Lesson 7: Creating and Solving Quadratic Equations in One Variable

Student Outcomes

- Students interpret word problems to create equations in one variable and solve them (i.e., determine the solution set) using factoring and the *zero product property*.

MP.2

Throughout this lesson, students are presented with a verbal description where a relationship can be modeled algebraically. Students must make sense of the quantities presented and decontextualize the verbal description to solve the basic one-variable quadratic equations. Then, they contextualize their solutions to interpret results and answer the questions posed by the examples.

Lesson Notes

Students wrote and solved algebraic expressions and equations based on verbal statements for linear and exponential equations in Modules 1 and 3. In this lesson, students apply the same ideas to solving quadratic equations by writing expressions to represent side lengths and solving equations when given the area of a rectangle or other polygons (**A-SSE.B.3a**). **A-CED.A.1** is central to the concepts of this lesson, as students create equations from a context and solve using the techniques they have developed in the early lessons of this module.

Classwork

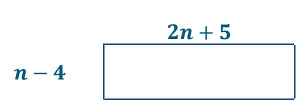
Opening Exercise (5 minutes)

Read the following prompt out loud and have students take notes. Then, have them work with a partner or small group to find the unknown number.

Opening Exercise

The length of a rectangle is 5 in. more than twice a number. The width is 4 in. less than the same number. The perimeter of the rectangle is 44 in. Sketch a diagram of this situation, and find the unknown number.

$$2l + 2w = P$$
$$2(2n+5) + 2(n-4) = 44$$
$$6n + 2 = 44$$
$$n = 7$$



Example 1 (5 minutes)

Review the Opening Exercise with students. Then, ask students: What if, instead of the perimeter, we knew the area?

> **Example 1**
>
> **The length of a rectangle is 5 in. more than twice a number. The width is 4 in. less than the same number. If the area of the rectangle is 15 in^2, find the unknown number.**
>
> $$lw = A$$
> $$(2n+5)(n-4) = 15$$
> $$2n^2 - 3n - 20 = 15$$
> $$2n^2 - 3n - 35 = 0$$
> $$(2n+7)(n-5) = 0$$
> $$n = 5 \text{ or } -\frac{7}{2}.$$
>
> *For this context (area), only positive values make sense. So, only $n = 5$ is possible.*

Give students a few minutes to find a solution. Then, either ask a student to demonstrate the solution on the board or present it to the class yourself.

Example 2 (5 minutes)

Another way to relate expressions to the area of a rectangle is through proportion. Show students the following example:

> **Example 2**
>
> **A picture has a height that is $\frac{4}{3}$ its width. It is to be enlarged so that the ratio of height to width remains the same, but the area is 192 in^2. What are the dimensions of the enlargement?**
>
> *Let $4x$ to $3x$ represent the ratio of height to width. $A = (h)(w)$, so we have*
>
> $$(4x)(3x) = 192$$
> $$12x^2 = 192$$
> $$x = 4 \text{ or } -4,$$
>
> *which means that $h = 16$ and $w = 12$ because only positive values make sense in the context of area. Therefore, the dimensions of the enlargement are 16 inches and 12 inches.*

Give students a few minutes to find a solution. Then, either have a student demonstrate the solution on the board or present it to the class yourself.

Exercises 1–6 (20 minutes)

The exercises below are scaffolded, beginning with the most basic that follow directly from those above, and culminating with exercises that require deeper reasoning and interpretation from students. If time does not permit assigning all of these exercises in class, you should select the number and level of difficulty that fit the needs of your students.

Exercise 1–6

Solve the following problems. Be sure to indicate if a solution is to be rejected based on the contextual situation.

1. **The length of a rectangle is 4 cm more than 3 times its width. If the area of the rectangle is $15\ \text{cm}^2$, find the width.**

$$(4+3w)(w)=15$$
$$3w^2+4w-15=0$$
$$(w+3)(3w-5)=0$$
$$w=\frac{5}{3}\ or\ -3$$

However, in this context only the positive value makes sense. Therefore, the width of the rectangle is $\frac{5}{3}$ cm.

2. **The ratio of length to width in a rectangle is $2:3$. Find the length of the rectangle when the area is $150\ \text{in}^2$.**

$$(2x)(3x)=150$$
$$6x^2-150=0$$
$$6(x^2-25)=0$$
$$6(x+5)(x-5)=0$$
$$x=5\ or\ -5$$

In this context, only positive values make sense, which means $x=5$. Therefore, the length of the rectangle is 10 inches.

