

Lesson 12: Completing the Square

Classwork

Opening Exercise

Rewrite each expression by completing the square.

a. $z^2 - 5z + 8$

b. $x^2 + 0.6x + 1$

Example 1

Now complete the square for $2x^2 + 16x + 3$.

Example 2

Business Application Vocabulary

UNIT PRICE (PRICE PER UNIT): The price per item a business sets to sell its product, which is sometimes represented as a linear expression.

QUANTITY: The number of items sold, sometimes represented as a linear expression.

REVENUE: The total income based on sales (but without considering the cost of doing business).

UNIT COST (COST PER UNIT) OR PRODUCTION COST: The cost of producing one item, sometimes represented as a linear expression.

PROFIT: The amount of money a business makes on the sale of its product. Profit is determined by taking the total revenue (the quantity sold multiplied by the price per unit) and subtracting the total cost to produce the items (the quantity sold multiplied by the production cost per unit): $\text{Profit} = \text{Total Revenue} - \text{Total Production Costs}$.

The following business formulas will be used in this and the remaining lessons in the module:

Total Production Costs = (cost per unit)(quantity of items sold)

Total Revenue = (price per unit)(quantity of items sold)

Profit = Total Revenue – Total Production Costs

Now solve the following problem:

A certain business is marketing its product and has collected data on sales and prices for the past few years. The company determined that when it raised the selling price of the product, the number of sales went down. The cost of producing a single item is \$10.

- a. Using the data the company collected in this table, determine a linear expression to represent the quantity sold, q .

Selling Price (s)	Quantity Sold (q)
10	1,000
15	900
20	800
25	700

- b. Now find an expression to represent the profit function, P .

Exercises

For Exercises 1–5, rewrite each expression by completing the square.

1. $3x^2 + 12x - 8$

2. $4p^2 - 12p + 13$

3. $\frac{1}{2}y^2 + 3y - 4$

4. $1.2n^2 - 3n + 6.5$

5. $\frac{1}{3}v^2 - 4v + 10$

6. A fast food restaurant has determined that its price function is $3 - \frac{x}{20,000}$, where x represents the number of hamburgers sold.

a. The cost of producing x hamburgers is determined by the expression $5,000 + 0.56x$. Write an expression representing the profit for selling x hamburgers.

b. Complete the square for your expression in part (a) to determine the number of hamburgers that need to be sold to maximize the profit, given this price function.

Lesson Summary

Here is an example of completing the square of a quadratic expression of the form $ax^2 + bx + c$.

$$3x^2 - 18x - 2$$

$$3(x^2 - 6x) - 2$$

$$3(x^2 - 6x + 9) - 3(9) - 2$$

$$3(x - 3)^2 - 3(9) - 2$$

$$3(x - 3)^2 - 29$$

Problem Set

Rewrite each expression by completing the square.

1. $-2x^2 + 8x + 5$

2. $2.5k^2 - 7.5k + 1.25$

3. $\frac{4}{3}b^2 + 6b - 5$

4. $1,000c^2 - 1,250c + 695$

5. $8n^2 + 2n + 5$