Lesson 8: Modeling a Context from a Verbal Description

Classwork

Example 1

Christine has $\$500$ to deposit in a savings account, and she is trying to decide between two banks. Bank A offers 10% annual interest compounded quarterly. Rather than compounding interest for smaller accounts, Bank B offers to add $\$15$ quarterly to any account with a balance of less than $\$1{,}000$ for every quarter, as long as there are no withdrawals. Christine has decided that she will neither withdraw, nor make a deposit for a number of years.

Develop a model that will help Christine decide which bank to use.

Example 2

Alex designed a new snowboard. He wants to market it and make a profit. The total initial cost for manufacturing set-up, advertising, etc. is $\$500{,}000$, and the materials to make the snowboards cost $\$100$ per board.

The demand function for selling a similar snowboard is $D(p) = 50{,}000 - 100p$, where $p =$ selling price of each snowboard.

a. Write an expression for each of the following. Let p represent the selling price:

Demand Function (number of units that will sell)

Revenue (number of units that will sell, price per unit, p)

Total Cost (cost for producing the snowboards)

b. Write an expression to represent the profit.

c. What is the selling price of the snowboard that will give the maximum profit?

d. What is the maximum profit Alex can make?

Exercises

Alvin just turned 16 years old. His grandmother told him that she will give him $\$10{,}000$ to buy any car he wants whenever he is ready. Alvin wants to be able to buy his dream car by his 21st birthday, and he wants a 2009 Avatar Z, which he could purchase today for $\$25{,}000$. The car depreciates (reduces in value) at a rate is 15% per year. He wants to figure out how long it would take for his $\$10{,}000$ to be enough to buy the car, without investing the $\$10{,}000$.

1. Write the function that models the depreciated value of the car after n number of years.

After n years	Value of the Car
1	
2	
3	
4	
5	
6	

a. Will he be able to afford to buy the car when he turns 21? Explain why or why not.

b. Given the same rate of depreciation, after how many years will the value of the car be less than $\$5{,}000$?

c. If the same rate of depreciation were to continue indefinitely, after how many years would the value of the car be approximately $\$1$?

2. Sophia plans to invest $\$1{,}000$ in each of three banks.

   Bank A offers an annual interest rate of 12%, compounded annually.

   Bank B offers an annual interest rate of 12%, compounded quarterly.

   Bank C offers an annual interest rate of 12%, compounded monthly.

   a. Write the function that describes the growth of investment for each bank in n years.

   b. How many years will it take to double her initial investment for each bank? (Round to the nearest whole dollar.)

Year	Bank A	Bank B	Bank C
Year 1			
Year 2			
Year 3			
Year 4			
Year 5			
Year 6			
Year 7			

   c. Sophia went to Bank D. The bank offers a "double your money" program for an initial investment of $\$1{,}000$ in five years, compounded annually. What is the annual interest rate for Bank D?

Lesson Summary

- We can use the full modeling cycle to solve real-world problems in the context of business and commerce (e.g., compound interest, revenue, profit, and cost) and population growth and decay (e.g., population growth, depreciation value, and half-life) to demonstrate linear, exponential and quadratic functions described verbally through using graphs, tables, or algebraic expressions to make appropriate interpretations and decisions.
- Sometimes a graph or table is the best model for problems that involve complicated function equations.

Problem Set

1. Maria invested $\$10{,}000$ in the stock market. Unfortunately, the value of her investment has been dropping at an average rate of 3% each year.
   a. Write the function that best models the situation.
   b. If the trend continues, how much will her investment be worth in 5 years?
   c. Given the situation, what should she do with her investment?

2. The half-life of the radioactive material in Z-Med, a medication used for certain types of therapy, is 2 days. A patient receives a 16 mCi dose (millicuries, a measure of radiation) in his treatment. (*Half-life* means that the radioactive material decays to the point where only half is left.)
   a. Make a table to show the level of Z-Med in the patient's body after n days.

Number of days	Level of Z-Med in patient
0	
2	
4	
6	
8	
10	

   b. Write an equation for $f(n)$ to model the half-life of Z-Med for n days. (Be careful here. Make sure that the formula works for both odd and even numbers of days.)
   c. How much radioactive material from Z-Med is left in the patient's body after 20 days of receiving the medicine?

3. Suppose a male and a female of a certain species of animal were taken to a deserted island. The population of this species quadruples (multiplies by 4) every year. Assume that the animals have an abundant food supply and that there are no predators on the island.

   a. What is an equation that can be used to model the population of the species?

   b. What will the population of the species be after 5 years?

After n years	Population
0	
1	
2	
3	
4	
5	

   c. Write an equation to find how many years it will take for the population of the animals to exceed 1 million. Find the number of years, either by using the equation or a table.

4. The revenue of a company for a given month is represented as $R(x) = 1{,}500x - x^2$ and its costs as $C(x) = 1{,}500 + 1{,}000x$. What is the selling price, x, of its product that would yield the maximum profit? Show or explain your answer.

After n years	Population
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	