



Lesson 2: Construct an Equilateral Triangle

Student Outcomes

- Students apply the equilateral triangle construction to more challenging problems.
- Students communicate mathematical concepts clearly and concisely.

Lesson Notes

Lesson 2 directly builds on the notes and exercises from Lesson 1. The continued lesson allows the class to review and assess understanding from Lesson 1. By the end of this lesson, students should be able to apply their knowledge of how to construct an equilateral triangle to more difficult constructions and to write clear and precise steps for these constructions.

Students critique each other's construction steps in the Opening Exercise; this is an opportunity to highlight Mathematical Practice 3. Through the critique, students experience how a lack of precision affects the outcome of a construction. Be prepared to guide the conversation to overcome student challenges, perhaps by referring back to the Euclid piece from Lesson 1 or by sharing your own writing. Remind students to focus on the vocabulary they are using in the directions because it will become the basis of writing proofs as the year progresses.

In the Exploratory Challenges, students construct three equilateral triangles, two of which share a common side. Allow students to investigate independently before offering guidance. As students attempt the task, ask them to reflect on the significance of the use of circles for the problem.

Classwork

Opening Exercise (5 minutes)

Students should test each other's instructions for the construction of an equilateral triangle. The goal is to identify errors in the instructions or opportunities to make the instructions more concise.

Opening Exercise

You will need a compass, a straightedge, and another student's Problem Set.

Directions:

Follow the directions from another student's Problem Set write-up to construct an equilateral triangle.

- What kinds of problems did you have as you followed your classmate's directions?
- Think about ways to avoid these problems. What criteria or expectations for writing steps in constructions should be included in a rubric for evaluating your writing? List at least three criteria.

Discussion (5 minutes)

MP.5

- What are common errors? What are concrete suggestions to help improve the instruction-writing process?
 - *Correct use of vocabulary, simple and concise steps (making sure each step involves just one instruction), and clear use of labels.*

It is important for students to describe objects using correct terminology instead of pronouns. Instead of “it” and “they,” perhaps “the center” and “the sides” should be used.

Exploratory Challenge 1 (15 minutes)

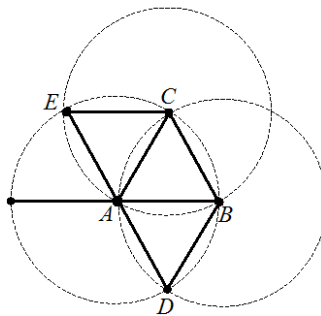
Exploratory Challenge 1

You will need a compass and a straightedge.

Using the skills you have practiced, construct three equilateral triangles, where the first and second triangles share a common side and the second and third triangles share a common side. Clearly and precisely list the steps needed to accomplish this construction.

Switch your list of steps with a partner, and complete the construction according to your partner’s steps. Revise your drawing and list of steps as needed.

Construct three equilateral triangles here:



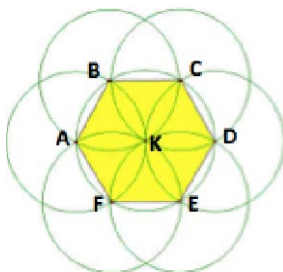
1. Draw a segment $\overline{AB}$.
2. Draw circle A: center A, radius $\overline{AB}$.
3. Draw circle B: center B, radius $\overline{BA}$.
4. Label one intersection as C; label the other intersection as D.
5. Draw circle C: center C, radius $\overline{CA}$.
6. Label the unlabeled intersection of circle C with circle A (or the unlabeled intersection of circle C with circle B) as E.
7. Draw all segments that are congruent to $\overline{AB}$ between the labeled points.

There are many ways to address Step 7; students should be careful to avoid making a blanket statement that would allow segment $\overline{BE}$ or $\overline{CD}$.

Exploratory Challenge 2 (16 minutes)

Exploratory Challenge 2

On a separate piece of paper, use the skills you have developed in this lesson to construct a regular hexagon. Clearly and precisely list the steps needed to accomplish this construction. Compare your results with a partner, and revise your drawing and list of steps as needed.



1. Draw circle K : center K , any radius.
2. Pick a point on the circle; label this point A .
3. Draw circle A : center A , radius AK .
4. Label the intersections of circle A with circle K as B and F .
5. Draw circle B : center B , radius BK .
6. Label the intersection of circle B with circle K as C .
7. Continue to treat the intersection of each new circle with circle K as the center of a new circle until the next circle to be drawn is circle A .
8. Draw $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EF}$, $\overline{FA}$.

Can you repeat the construction of a hexagon until the entire sheet is covered in hexagons (except the edges, which will be partial hexagons)?

Yes, this result resembles wallpaper, tile patterns, etc.

Exit Ticket (5 minutes)

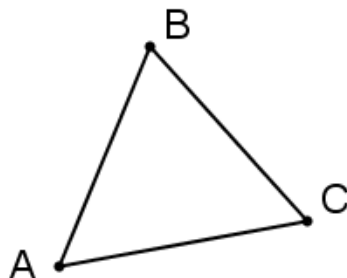
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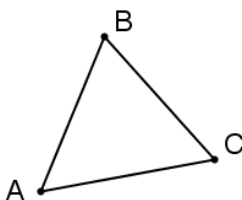
Exit Ticket

$\triangle ABC$ is shown below. Is it an equilateral triangle? Justify your response.



Exit Ticket Sample Solution

$\triangle ABC$ is shown below. Is it an equilateral triangle? Justify your response.



The triangle is not equilateral. Students may prove this by constructing two intersecting circles using any two vertices as the given starting segment. The third vertex will not be one of the two intersection points of the circles.

Problem Set Sample Solution

Why are *circles* so important to these constructions? Write out a concise explanation of the importance of circles in creating equilateral triangles. Why did Euclid use *circles* to create his equilateral triangles in Proposition 1? How does construction of a circle ensure that all relevant segments will be of equal length?

The radius of equal-sized circles, which must be used in construction of an equilateral triangle, does not change. This consistent length guarantees that all three side lengths of the triangle are equal.