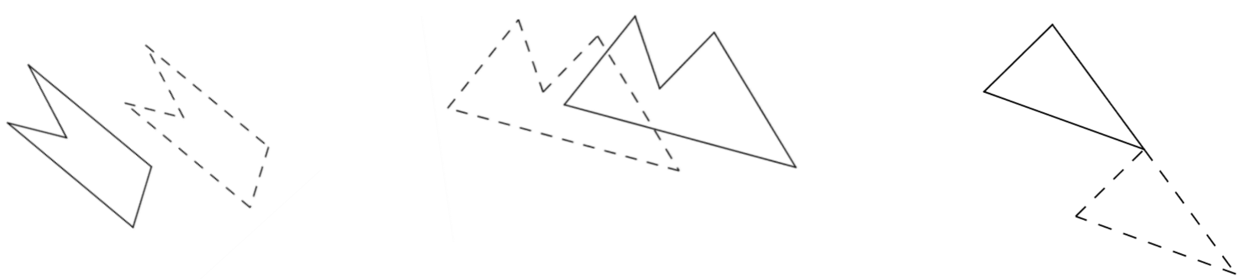
Note: The parallel line construction above shows a quick way to find the point Q in part 2 of the definition of translation!

In the figure to the right, quadrilateral $ABCD$ has been translated the length and direction of vector $\overrightarrow{CC'}$. Notice that the distance and direction from each vertex to its corresponding vertex on the image are identical to that of $\overrightarrow{CC'}$.



Example 1

Draw the vector that defines each translation below.

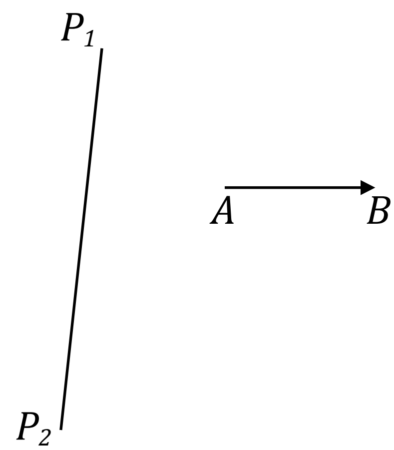


Finding the vector is relatively straightforward. Applying a vector to translate a figure is more challenging. To translate a figure, we must construct parallel lines to the vector through the vertices of the original figure and then find the points on those parallel lines that are the same direction and distance away as given by the vector.

Example 2

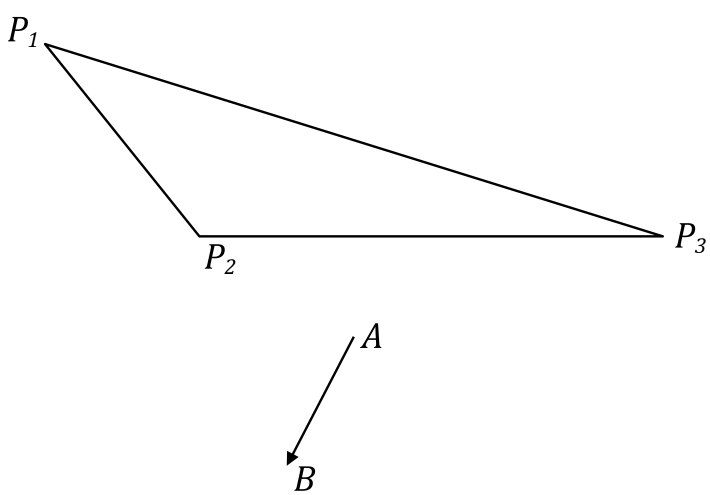
Use your compass and straightedge to apply $T_{\overrightarrow{AB}}$ to segment P_1P_2.

Note: Use the steps from the Exploratory Challenge *twice* for this question, creating two lines parallel to $\overrightarrow{AB}$: one through P_1 and one through P_2.



Example 3

Use your compass and straightedge to apply $T_{\overrightarrow{AB}}$ to $\triangle P_1P_2P_3$.



Relevant Vocabulary

__Parallel__: Two lines are *parallel* if they lie in the same plane and do not intersect. Two segments or rays are parallel if the lines containing them are parallel lines.

Lesson Summary

A translation carries segments onto segments of equal length.

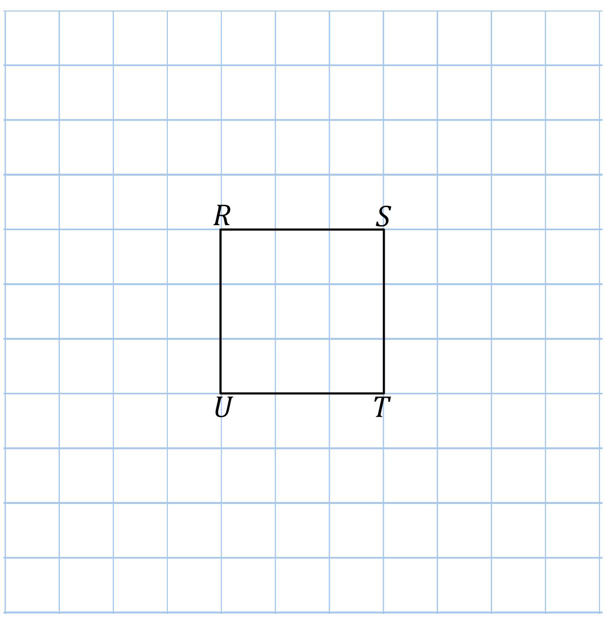
A translation carries angles onto angles of equal measure.

