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## Lesson 5: Identifying Proportional and Non-Proportional

## Relationships in Graphs

## Student Outcomes

- Students decide whether two quantities are proportional to each other by graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- Students study examples of quantities that are proportional to each other as well as those that are not.


## Classwork

## Opening Exercise (5 minutes)

Give students the ratio table and ask them to identify if the two quantities are proportional to each other and to give reasoning for their answers.

## Opening Exercise

Isaiah sold candy bars to help raise money for his scouting troop. The table shows the amount of candy he sold compared to the money he received.

| $x$ <br> Candy Bars Sold | $y$ <br> Money Received (\$) |
| :---: | :---: |
| 2 | 3 |
| 4 | 5 |
| 8 | 9 |
| 12 | 12 |

Is the amount of candy bars sold proportional to the money Isaiah received? How do you know?
The two quantities are not proportional to each other because a constant describing the proportion does not exist.

## Exploratory Challenge (9 minutes): From a Table to Graph

Prompt students to create another ratio table that contains two sets of quantities that are proportional to each other using the first ratio on the table.

Present a coordinate grid and ask students to recall standards from Grades 5 and 6 on the following: coordinate plane, $x$-axis, $y$-axis, origin, quadrants, plotting points, and ordered pairs.

As a class, ask students to express the ratio pairs as ordered pairs.
Questions to discuss:

- What is the origin and where is it located?
- The origin is the intersection of the $x$-axis and the $y$-axis, at the ordered pair $(0,0)$.
- Why are we going to focus on Quadrant 1?
- Since we are measuring or counting quantities (number of candy bars sold and amount of money), the numbers in our ratios will be positive. Both the $x$-coordinates and the $y$-coordinates are positive in Quadrant 1.
- What should we label the $x$-axis and $y$-axis?
- The $x$-axis should be labeled the number of candy bars sold, and the $y$-axis should be labeled the amount of money received.
- Could it be the other way around?
- No, the amount of money received depends on the number of candy bars being sold. The dependent variable should be labeled on the y-axis. Therefore, the amount of money should be labeled on the $y$ axis.
- How should we note that on the table?
- The first value in each of the pairs is the $x$-coordinate (the independent variable) and the second values in each of the pairs is the $y$-coordinate (the dependent variable).
- How do we plot the first ratio pair?
- If the relationship is 3: 2, where 3 represents 3 candy bars sold and 2 represents 2 dollars received, then from the origin, we move 3 units to the right on the $x$-axis and move up 2 units on the $y$-axis.
- When we are plotting a point, where do we count from?
- The origin, $(0,0)$

Have students plot the rest of the points and use a ruler to join the points.

- What observations can you make about the arrangement of the points?
- The points all fall on a line.
- Do we extend the line in both directions? Explain why or why not.
- Technically, the line for this situation should start at $(0,0)$ to represent 0 dollars for 0 candy bars and extend infinitely in the positive direction because the more candy bars Isaiah sells, the more money he makes.
- Would all proportional relationships pass through the origin? Think back to those discussed in previous lessons.

Take a few minutes for students to share some of the context of previous examples and whether ( 0,0 ) would always be included on the line that passes through the pairs of points in a proportional relationship.

- Yes, it should always be included for proportional relationships. For example, if a worker works zero hours, then he or she would get paid zero dollars, or if a person drives zero minutes, the distance covered is zero miles.
- What can you infer about graphs of two quantities that are proportional to each other?
- The points will appear to be on a line that goes through the origin.
- Why do you think the points appear on a line?
- Each candy bar is being sold for \$1.50; therefore, 1.5 is the unit rate and also the constant of the proportion. This means that for every increase of 1 on the $x$-axis, there will be an increase of the same proportion (the constant, 1.5) on the y-axis. When the points are connected, a line is formed. Each point may not be part of the set of ratios; however, the line would pass through all of the points that do exist in the set of ratios.

Complete "Important Note" with class. In a proportional relationship, the points will all appear on a line going through the origin.

Example 1: From a Table to Graph
Using the ratio provided, create a table that shows money received is proportional to the number of candy bars sold. Plot the points in your table on the grid.

| $x$ <br> Candy Bars Sold | $y$ <br> Money Received (\$) |
| :---: | :---: |
| 2 | 3 |
| 4 | 6 |
| 6 | 9 |
| 8 | 12 |



## Important Note:

Characteristics of graphs of proportional relationships:

1. Points appear on a line.
2. The line goes through the origin.

## Example 2 (8 minutes)

Have students plot ordered pairs for all the values of the Opening Exercise.

- Does the ratio table represent quantities that are proportional to each other?
- No, not all the quantities are proportional to each other.
- What can you predict about the graph of this ratio table?
- The points will not appear on a line and will not go through the origin.
- Was your prediction correct?
- My prediction was partly correct. The majority of the points appear on a line that goes through the origin.
- From this example, what is important to note about graphs of two quantities that are not proportional to each other?
- The graph could go through the origin, but if it does not lie in a straight line, it does not represent two quantities that are proportional to each other.


## Example 2

Graph the points from the Opening Exercise.

| $x$ <br> Candy Bars Sold | $y$ <br> Money Received (\$) |
| :---: | :---: |
| 2 | 3 |
| 4 | 5 |
| 8 | 9 |
| 12 | 12 |



## Example 3 (8 minutes)

Have students plot the points from Example 3.