3. **One base of a trapezoid is 4 in. more than twice the length of the second base. The height of the trapezoid is 2 in. less than the second base. If the area of the trapezoid is $4\ \text{in}^2$, find the dimensions of the trapezoid.**
(Note: The area of a trapezoid is $A=\frac{1}{2}(b_1+b_2)h$.)

$$A=\frac{1}{2(b_1+b_2)h}$$
$$4=\frac{1}{2}(2b_2+4+b_2)(b_2-2)$$
$$4=\left(\frac{3}{2}b_2+2\right)(b_2-2)$$
$$\frac{3}{2}{b_2}^2-b_2-8=0$$
$$\left(\frac{3}{2}b_2-4\right)(b_2+2)=0$$
$$b_2=\frac{8}{3}\ or\ -2.$$

However, only positive values make sense in this context, so b_1 is $\frac{28}{3}$ in., b_2 is $\frac{8}{3}$ in., and h is $\frac{2}{3}$ in.

4. **A garden measuring 12 m by 16 m is to have a pedestrian pathway that is w meters wide installed all the way around it, increasing the total area to 285 m^2. What is the width, w, of the pathway?**

$$(12+2w)(16+2w)=285$$
$$4w^2+56w-93=0$$
$$(2w+31)(2w-3)=0$$
$$w=\frac{3}{2} \text{ or } -\frac{31}{2}$$

However, only the positive value makes sense in this context, so the width of the pathway is $\frac{3}{2}$ m.

5. **Karen wants to plant a garden and surround it with decorative stones. She has enough stones to enclose a rectangular garden with a perimeter of 68 ft., and she wants the garden to cover 240 ft^2. What is the length and width of her garden?**

$$68=2l+2w$$
$$w=34-l$$
$$240=(l)(34-l)$$
$$l^2-34l+240=0$$
$$(l-10)(l-24)=0$$
$$l=10 \text{ or } 24$$

Important to notice here is that both solutions are positive and could represent the length. Because length and width are arbitrary distinctions here, the garden measures 24 ft. $\times$ 10 ft., with either quantity representing the width and the other representing the length.

For Exercise 6, a discussion on how to identify algebraically an unknown odd number may be necessary. If the students have not worked on problems using consecutive integers, this one might be tricky for some. It is likely that they can come up with at least one set of numbers to fit the description without using algebra. If time permits, let them explore the possibilities for an algebraic method of solving this problem. A discussion of the general expressions used to represent odd integers will be important for all students.

Have students read Exercise 6 and then ask the following:

- How will you name the odd integers?
  - *If we call the first one n, then the next one would be $n+2$. Or, if we call the first one $2n-1$, the next one would be $2n+1$.*

6. **Find two consecutive odd integers whose product is 99. (Note: There are two different pairs of consecutive odd integers and only an algebraic solution will be accepted.)**

Let n represent the first odd integer and $n+2$ represent the subsequent odd integer. The product is $n(n+2)$, which must equal 99.

$$n^2 + 2n - 99 = 0$$
$$(n-9)(n+11) = 0$$
$$n = 9 \text{ or } n = -11$$

If $n = 9$, then $n + 2 = 11$, so the numbers could be 9 and 11. Or if $n = -11$, then $n + 2 = -9$, so the numbers could be -11 and -9.

OR

Let $2n - 1$ represent the first odd integer and $2n + 1$ represent the subsequent odd integer. The product is $4n^2 - 1$, which must equal 99.

$$4n^2 - 1 = 99$$
$$4n^2 = 100$$
$$n^2 = 25$$
$$n = \pm 5$$

The two consecutive pairs of integers would be

$2(5) - 1 = 9; 2(5) + 1 = 11$

AND

$2(-5) - 1 = -11; 2(5) + 1 = -9.$

7. **Challenge: You have a 500-foot roll of chain link fencing and a large field. You want to fence in a rectangular playground area. What are the dimensions of the largest such playground area you can enclose? What is the area of the playground?**

$2w + 2l = 500$, so $w + l = 250$, and $l = 250 - w$. $A = (l)(w)$, so $(250 - w)(w) = 0$ gives us roots at $w = 0$ and $w = 250$. This means the vertex of the equation is $w = 125 \rightarrow l = 250 - 125 = 125$. The area of the playground will be $125 \times 125 = 15,625 \text{ ft}^2$.