Problem Set

Translate each figure according to the instructions provided.

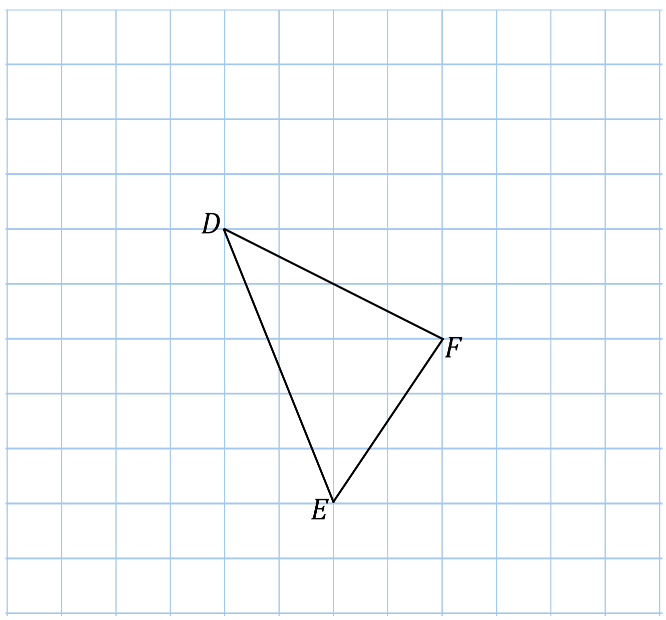
1. 2 units down and 3 units left.

   Draw the vector that defines the translation.

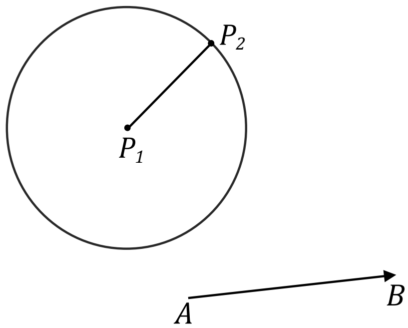


2. 1 unit up and 2 units right.

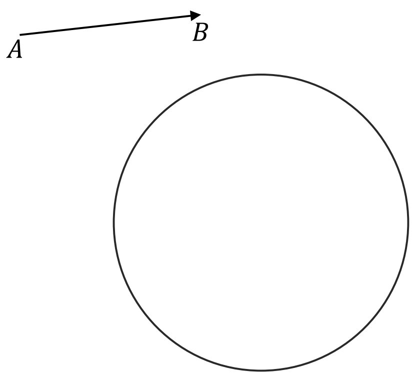
   Draw the vector that defines the translation.



3. Use your compass and straightedge to apply $T_{\overrightarrow{AB}}$ to the circle below (center P_1, radius $\overline{P_1P_2}$).

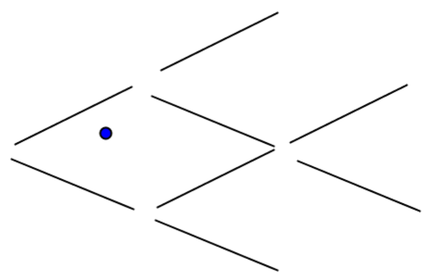


4. Use your compass and straightedge to apply $T_{\overrightarrow{AB}}$ to the circle below.
Hint: You will need to first find the center of the circle. You can use what you learned in Lesson 4 to do this.

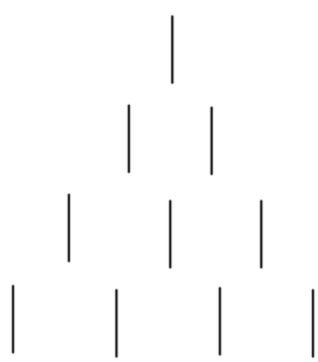


Two classic toothpick puzzles appear below. Solve each puzzle.

5. Each segment on the fish represents a toothpick. Move (translate) exactly three toothpicks and the eye to make the fish swim in the opposite direction. Show the translation vectors needed to move each of the three toothpicks and the eye.



6. Again, each segment represents a single toothpick. Move (translate) exactly three toothpicks to make the "triangle" point downward. Show the translation vectors needed to move each of the three toothpicks.



7. Apply $T_{\overrightarrow{GH}}$ to translate $\triangle ABC$.