- How are the graphs of the data in Examples 1 and 3 similar? How are they different?
- In both graphs, the points appear on a line. One graph is steeper than the other. The graph in Example 1 begins at the origin, but the graph in Example 3 does not.
- What do you know about the ratios before you graph them?
- The quantities are not proportional to each other.
- What can you predict about the graph of this ratio table?
- The points will not appear on a line that goes through the origin.
- Was your prediction correct?
- No. The graph forms a line, but the line does not go through the origin.
- What are the similarities of the graphs of two quantities that are proportional to each other and the graphs of two quantities that are not proportional?
- Both graphs can have points that appear on a line, but the graph of the quantities that are proportional to each other must also go through the origin.


## Example 3

Graph the points provided in the table below and describe the similarities and differences when comparing your graph to the graph in Example 1.

| $x$ | $y$ |
| :---: | :---: |
| 0 | 6 |
| 3 | 9 |
| 6 | 12 |
| 9 | 15 |
| 12 | 18 |



## Similarities with Example 1:

The points of both graphs fall in a line.

Differences from Example 1:
The points of graph in Example 1 appear on a line that passes through the origin. The points of the graph in Example 3 appear on a line that does not pass through the origin.

## Closing (5 minutes)

- How are proportional quantities represented on a graph?
- They are represented on a graph where the points appear on a line that passes through the origin.
- What is a common mistake that someone might make when deciding whether a graph of two quantities shows that they are proportional to each other?
- Both graphs can have points that appear on a line, but the graph of the quantities that are proportional to each other also goes through the origin. In addition, the graph could go through the origin, but the points do not appear on a line.


## Lesson Summary

When two proportional quantities are graphed on a coordinate plane, the points appear on a line that passes through the origin.

## Exit Ticket (10 minutes)

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## Lesson 5: Identifying Proportional and Non-Proportional

## Relationships in Graphs

## Exit Ticket

1. The following table gives the number of people picking strawberries in a field and the corresponding number of hours that those people worked picking strawberries. Graph the ordered pairs from the table. Does the graph represent two quantities that are proportional to each other? Explain why or why not.

| $x$ | $y$ |
| :---: | :---: |
| 1 | 3 |
| 7 | 1 |
| 4 | 2 |


2. Use the given values to complete the table. Create quantities proportional to each other and graph them.

| $x$ | $y$ |
| :---: | :---: |
|  |  |
| 4 | 2 |
|  |  |


3. a. What are the differences between the graphs in Problems 1 and 2?
b. What are the similarities in the graphs in Problems 1 and 2?
c. What makes one graph represent quantities that are proportional to each other and one graph not represent quantities that are proportional to each other in Problems 1 and 2?

## Exit Ticket Sample Solutions

1. The following table gives the number of people picking strawberries in a field and the corresponding number of hours that those people worked picking strawberries. Graph the ordered pairs from the table. Does the graph represent two quantities that are proportional to each other? Why or why not?

Although the points fall on a line, the line does not pass through the origin, so the graph does not represent two quantities that are proportional to each other.

| $x$ | $y$ |
| :---: | :---: |
| 1 | 3 |
| 7 | 1 |
| 4 | 2 |


2. Use the given values to complete the table. Create quantities proportional to each other and graph.

| $x$ | $y$ |
| :---: | :---: |
| 2 | 1 |
| 4 | 2 |
| 6 | 3 |


3. a. What are the differences between the graphs in Problems 1 and 2?

The graph in Problem 1 forms a line that slopes downward while the graph in Problem 2 slopes upward.
b. What are the similarities in the graphs in Problems 1 and 2?

Both graphs form lines, and both graphs include the point $(4,2)$.
c. What makes one graph represent quantities that are proportional to each other and one graph not represent quantities that are proportional to each other in Problems 1 and 2?

Although both graphs form lines, the graph that represents quantities that are proportional to each other needs to pass through the origin.

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## Problem Set Sample Solutions

1. Determine whether or not the following graphs represent two quantities that are proportional to each other. Explain your reasoning.
a.


This graph represents two quantities that are proportional to each other because the points appear on a line, and the line that passes through the points would also pass through the origin.
b.


Even though the points appear on a line, the line does not go through the origin. Therefore, this graph does not represent a proportional relationship.
c.

"Even though it goes through the origin, this graph does not show a proportional relationship because the points do not appear on one line.

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2. Create a table and a graph for the ratios $2: 22,3$ to 15 , and $1: 11$. Does the graph show that the two quantities are proportional to each other? Explain why or why not.

This graph does not because the points do not appear on a line that goes through the origin.

| $x$ | $y$ |
| :---: | :---: |
| 2 | 22 |
| 3 | 15 |
| 1 | 11 |


3. Graph the following tables and identify if the two quantities are proportional to each other on the graph. Explain why or why not.
a.

| $x$ | $y$ |
| :---: | :---: |
| 3 | 1 |
| 6 | 2 |
| 9 | 3 |
| 12 | 4 |



Yes, because the graph of the relationship is a straight line that passes through the origin
b.

| $x$ | $y$ |
| :---: | :---: |
| 1 | 4 |
| 2 | 5 |
| 3 | 6 |
| 4 | 7 |



No, because the graph does not pass through the origin.