Scaffolding:

For students who enjoy a challenge, let them try Exercise 7 as a preview of coming attractions. They may use tables or graphs to find the solution. These concepts will be addressed in a later lesson.

Closing (5 minutes)

Choose two exercises from this lesson that students struggled the most with and review them quickly on the board. Demonstrate the best strategies for solving.

Lesson Summary

When provided with a verbal description of a problem, represent the scenario algebraically. Start by identifying the unknown quantities in the problem and assigning variables. For example, write expressions that represent the length and width of an object.

Solve the equation using techniques previously learned, such as factoring and using the zero product property. The final answer should be clearly stated and should be reasonable in terms of the context of the problem.

Exit Ticket (5 minutes)

Name ____ Date____________________

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Exit Ticket

1. The perimeter of a rectangle is 54 cm. If the length is 2 cm more than a number, and the width is 5 cm less than twice the same number, what is the number?

2. A plot of land for sale has a width of x ft. and a length that is 8 ft. less than its width. A farmer will only purchase the land if it measures $240\ \text{ft}^2$. What value for x will cause the farmer to purchase the land?

Exit Ticket Sample Solutions

1. **The perimeter of a rectangle is 54 cm. If the length is 2 cm more than a number, and the width is 5 cm less than twice the same number, what is the number?**

$$2l + 2w = P$$
$$2(n+2) + 2(2n-5) = 54$$
$$6n - 6 = 54$$
$$n = 10$$

2. **A plot of land for sale has a width of x ft. and a length that is 8 ft. less than its width. A farmer will only purchase the land if it measures 240 ft^2. What value for x will cause the farmer to purchase the land?**

$$(x)(x-8) = 240$$
$$x^2 - 8x - 240 = 0$$
$$(x-20)(x+12) = 0$$
$$x = 20 \text{ or } x = -12$$

Since the answer cannot be negative, the answer is $x = 20$. The farmer will purchase the land if the width is 20 ft.

Problem Set Sample Solutions

Solve the following problems.

1. **The length of a rectangle is 2 cm less than its width. If the area of the rectangle is 35 cm^2, find the width.**

$$(w-2)(w) = 35$$
$$w^2 - 2w - 35 = 0$$
$$(w+5)(w-7) = 0$$
$$w = 7 \text{ or } -5$$

However, since the measurement can only be positive, the width is 7 cm.

2. **The ratio of length to width (measured in inches) in a rectangle is $4:7$. Find the length of the rectangle if the area is known to be 700 in^2.**

$$(4x)(7x) = 700$$
$$28x^2 - 700 = 0$$
$$28(x^2 - 25) = 0$$
$$28(x+5)(x-5) = 0$$
$$x = 5 \text{ or } -5$$

However, the measure can only be positive, which means $x = 5$, and the length is 20 inches.

3. **One base of a trapezoid is three times the length of the second base. The height of the trapezoid is 2 in. smaller than the second base. If the area of the trapezoid is 30 in^2, find the lengths of the bases and the height of the trapezoid.**

$$A = \frac{1}{2}(b_1 + b_2)h$$
$$30 = \frac{1}{2}(3b + b)(b - 2)$$
$$30 = (2b)(b - 2)$$
$$2b^2 - 4b - 30 = 0$$
$$(2b - 10)(b + 3) = 0$$
$$b_2 = 5 \text{ or } -3$$

However, only the positive value makes sense, so $b_1 = 15$ in., $b_2 = 5$ in., and $h = 3$ in.

4. **A student is painting an accent wall in his room where the length of the wall is 3 ft. more than its width. The wall has an area of 130 ft^2. What are the length and the width, in feet?**

$$(w + 3)(w) = 130$$
$$w^2 + 3w - 130 = 0$$
$$(w + 13)(w - 10) = 0$$
$$w = 10 \text{ or } -13$$

However, since the measure must be positive, the width is 10 feet, and the length is 13 ft.

5. **Find two consecutive even integers whose product is 80. (There are two pairs, and only an algebraic solution will be accepted.)**

$$(w)(w + 2) = 80$$
$$w^2 + 2w - 80 = 0$$
$$(w + 10)(w - 8) = 0$$
$$w = 8 \text{ or } -10$$

So, the consecutive even integers are 8 and 10 or -10 and -8.