

Lesson 2: Analyzing a Data Set

Classwork

Opening Exercise

When tables are used to model functions, we typically have just a few sample values of the function and therefore have to do some detective work to figure out what the function might be. Look at these three tables:

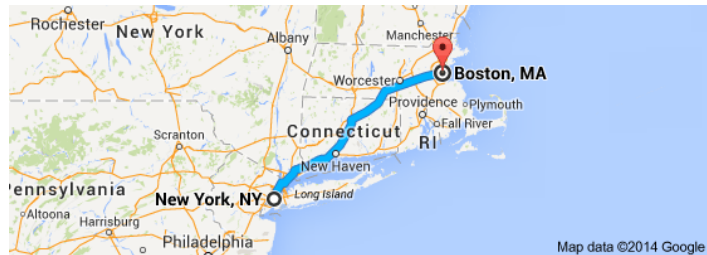
x	$f(x)$
0	6
1	12
2	18
3	24
4	30
5	36

x	$g(x)$
0	0
1	14
2	24
3	30
4	32
5	30

x	$h(x)$
0	1
1	3
2	9
3	27
4	81
5	243

Example 1

Noam and Athena had an argument about whether it would take longer to get from NYC to Boston and back by car or by train. To settle their differences, they made separate, non-stop round trips from NYC to Boston. On the trip, at the end of each hour, both recorded the number of miles they had traveled from their starting points in NYC. The tables below show their travel times, in hours, and the distances from their starting points, in miles. The first table shows Noam's travel time and distance from the starting point, and the second represents Athena's. Use both data sets to justify your answers to the questions below.



Time in Hours	Noam's Distance
0	0
1	55
2	110
3	165
4	220
5	165
6	110
7	55
8	0

Time in Hours	Athena's Distance
0	0
1	81
2	144
3	189
4	216
5	225
6	216
7	189
8	144
9	81
10	0

- Who do you think is driving, and who is riding the train? Explain your answer in the context of the problem.
- According to the data, how far apart are Boston and New York City? Explain mathematically.

- c. How long did it take each of them to make the round trip?
- d. According to their collected data, which method of travel was faster?
- e. What was the average rate of change for Athena for the interval from 3 to 4 hours? How might you explain that in the context of the problem?
- f. Noam believes a quadratic function can be used as a model for both data sets. Do you agree? Use and describe the key features of the functions represented by the data sets to support your answer.

Exercises

1. Explain why each function can or cannot be used to model the given data set.

a. $f(x) = 3x + 5$

b. $f(x) = -(x - 2)^2 + 9$

c. $f(x) = -x^2 + 4x - 5$

d. $f(x) = 3^x + 4$

e. $f(x) = (x - 2)^2 + 9$

f. $f(x) = -(x + 1)(x - 5)$

x	$f(x)$
0	5
1	8
2	9
3	8
4	5
5	0
6	-7

2. Match each table below to the function and the context, and explain how you made your decision.

A	B	C	D	E																																																												
<table><tr><th>x</th><th>y</th></tr><tr><td>1</td><td>9</td></tr><tr><td>2</td><td>18</td></tr><tr><td>3</td><td>27</td></tr><tr><td>4</td><td>18</td></tr><tr><td>5</td><td>9</td></tr></table>	x	y	1	9	2	18	3	27	4	18	5	9	<table><tr><th>x</th><th>y</th></tr><tr><td>1</td><td>12</td></tr><tr><td>2</td><td>24</td></tr><tr><td>3</td><td>36</td></tr><tr><td>4</td><td>48</td></tr><tr><td>5</td><td>60</td></tr></table>	x	y	1	12	2	24	3	36	4	48	5	60	<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>160</td></tr><tr><td>1</td><td>174</td></tr><tr><td>2</td><td>156</td></tr><tr><td>3</td><td>106</td></tr><tr><td>4</td><td>24</td></tr></table>	x	y	0	160	1	174	2	156	3	106	4	24	<table><tr><th>x</th><th>y</th></tr><tr><td>1</td><td>2</td></tr><tr><td>2</td><td>4</td></tr><tr><td>3</td><td>8</td></tr><tr><td>4</td><td>16</td></tr><tr><td>5</td><td>32</td></tr></table>	x	y	1	2	2	4	3	8	4	16	5	32	<table><tr><th>x</th><th>y</th></tr><tr><td>2</td><td>8</td></tr><tr><td>3</td><td>9</td></tr><tr><td>4</td><td>8</td></tr><tr><td>5</td><td>5</td></tr><tr><td>6</td><td>0</td></tr></table>	x	y	2	8	3	9	4	8	5	5	6	0
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Equations:

$$f(x) = 12x$$

$$h(x) = -9|x - 3| + 27$$

$$g(x) = -(x)(x - 6)$$

$$p(x) = 2^x$$

$$q(x) = -16x^2 + 30x + 160$$

Contexts:

1. The population of bacteria doubled every month, and the total population vs. time was recorded.
2. A ball was launched upward from the top of a building, and the vertical distance of the ball from the ground vs. time was recorded.
3. The height of a certain animal's vertical leap was recorded at regular time intervals of one second; the animal returned to ground level after six seconds.
4. Melvin saves the same amount of money every month. The total amount saved after each month was recorded.
5. Chris ran at a constant rate on a straight-line path and then returned at the same rate. His distance from his starting point was recorded at regular time intervals.

Lesson Summary

The following methods can be used to determine the appropriate model for a given data set as a linear, quadratic, or exponential function:

- If the first difference is constant, then the data set could be modeled by a linear function.
- If the second difference is constant, then the data set could be modeled by a quadratic function.
- If the subsequent y -values are multiplied by a constant, then the data set could be modeled by an exponential function.

Problem Set

- 1.
- a. Determine the function type that could be used to model the data set at the right and explain why.
 - b. Complete the data set using the special pattern of the function you described above.
 - c. If it exists, find the minimum or maximum value for the function model. If there is no minimum or maximum, explain why.

x	y
0	
1	10
2	0
3	-6
4	-8
5	
6	

- 2.
- a. Determine the function type that could be used to model the data set and explain why.
 - b. Complete the data set using the special pattern of the function you described above.
 - c. If it exists, find the minimum or maximum value for the function model. If there is no minimum or maximum, explain why.

x	y
-1	
0	
1	
2	16
3	64
4	256
5	1024

- 3.
- a. Determine the function type that could be used to model the data set and explain why.
 - b. Complete the data set using the special pattern of the function you described above.
 - c. If it exists, find the minimum or maximum value for the function model. If there is no minimum or maximum, explain why.

x	y
-1	
0	12
1	
2	24
3	
4	36
5	

4. Circle all the function types that could possibly be used to model a context if the given statement applies.

a. When x -values are at regular intervals, the first difference of y -values is not constant.

Linear Function

Quadratic Function

Exponential Function

Absolute Value Function

b. When x -values are at regular intervals, the second difference of y -values is not constant.

Linear Function

Quadratic Function

Exponential Function

Absolute Value Function

c. When x -values are at regular intervals, the quotient of any two consecutive y -values is a constant that is not equal to 0 or 1.

Linear Function

Quadratic Function

Exponential Function

Absolute Value Function

d. There may be up to two different x -values for $y = 0$.

Linear Function

Quadratic Function

Exponential Function

Absolute Value Function